



DEPARTMENT OF
STATISTICS

The Analysis of Recurrent Events: A Summary of Methodology

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Motivation

- Conventional analyses

- Examples

- Problems

Setting

- Recurrent Events

- Examples

Objectives

- Scientific Questions

Existing Models for Recurrent Events

- Mean Cumulative Function

- Time-to-Event

- Event rates

- Application

Considerations

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Considerations

Composite endpoints

What are composite endpoints?

Standard approach in many cardiovascular trials

- ▶ Include two or more types of related clinical events
- ▶ Increase event rate and avoid multiplicity
- ▶ Analysis focussed on time to first event

- ▶ Examples in cardiovascular trials:
 - CV death, MI and stroke in hypertension trials
 - CV death and HF hospitalisation in heart failure trials

EMPHASIS-HF

Zanad F *et al*, NEJM 2011;364:11-21

- ▶ Eplerenone vs. placebo in 2737 patients with mild HF
- ▶ NYHA class II
- ▶ Ejection fraction $\leq 35\%$
- ▶ Tested hypothesis that eplerenone would reduce the risk of death and the risk of hospitalisation

- ▶ **Primary outcome:** composite of death from cardiovascular disease or hospitalisation for heart failure
- ▶ Analysed as time to first event using **Cox proportional-hazards model**

Cox proportional-hazards model

Background

- ▶ Most commonly used regression model in survival analysis
- ▶ Hazard function: describes conditional probability of an event occurring at time t , given that the event has not yet occurred
 - Instantaneous risk/intensity
 - $$h(t) = \lim_{dt \rightarrow 0} \left\{ \frac{P(t \leq T < t+dt | T \geq t)}{dt} \right\}$$
- ▶ Models based on the hazard function can assess whether covariates have an effect on the hazard

Cox proportional-hazards model

Analysis strategy

In heart failure, analysis of composite endpoints proceeds in a standard manner:

Cox proportional-hazards model

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- ▶ Exploratory analysis using Kaplan-Meier
 - $t_{(1)} < t_{(2)} < t_{(3)} < \dots$: ordered event times
 - m_j : number at risk just before time $t_{(j)}$
 - d_j : number with event at time $t_{(j)}$
 - $\hat{S}(t) = \prod_{j=1}^k \left(\frac{m_j - d_j}{m_j} \right)$, $t_{(k)} \leq t < t_{(k+1)}$

Cox proportional-hazards model

Analysis strategy

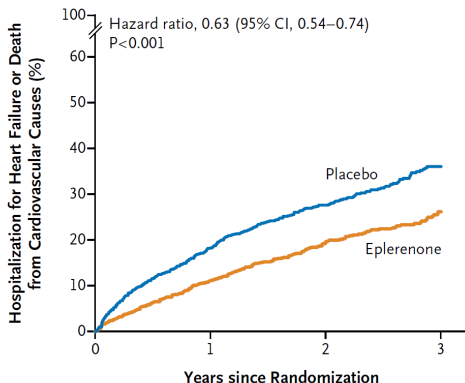
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 - $\hat{S}(t) = \prod_{j=1}^k \left(\frac{m_j - d_j}{m_j} \right)$, $t_{(k)} \leq t < t_{(k+1)}$

- ▶ Estimation using Cox proportional-hazards model
 - $h_i(t) = \exp\{\beta z_i\} h_0(t)$

EMPHASIS-HF

Zanad F *et al*, NEJM 2011;364:11-21



No. at Risk

Placebo	1373	848	512	199
Eplerenone	1364	925	562	232

CHARM-Preserved

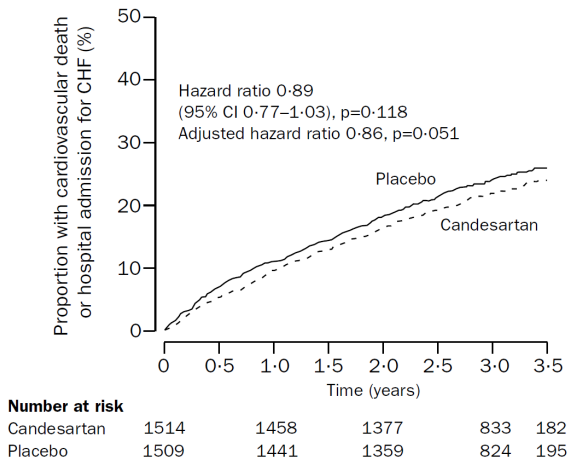
Yusuf S *et al*, The Lancet 2003;362:777-781

- ▶ CHARM: three parallel, independent trials
- ▶ Candesartan vs. placebo in 3021 patients with symptomatic heart failure
- ▶ CHARM-Preserved: preserved ejection fraction $\geq 40\%$

- ▶ Primary outcomes
 - Overall programme: all-cause mortality
 - Component trials: composite of death from cardiovascular disease or hospitalisation for heart failure
- ▶ Analysed as time to first event using Cox proportional-hazards model

CHARM-Preserved

Yusuf S *et al*, The Lancet 2003;362:777-781



Problems

What is wrong with composite endpoints?

Only first occurring endpoint is analysed

Furthermore...

- ▶ HF not characterised by a single event
- ▶ Chronic diseases characterised by recurrent events
- ▶ Repeat, non fatal events ignored

EMPHASIS-HF

Median follow-up: 25 months

HF Hospitalisations	Eplerenone (N=1364)	Placebo (N=1373)
≥ 1 admissions	186	277
≥ 2 admissions	67	110
All admissions	312	481
'Unused' admissions	126	204

CHARM-Preserved

Median follow-up: 37 months

HF Hospitalisations	Candesartan (N=1513)	Placebo (N=1508)
≥ 1 admissions	230	278
≥ 2 admissions	95	114
All admissions	392	547
'Unused' admissions	162	269

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Recurrent events

What are recurrent events?

Recurrent events involve repeat occurrences of the same type of event over time

Recurrent events

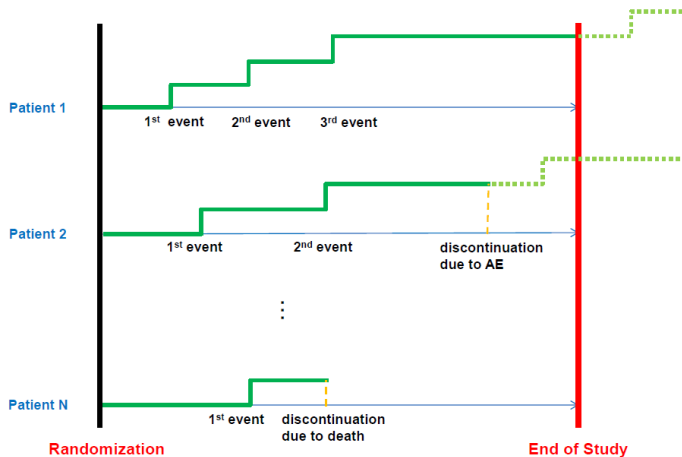
What are recurrent events?

Recurrent events involve repeat occurrences of the same type of event over time

Examples include:

- ▶ Heart failure hospitalisations in CV studies
- ▶ Exacerbations in COPD trials
- ▶ Seizures in epilepsy trials
- ▶ Asthma attacks in asthma trials

Patient profiles

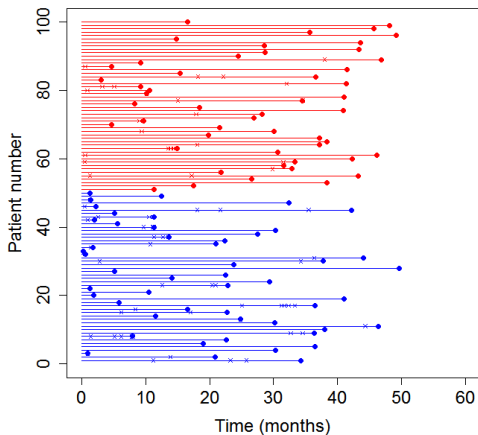


- ▶ We will consider indications where recurrent events are clinically meaningful
 - Treatment expected to impact first event
 - Treatment also expected to impact subsequent events
- ▶ Limit to case where censoring is non-informative
- ▶ We shall be focussing more on analysis methods, rather than design aspects

- ▶ Events are instantaneous, i.e. they have no duration
- ▶ Events do not affect trial conduct, e.g. no treatment switching after an event

EMPHASIS-HF

Patient profiles



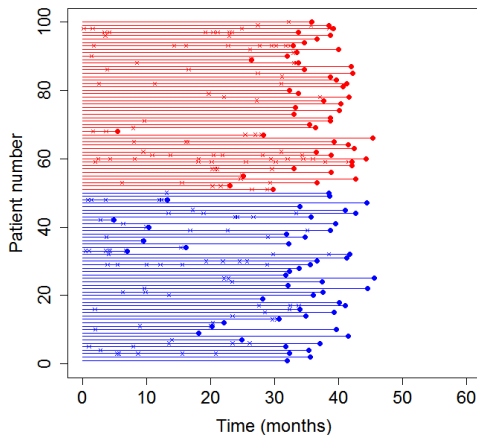
EMPHASIS-HF

Hospitalisation counts

	Eplerenone (N=1364)	Placebo (N=1373)
Follow-up years	2916.07	2830.91
Deaths	205	253
CV deaths	178	215
HF Hospitalisations:		
1	119	167
2	41	60
3	13	24
4	6	12
5	2	10
6	1	4
7	2	0
8	1	0
10	1	0
All admissions	312	481

CHARM-Preserved

Patient profiles



CHARM-Preserved

Hospitalisation counts

	Candesartan (N=1514)	Placebo (N=1509)
Follow-up years	4424.62	4374.03
Deaths	244	237
CV deaths	170	170
HF Hospitalisations:		
1	135	164
2	56	55
3	23	25
4	9	13
5	4	9
6	1	4
7	2	2
8	0	2
≥ 9	0	4
All admissions	392	547

Similarities

Heart failure clinical trials

- ▶ Repeated hospitalisations are an indicator for worsening condition
- ▶ Relatively long follow-up
- ▶ Staggered study entry
- ▶ No fixed follow-up time (fixed date)

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What are we interested in?

- ▶ Does the intervention decrease the event number over the study period compared to control?

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- ▶ What is the intervention effect on the number of higher-order events, e.g. 3rd event, compared to control?

Scientific questions

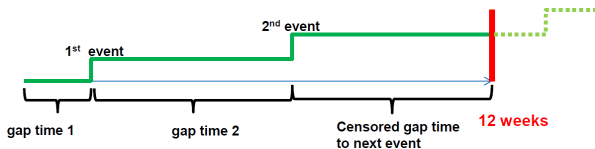
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- ▶ How many events does the intervention prevent, on average, compared to control?
- ▶ What is the intervention effect on the number of higher-order events, e.g. 3rd event, compared to control?
- ▶ What is the effect of intervention on the number of subsequent events among those who experienced a preceding event?

Need to decide which aspect of the recurrent event data process is of interest

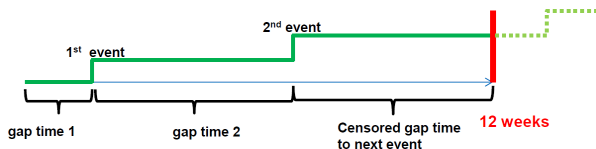
1. Cumulative number of events over a specified time period
 - Number of events by end of study events
2. Rate of events
 - Number of events per unit time
3. Time to event
 - Times to successive events
4. Gap times between successive events
 - Times between successive events

1. Cumulative number of events over a specified time period
 - Number of events by end of study: 2 events



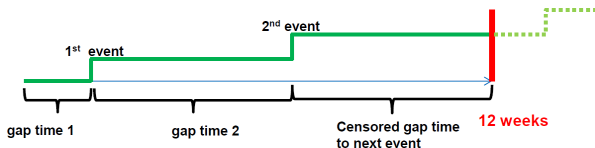
2. Rate of events

- Number of events per unit time: assuming constant rate leads to $1/6$ events per week



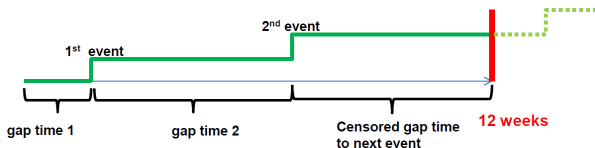
3. Time to event

- Times to successive events: time to 1st and 2nd event, time to 3rd event censored at 12 weeks



4. Gap times between successive events

- Times between successive events: gap times 1 & 2 and third gap time censored at 12 weeks



Recurrent event analysis

Comparison with time-to-event

- ▶ Time-to-event endpoints
 - Statistical approaches well established
 - Gold standard in many indications
 - Substantial experience in regulatory assessment

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▶ Recurrent event endpoints

- Statistical approaches more complex
- Less regulatory experience
- Good experience in some indications do exist (e.g. MS and asthma)

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Comparison with time-to-event

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► Recurrent event endpoints

- Statistical approaches more complex
- Less regulatory experience
- Good experience in some indications do exist (e.g. MS and asthma)
- More efficient as information beyond the first event is used

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Recurrent Events

Existing Methodology

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- ▶ Non-parametric estimator for mean cumulative function
 - ▶ Time-to-event approaches

 - ▶ Methods based on event rates

Recurrent Events

Existing Methodology

- ▶ Non-parametric estimator for mean cumulative function
- ▶ Time-to-event approaches
 - WLW: cumulative time from randomisation to events
 - PWP: analyses gap times, conditional risk sets
 - Andersen-Gill: extension of Cox proportional-hazards model
- ▶ Methods based on event rates

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 - WLW: cumulative time from randomisation to events
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 - Andersen-Gill: extension of Cox proportional-hazards model
- ▶ Methods based on event rates
 - Poisson: total events divided by follow-up
 - Negative Binomial: individual Poisson rates which vary according to Gamma

Mean Cumulative Function

Notation and derivation

- ▶ $N(t)$: Counting process, i.e. number of events a subject has experienced by time t
- ▶ Arbitrary MCF: $\mu(t) = \mathbb{E}\{N(t)\}$

How do we estimate $\mu(t) = \mathbb{E}\{N(t)\}$?

Mean Cumulative Function

Notation and derivation

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- ▶ $dN(t)$: jump of N over a small time interval $[t, t + dt)$

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- ▶ $t_{(1)}, t_{(2)}, \dots, t_{(H)}$: H distinct event times across all n patients

Mean Cumulative Function

Notation and derivation

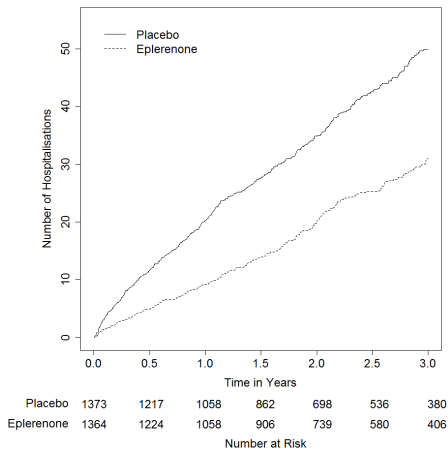
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Nelson-Aalen estimator for the MCF is given by:

$$\hat{\mu}(t) = \sum_{\{h|t_{(h)} \leq t\}} \frac{dN_{\Sigma}(t_{(h)})}{Y_{\Sigma}(t_{(h)})}$$

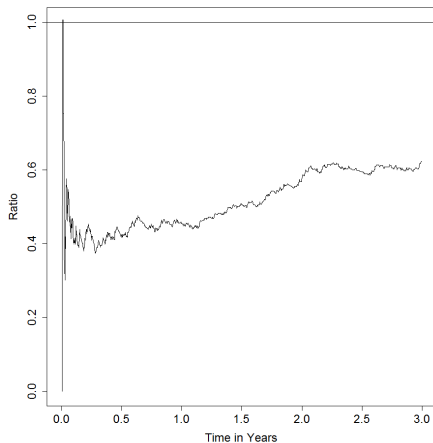
EMPHASIS-HF

Mean cumulative function



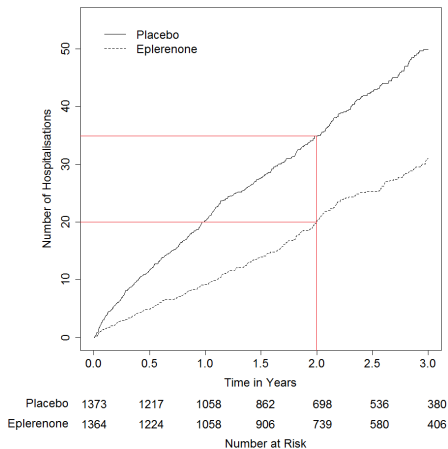
EMPHASIS-HF

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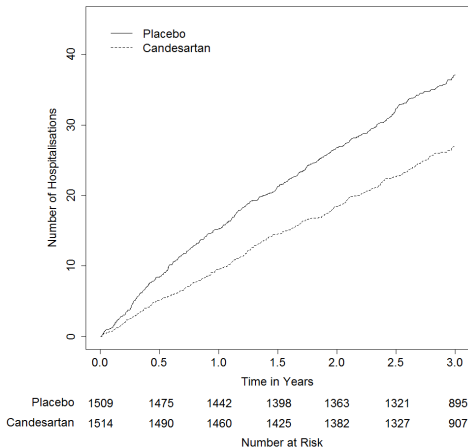
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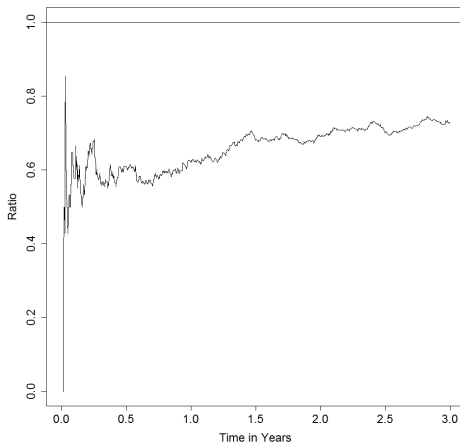
CHARM-Preserved

Mean cumulative function



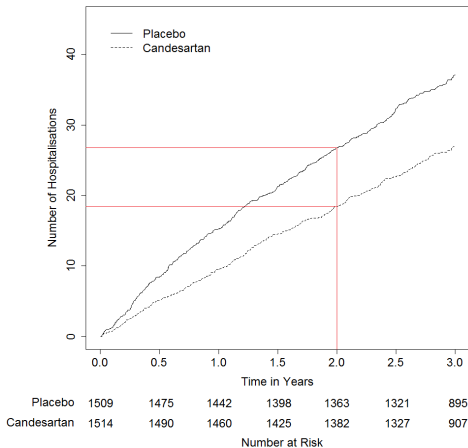
CHARM-Preserved

Mean cumulative function



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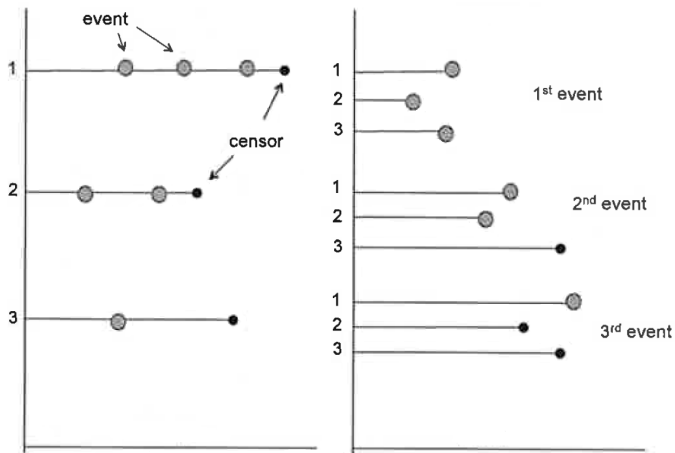
Mean cumulative function



- ▶ Interested in first K events
- ▶ Analyse each time ordered event using a Cox proportional-hazards model
- ▶ Estimate test statistic or hazard ratio for each time ordered event
- ▶ Combine K estimates using optimal weights or $1/\text{variance}$

WLW (Wei-Lin-Weissfeld)

Patient profiles



EMPHASIS-HF

Application

	HR	95% CI	<i>p</i> -value
1st HFH	0.63	(0.53,0.76)	< 0.001
2nd HFH	0.58	(0.43,0.79)	< 0.001
3rd HFH	0.50	(0.31,0.80)	0.004

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- ▶ 463 had at least 1 HFH
- ▶ 177 had at least 2 HFH
- ▶ 76 had at least 3 HFH

EMPHASIS-HF

Hospitalisation counts

	Eplerenone (N=1364)	Placebo (N=1373)
Follow-up years	2916.07	2830.91
Deaths	205	253
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HF Hospitalisations:		
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All admissions	312	481

CHARM-Preserved Application

	HR	95% CI	<i>p</i> -value
1st HFH	0.80	(0.68,0.96)	0.015
2nd HFH	0.82	(0.62,1.07)	0.146
3rd HFH	0.65	(0.43,0.97)	0.036

- ▶ Preserves randomisation
- ▶ Analyses cumulative effect of treatment on hospitalisations from randomisation
 - Effect on second includes effect on first
 - Difficult to interpret global treatment effects
- ▶ Semi-parametric approach: no assumption on baseline hazard needed
- ▶ Can't analyse all hospitalisations due to small numbers for higher order events
- ▶ Need to specify K in advance
- ▶ Subjects considered to be at risk for event k , even if they haven't experienced event $k - 1$

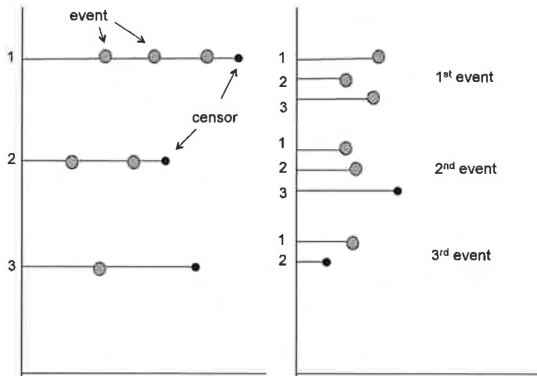
PWP (Prentice-Williams-Peterson)

Analysis method

- ▶ Analyses gap times between different failures
- ▶ Subject not at risk of second event until they've had a first
 - Conditional risk set for event k made up of all subjects who have had event $k - 1$
- ▶ Analyse each time ordered event using a Cox proportional-hazards model
- ▶ Estimate test statistic or hazard ratio for each time ordered event
- ▶ Combine K estimates using optimal weights or $1/\text{variance}$

PWP (Prentice-Williams-Peterson)

Patient profiles



CHARM-Preserved Application

	HR	95% CI	<i>p</i> -value
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- ▶ 508 had at least 1 HFH
- ▶ 209 had at least 2 HFH
- ▶ 98 had at least 3 HFH

PWP (Prentice-Williams-Peterson)

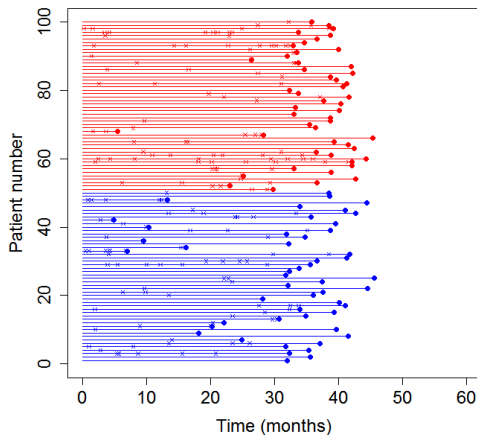
Properties

- ▶ Semi-parametric approach: no assumption on baseline hazard needed
- ▶ Conditional risk sets better reflect true disease progression
- ▶ Doesn't assume common baseline hazard for each gap time
- ▶ Can't analyse all hospitalisations due to small numbers for higher order events
- ▶ Need to specify K in advance
- ▶ Parameters for each of the k events need to be interpreted conditionally: treatment comparisons are not protected through randomisation
- ▶ Difficult to interpret global treatment effects

- ▶ Extension of Cox proportional-hazards model (proportional-intensity)
 - $\lambda(t) = \exp\{\beta z_i\} \lambda_0(t)$
 - $\lambda_0(t)$: baseline intensity function
- ▶ Each gap time contributes to the likelihood
- ▶ Gives a intensity/hazard ratio for recurrent events
- ▶ Assumes that events are independent
 - Robust standard errors accommodate heterogeneity

CHARM-Preserved

Patient profiles



Andersen-Gill

Properties

- ▶ Semi-parametric approach: no assumption on baseline hazard needed
- ▶ Can analyse all hospitalisations for all individuals
- ▶ Assumes common baseline hazard for each gap time
- ▶ Proportionality assumption may be too strong in practice
 - Intensity/hazard ratio assumed to be constant through time and common across recurrent events

Poisson

Analysis method

- ▶ Commonly used for event rates
- ▶ Simple: total number of events divided by total follow-up in each group
- ▶ Gives a rate ratio for recurrent events
- ▶ Assumes that all events are independent
- ▶ Perform a Poisson regression on the count data, adjusting for treatment and including an offset for time in the study

Negative Binomial

Analysis method

- ▶ Events within an individual related - naturally accommodated by negative binomial
- ▶ Each individual has their own individual Poisson hospitalisation rate
- ▶ Poisson rates vary according to Gamma
- ▶ Straightforward to implement
- ▶ Does not require complex data files
- ▶ Perform a negative binomial regression on the count data, adjusting for treatment and including an offset for time in the study

Negative Binomial Properties

- ▶ Simple and naturally allows for overdispersion
- ▶ Correlation of events with the same individual is accounted for through the inclusion of a random effect term
- ▶ Poisson process assumption for the conditional counting process may not hold
- ▶ Constant baseline assumption may be too strong in practice
 - Could assume other parametric models for conditional counting process
- ▶ Rate ratio also assumed to be constant over time and common across recurrent events

EMPHASIS-HF

Application

	HR	95% CI	<i>p</i> -value
Composite	0.69	(0.59,0.81)	< 0.001

	RR	95% CI	<i>p</i> -value
Poisson	0.63	(0.55,0.73)	< 0.001
Negative binomial	0.53	(0.42,0.66)	< 0.001

CHARM-Preserved Application

	HR	95% CI	<i>p</i> -value
Adjudicated composite	0.89	(0.77,1.03)	0.118
Unadjudicated composite	0.86	(0.74,1.00)	0.050

	RR	95% CI	<i>p</i> -value
Poisson	0.71	(0.62,0.81)	< 0.001
Negative binomial	0.68	(0.54,0.85)	< 0.001
Andersen-Gill	0.71	(0.57,0.88)	0.002

EMPHASIS-HF

Summary

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WLW 3rd HFH	0.65	(0.43,0.97)	0.036
PWP 1st HFH	0.80	(0.68,0.96)	0.015
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PWP 3rd HFH	0.68	(0.46,1.02)	0.066
Poisson	0.71	(0.62,0.81)	< 0.001
Negative binomial	0.68	(0.54,0.85)	< 0.001
Andersen-Gill	0.71	(0.57,0.88)	0.002

- ▶ Treatment acts on incidence of first hospitalisations and on recurrences

- ▶ EMPHASIS-HF
 - Poisson for firsts: 0.65 (0.54-0.73, $P < 0.001$)
 - Negative binomial for repeats: 0.52 (0.33-0.82, $P = 0.004$)

- ▶ CHARM-Preserved
 - Poisson for firsts: 0.82 (0.69-0.97, $P = 0.025$)
 - Negative binomial for repeats: 0.58 (0.39-0.87, $P = 0.009$)

Motivation

- Conventional analyses

- Examples

- Problems

Setting

- Recurrent Events

- Examples

Objectives

- Scientific Questions

Existing Models for Recurrent Events

- Mean Cumulative Function

- Time-to-Event

- Event rates

- Application

Considerations

Scientific questions

What are we interested in?

- ▶ Does the intervention decrease the event number over the study period compared to control?
- ▶ How many events does the intervention prevent, on average, compared to control?
- ▶ What is the intervention effect on the number of higher-order events, e.g. 3rd event, compared to control?
- ▶ What is the effect of intervention on the number of subsequent events among those who experienced a preceding event?

Statistical Considerations

Summary

► Modelling framework

- Fully parametric
- Semi-parametric
- Non-parametric

Event rate

- Constant
- Time-varying
- Unspecified

► Overdispersion

► Censoring

- Informative censoring assumption
More hospitalisations → increased risk of death
- Terminal event