Due Monday 10/29/18 at 11:00am Name ______________________________

Homework Chapter 28: Magnetic Fields

28.03 An electron that has an instantaneous velocity of $\vec{\mathbf{v}} = (2.0\times10^6\ \text{m/s})\hat{\mathbf{i}} + (3.0\times10^6\ \text{m/s})\hat{\mathbf{j}}$ is moving through the uniform magnetic field $\vec{\mathbf{B}} = (0.030\ \text{T})\hat{\mathbf{i}} - (0.15\ \text{T})\hat{\mathbf{j}}$ (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.

3. (a) The force on the electron is

$$\vec{F}_B = q\vec{v}\times\vec{B} = q(v_x\hat{\text{i}} + v_y\hat{\text{j}})\times(B_x\hat{\text{i}} + B_y\hat{j}) = q(v_xB_y - v_yB_x)\hat{\text{k}}$$
$$= (-1.6\times10^{-19}\text{C})\left[(2.0\times10^6\ \text{m/s})(-0.15\ \text{T}) - (3.0\times10^6\ \text{m/s})(0.030\ \text{T})\right]$$
$$= (6.2\times10^{-14}\ \text{N})\hat{\text{k}}.$$

should be 6.2×10^{-14} N

Thus, the magnitude of $\vec{F}_B$ is 6.2×10^{14} N, and $\vec{F}_B$ points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -(6.2\times10^{-14}\ \text{N})\hat{\text{k}}$.

28.05 An electron moves through a uniform magnetic field given by $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + (3.0B_x)\hat{\mathbf{j}}$. At a particular instant, the electron has velocity $\vec{\mathbf{v}} = (2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ m/s and the magnetic force acting on it is $(6.4\times10^{-19}\ \text{N})\hat{\mathbf{k}}$. Find B_x.

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q(v_xB_y - v_yB_x)\hat{\text{k}} = q(v_x(3B_x) - v_yB_x)\hat{\text{k}}$$

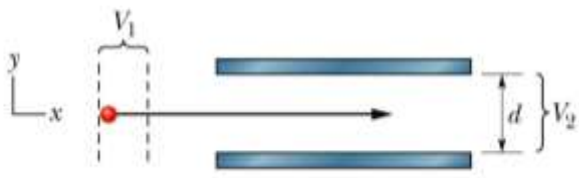
where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\hat{\text{k}}$ where $F_z = 6.4\times10^{-19}$ N, then we are led to the condition

$$q(3v_x - v_y)B_x = F_z \quad\Rightarrow\quad B_x = \frac{F_z}{q(3v_x - v_y)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6\times10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times10^{-19}\ \text{N}}{(-1.6\times10^{-19}\text{C})[3(2.0\ \text{m/s}) - 4.0\ \text{m}]} = -2.0\ \text{T}.$$

28.09 In Fig. 28-32, an electron accelerated from rest through potential difference V_1 = 1.00 kV enters the gap between two parallel plates having separation d = 20.0 mm and potential difference V_2 = 100 V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?



9. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$. Note that $\vec{v} \perp \vec{B}$ so $\left|\vec{v} \times \vec{B}\right| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100\ \text{V}/(20\times10^{-3}\,\text{m}) = E = 5000\ \text{V/m}}{\underbrace{\sqrt{2\left(1.0\times10^{3}\,\text{V}\right)\left(1.60\times10^{-19}\,\text{C}\right)/\left(9.11\times10^{-31}\,\text{kg}\right)}}_{v\,=\,1.874\times10^{7}\ \text{m/s}}} = 2.67\times10^{-4}\,\text{T}.$$

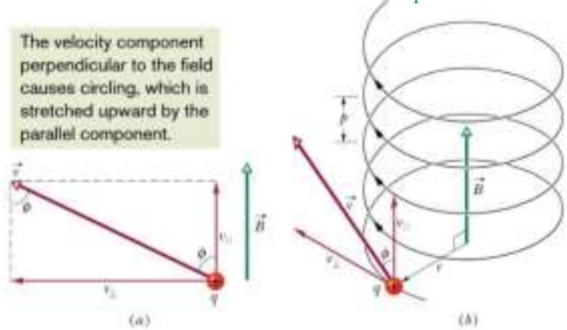
In unit-vector notation, $\vec{B} = -(2.67\times10^{-4}\ \text{T})\hat{\text{k}}$.

28.23 What uniform magnetic field, applied perpendicular to a beam of electrons moving at 1.30×10^6 m/s, is required to make the electrons travel in a circular arc of radius 0.350 m?

23. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.30\times10^{6}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.350\ \text{m}\right)} = 2.11\times10^{-5}\ \text{T}.$$

28.29 An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T. The pitch of the path is 6.00 μm, and the magnitude of the magnetic force on the electron is 2.00×10^{-15} N. What is the electron's speed?



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29. Reference to Fig. 28-11 is very useful for interpreting this problem. The distance traveled parallel to $\vec{B}$ is $d_{\parallel} = v_{\parallel}T = v_{\parallel}(2\pi m_e/|q|B)$ using Eq. 28-17. Thus,

$$v_{\parallel} = \frac{d_{\parallel}eB}{2\pi m_e} = 50.3 \text{ km/s}$$

using the values given in this problem. Also, since the magnetic force is $|q|Bv_{\perp}$, then we find $v_{\perp} = 41.7$ km/s. The speed is therefore $v = \sqrt{v_{\perp}^2 + v_{\parallel}^2} = 65.3$ km/s.

28.39 A horizontal power line carries a current of 5000 A from south to north. Earth's magnetic field (60.0 μT) is directed toward the north and inclined downward at 70.0° to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earth's field.

39. **THINK** The magnetic force on a wire that carries a current i is given by $\vec{F}_B = i\vec{L} \times \vec{B}$, where $\vec{L}$ is the length vector of the wire and $\vec{B}$ is the magnetic field.

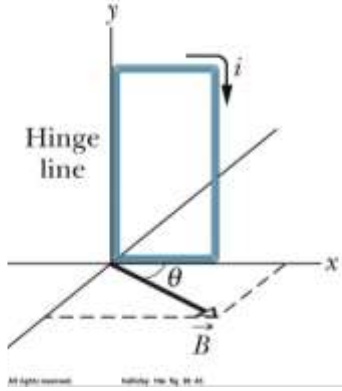
EXPRESS The magnitude of the magnetic force on the wire is given by $F_B = iLB \sin\phi$, where ϕ is the angle between the current and the field.

ANALYZE (a) With $\phi = 70°$, we have

$$F_B = (5000\,\text{A})(100\,\text{m})(60.0\times10^{-6}\,\text{T})\sin 70° = 28.2\,\text{N}.$$

(b) We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L} \times \vec{B}$ to show that the force is to the west.

LEARN From the expression $\vec{F}_B = i\vec{L} \times \vec{B}$, we see that the magnetic force acting on a current-carrying wire is a maximum when $\vec{L}$ is perpendicular to $\vec{B}$ $(\phi = 90°)$, and is zero when $\vec{L}$ is parallel to $\vec{B}$ $(\phi = 0°)$.



28.49 Figure 28-45 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm. It carries a current of 0.10 A and is hinged along one long side. It is mounted in the *xy* plane, at angle $\theta = 30°$ to the direction of a uniform magnetic field of magnitude 0.50 T. In unit-vector notation, what is the torque acting on the coil about the hinge line?

49. **THINK** Magnetic forces on the loop produce a torque that rotates it about the hinge line. Our applied field has two components: $B_x > 0$ and $B_z > 0$.

EXPRESS Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of $\vec{B}$ which is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the *y* axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the *y* axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Fig. 28-45 carrying current in the $-y$ direction.

Now, the B_z component will produce a force on this straight segment which points in the $-x$ direction (back toward the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $F = NiLB_x$ which points (by the right-hand rule) in the $+z$ direction.

ANALYZE Since the action of the force *F* is perpendicular to the plane of the coil, and is located a distance *x* away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)(x) = NiLxB\cos\theta = (20)(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30° = 0.0043\text{ N}\cdot\text{m}.$$

Since $\vec{\tau} = \vec{r}\times\vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times10^{-3}\text{ N}\cdot\text{m})\hat{j}$

LEARN An alternative way to do this problem is through the use of Eq. 28-37: $\vec{\tau} = \vec{\mu}\times\vec{B}$. The magnetic moment vector is

$$\vec{\mu} = -(NiA)\,\hat{k} = -(20)(0.10\,\text{A})(0.0050\,\text{m}^2)\hat{k} = -(0.01\,\text{A}\cdot\text{m}^2)\hat{k}.$$

The torque on the loop is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = (-\mu\,\hat{k})\times(B\cos\theta\,\hat{i} + B\sin\theta\,\hat{k}) = -(\mu B\cos\theta)\hat{j}\\ &= -(0.01\,\text{A}\cdot\text{m}^2)(0.50\text{ T})\cos 30°\hat{j}\\ &= (-4.3\times10^{-3}\text{ N}\cdot\text{m})\hat{j}.\end{aligned}$$

28.58 The magnetic dipole moment of Earth has magnitude 8.00×10^{22} J/T. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500 km, calculate the current they produce.

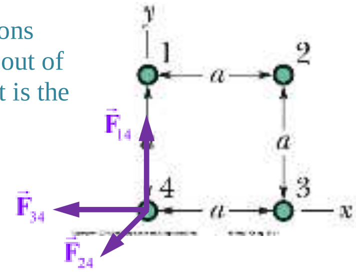
SKIP THIS QUESTION

58. From $\mu = NiA = i\pi r^2$ we get

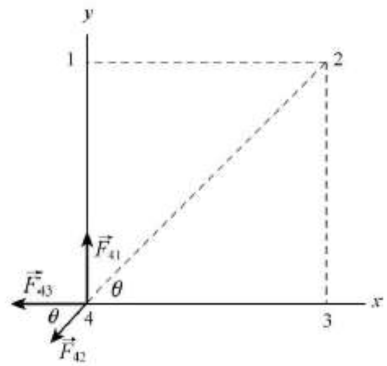
$$i = \frac{\mu}{\pi r^2} = \frac{8.00\times10^{22}\text{ J/T}}{\pi(3500\times10^3\text{ m})^2} = 2.08\times10^9\text{ A}.$$

Homework Chapter 29: Magnetic Fields Due to Currents

29.37 In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 13.5 cm. Each wire carries 7.50 A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 4?



37. We use Eq. 29-13 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown on the right.



The components of $\vec{F}_4$ are given by

$$F_{4x} = -F_{43} - F_{42}\cos\theta = -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} = -\frac{3\mu_0 i^2}{4\pi a}$$

and

$$F_{4y} = F_{41} - F_{42}\sin\theta = \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} = \frac{\mu_0 i^2}{4\pi a}.$$

Thus,

$$F_4 = \left(F_{4x}^2 + F_{4y}^2\right)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} = \frac{\sqrt{10}\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(7.50\text{A})^2}{4\pi(0.135\text{m})}$$
$$= 1.32\times10^{-4}\,\text{N/m}$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

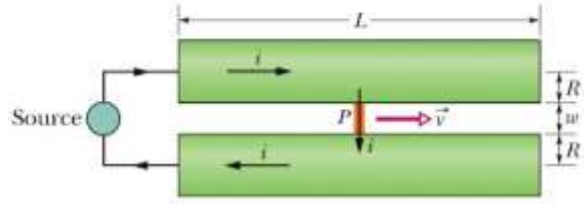
$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162°.$$

In unit-vector notation, we have

$$\vec{F}_1 = (1.32\times10^{-4}\,\text{N/m})[\cos162°\hat{\text{i}} + \sin162°\hat{\text{j}}] = (-1.25\times10^{-4}\,\text{N/m})\hat{\text{i}} + (4.17\times10^{-5}\,\text{N/m})\hat{\text{j}}$$

29.88 Figure 29-89 is an idealized schematic drawing of a rail gun. Projectile P sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let w be the distance between the rails, R the radius of each rail, and i the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$F = \frac{i^2\mu_0}{2\pi}\ln\frac{w+R}{R}$$



(b) If the projectile starts from the left end of the rails at rest, find the speed v at which it is expelled at the right. Assume that $i = 450$ kA, $w = 12$ mm, $R = 6.7$ cm, $L = 4.0$ m, and the projectile mass is 10 g..

88. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 29-12 to find the magnetic force on the segment, and Eq. 29-7 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\mathrm{i}}$ direction, and the current in rail 2 is in the $-\hat{\mathrm{i}}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{\mathrm{k}}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\mathrm{j}}$ direction) is given below. The coordinate origin is at the ~~bottom of the projectile.~~ center of bottom rail (wire 2).

$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 = i\,dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_1 + dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_2 = i\left[B_1 + B_2\right]\hat{\mathrm{i}}\,dy$$

$$= i\left[\frac{\mu_0 i}{4\pi(2R+w-y)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\mathrm{i}}\,dy.$$

Skipped integration steps:

$$= \frac{\mu_0 i^2}{4\pi}\left[-\ln(2R+w-y)+\ln y\right]_R^{R+w} = \frac{\mu_0 i^2}{4\pi}\left[\ln\frac{y}{2R+w-y}\right]_R^{R+w}$$

$$= \frac{\mu_0 i^2}{4\pi}\left[\ln\frac{R+w}{2R+w-(R+w)} - \ln\frac{R}{2R+w-R}\right]$$

$$= \frac{\mu_0 i^2}{4\pi}\left[\ln\frac{R+w}{R} - \ln\frac{R}{R+w}\right] \text{ but } \ln x^{-1} = -\ln x \text{ so}$$

$$= \frac{\mu_0 i^2}{4\pi}\left[\ln\frac{R+w}{R} + \ln\frac{R+w}{R}\right] = \frac{\mu_0 i^2}{2\pi}\ln\frac{R+w}{R}$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R+w-y} + \frac{1}{y}\right)dy\,\hat{\mathrm{i}} = \frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)\hat{\mathrm{i}}.$$

(b) Using the work-energy theorem, we have

$$\Delta K = \tfrac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL.$$

Thus, the final speed of the projectile is

$$v_f = \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)L\right]^{1/2}$$

$$= \left[\frac{2(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(450\times10^3\,\mathrm{A})^2\ln(1+1.2\,\mathrm{cm}/6.7\,\mathrm{cm})(4.0\,\mathrm{m})}{2\pi(10\times10^{-3}\,\mathrm{kg})}\right]^{1/2}$$

$$= 2.3\times10^3\,\mathrm{m/s}.$$ = MACH 6.74!