Lecture 3: Microscopic models of the dispersion

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- Origins of the dispersion
  - Dielectric resonances: bounded (polarization) charges
    - Elastically bounded electrons (exciton polaritons)
    - Polar lattice vibrations (phonon polaritons)

$$m\ddot{\mathbf{r}} = -m\gamma\dot{\mathbf{r}} - k\mathbf{r} - e\mathbf{E}$$

  - Metallic resonances: free charges
    - Conductivity in metals (underlying lattice resonances are much weaker)

$$m\ddot{\mathbf{r}} = -m\gamma\dot{\mathbf{r}} - e\mathbf{E}$$

Electromagnetic force on charges

$$f = -e\left(\boldsymbol{E} + \boldsymbol{v} \wedge \boldsymbol{B}\right) = -e\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \wedge \left(\boldsymbol{s} \wedge \boldsymbol{E}\right)\right) \approx -e\boldsymbol{E}$$

- Only electric part in the non-relativistic approximation

Dielectric resonance

$$\ddot{\boldsymbol{P}} + \gamma\dot{\boldsymbol{P}} + \frac{k}{m}\boldsymbol{P} = \frac{Ne^2}{m}\boldsymbol{E}_{loc}$$

$$(\boldsymbol{P} = -Ne\boldsymbol{r}), \quad \boldsymbol{E}_{loc} = \frac{\boldsymbol{P}}{3\varepsilon_0} + \boldsymbol{E}$$

$$\ddot{\boldsymbol{P}} + \gamma\dot{\boldsymbol{P}} + \omega_0^2\boldsymbol{P} = \frac{Ne^2}{m}\boldsymbol{E}$$

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{Ne^2}{3m\varepsilon_0}}$$

Electric field is decomposed to harmonic waves:

$$\boldsymbol{P} = \frac{Ne^2/m}{\omega_0^2 - \omega^2 - i\omega\gamma}\boldsymbol{E}$$

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon_0\left(1 + \frac{Ne^2}{m\varepsilon_0}\frac{1}{\omega_0^2 - \omega^2 + i\omega\gamma}\right)\boldsymbol{E} = \varepsilon\boldsymbol{E}$$

Dispersion near dielectric resonances

$$\varepsilon_r = 1 + \frac{Ne^2}{m\varepsilon_0} \frac{1}{\omega_0^2 - \omega^2 + i\omega\gamma} = 1 + \frac{\omega_L^2 - \omega_0^2}{\omega_0^2 - \omega^2 + i\omega\gamma}, \qquad \varepsilon_{stat} = \frac{\omega_L^2}{\omega_0^2}$$

Small damping ($\omega_0 >> \gamma$):

1) Low-frequency regime ($0 \le \omega \le \omega_0 - 2\gamma$)
2) Vicinity of the resonance ($\omega_0 - 2\gamma \le \omega \le \omega_0 + 2\gamma$)
3) High reflectivity regime (Reststrallung band) ($\omega_0 + 2\gamma \le \omega \le \omega_L$)
4) High-frequency regime ($\omega \ge \omega_L$)

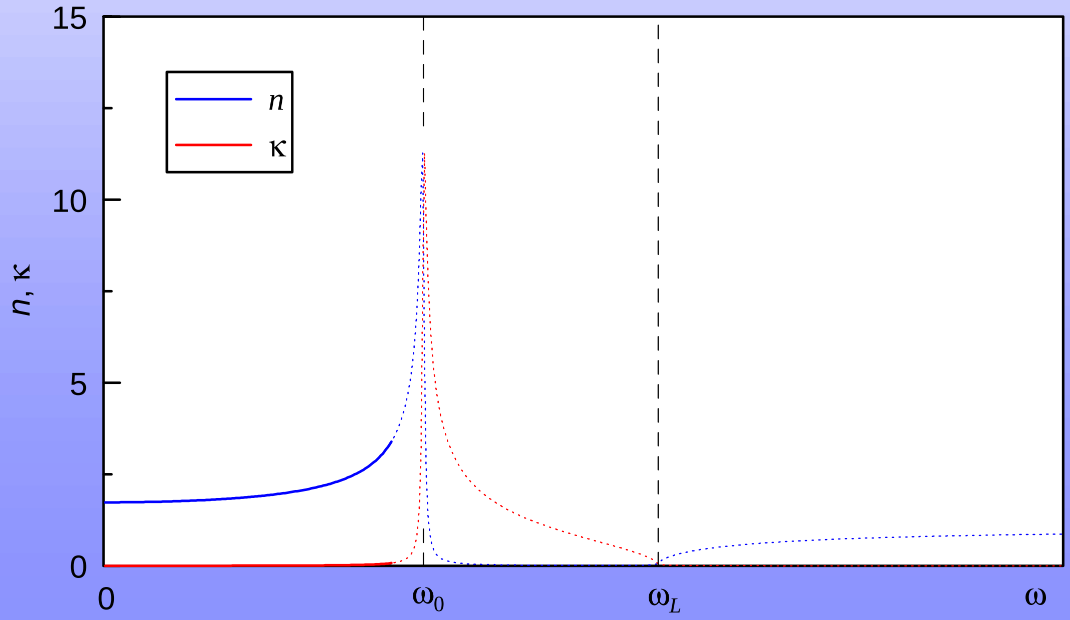
Dispersion: dielectric resonances

1) Low-frequency regime ($0 \le \omega \le \omega_0 - 2\gamma$), $n^2 = \varepsilon_r$, $\kappa = 0$, normal dispersion

$$\varepsilon_r = 1 + \frac{\omega_L^2 - \omega_0^2}{\omega_0^2 - \omega^2 + i\omega\gamma} \approx \varepsilon_{stat} + \frac{\omega^2}{\omega_0^2}(\varepsilon_{stat} - 1)$$

$$\varepsilon_r' = n^2 - \kappa^2$$

$$\varepsilon_r'' = 2n\kappa$$

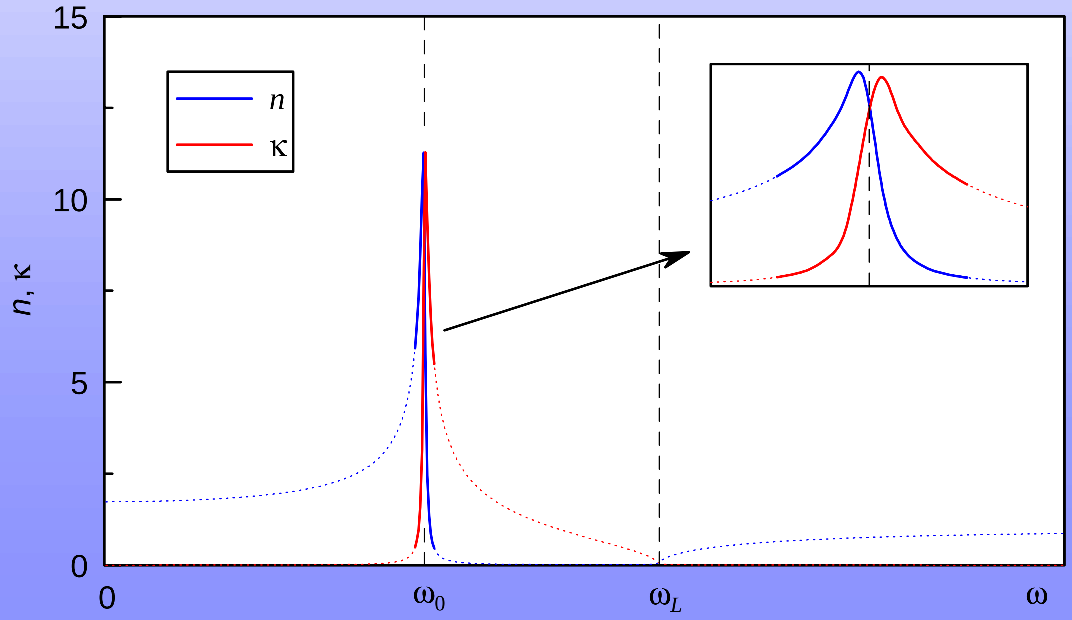


Dispersion: dielectric resonances

2) Vicinity of the resonance ($\omega_0 - 2\gamma \le \omega \le \omega_0 + 2\gamma$), anomalous dispersion

$$\varepsilon_r = 1 + \frac{\omega_L^2 - \omega_0^2}{\omega_0^2 - \omega^2 + i\omega\gamma} \approx 1 - \frac{\omega_L^2 - \omega_0^2}{\omega_0} \frac{2\Delta + i\gamma}{4\Delta^2 + \gamma^2}$$

$$\varepsilon_r' = n^2 - \kappa^2$$
$$\varepsilon_r'' = 2n\kappa$$

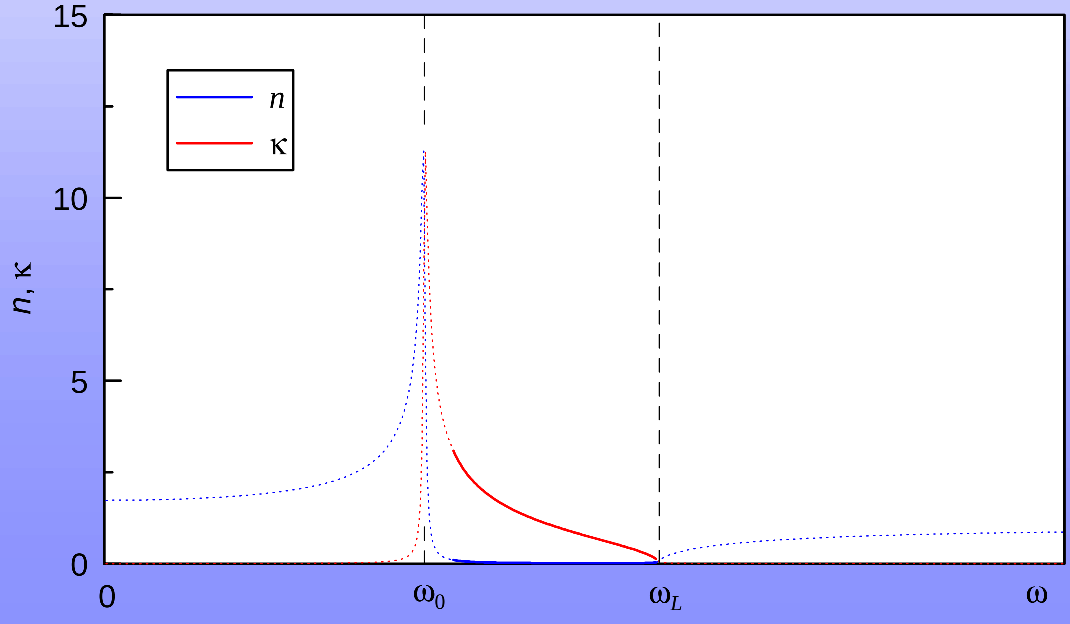


Dispersion: dielectric resonances

3) High reflectivity regime ($\omega_0 + 2\gamma \leq \omega \leq \omega_L$), $\kappa^2 = \varepsilon_r$, $n = 0$

$$\varepsilon_r = 1 + \frac{\omega_L^2 - \omega_0^2}{\omega_0^2 - \omega^2 + i\omega\gamma} \approx 1 - \left|\frac{\omega_L^2 - \omega_0^2}{\omega^2 - \omega_0^2}\right| < 0$$

$$\varepsilon_r' = n^2 - \kappa^2$$
$$\varepsilon_r'' = 2n\kappa$$

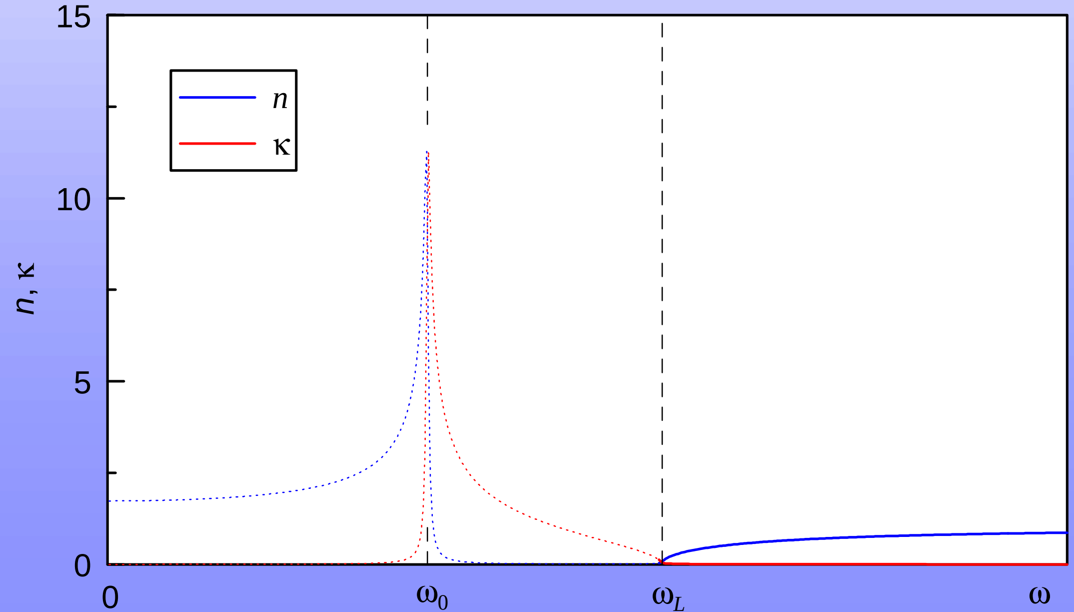


Dispersion: dielectric resonances

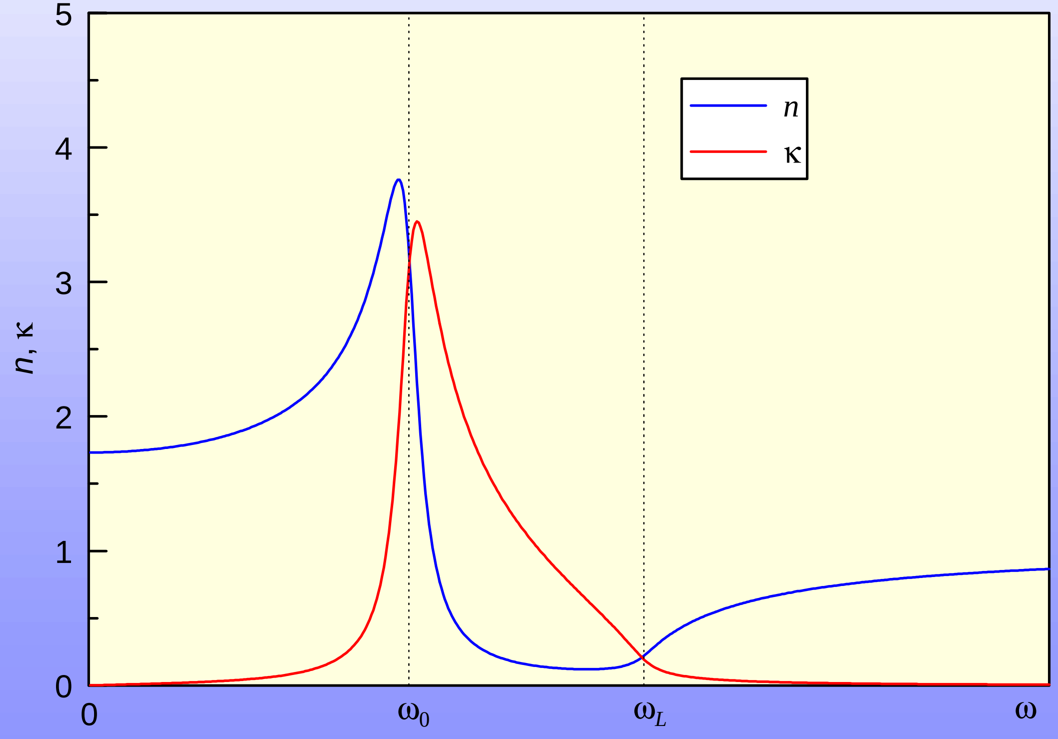
4) High-frequency regime ($\omega \geq \omega_L$), normal dispersion

$$\varepsilon_r = 1 + \frac{\omega_L^2 - \omega_0^2}{\omega_0^2 - \omega^2 + i\omega\gamma} \approx 1 - \left|\frac{\omega_L^2 - \omega_0^2}{\omega^2 - \omega_0^2}\right| > 0$$

$$\varepsilon_r' = n^2 - \kappa^2$$
$$\varepsilon_r'' = 2n\kappa$$

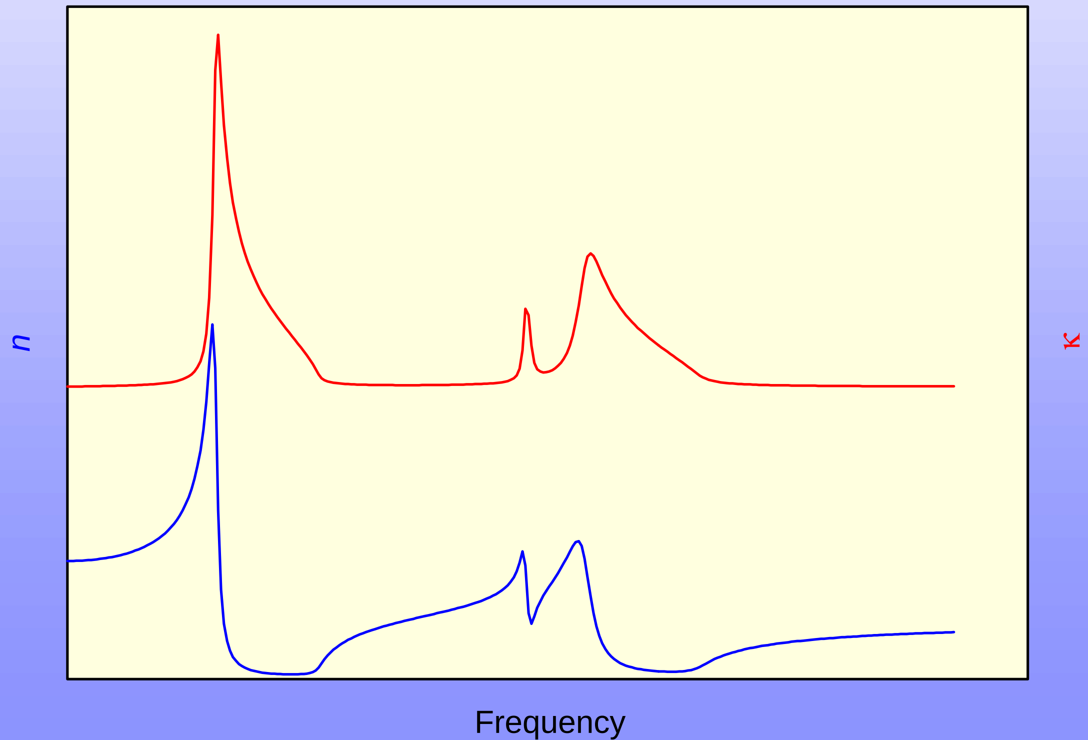


Dielectric resonance: stronger damping



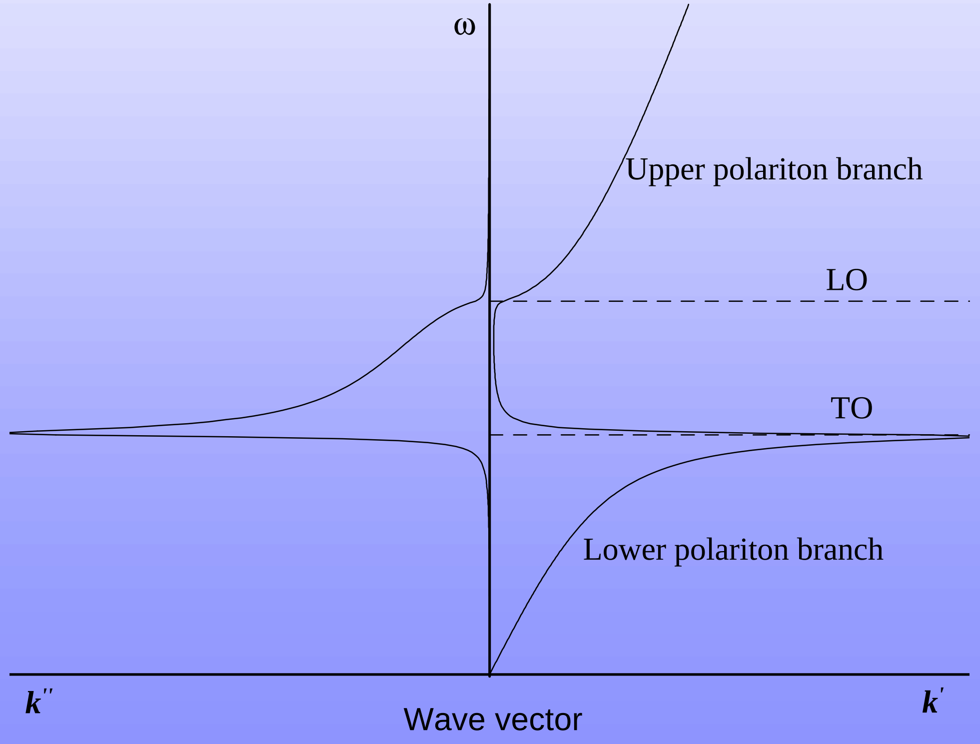
Several dielectric resonances

$$\varepsilon = \varepsilon_0 \left(1 + \sum_j \frac{f_j}{\omega_j^2 - \omega^2 + i\omega\gamma_j} \right)$$



Polariton concept

Polariton equation: $k = \frac{\omega}{c} n(\omega)$



Metallic resonance

$$\ddot{\boldsymbol{P}} + \gamma \dot{\boldsymbol{P}} + \frac{k}{m}\boldsymbol{P} = \frac{Ne^2}{m}\boldsymbol{E}_{loc} \qquad (\boldsymbol{P} = -Ne\boldsymbol{r}), \quad \boldsymbol{E}_{loc} = \frac{\boldsymbol{P}}{3\varepsilon_0} + \boldsymbol{E}$$

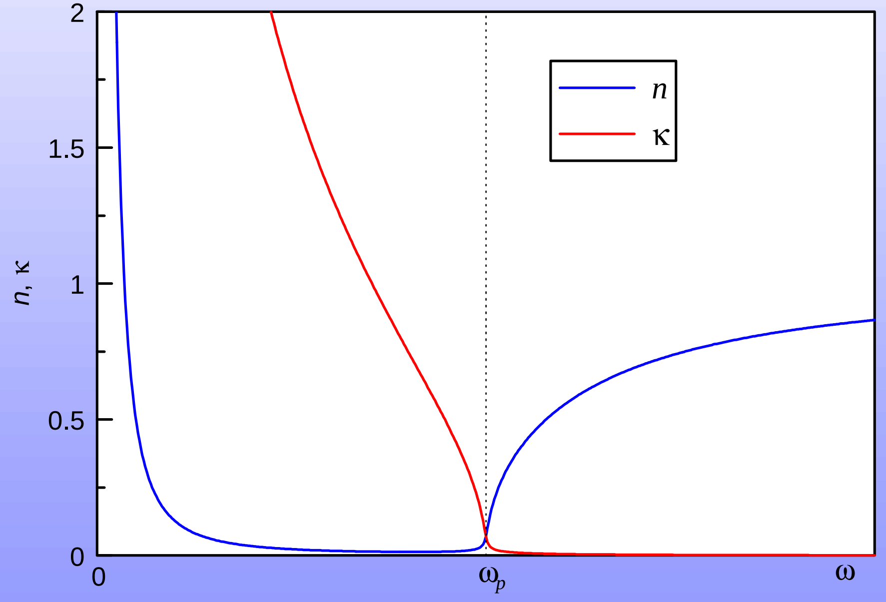
$$\frac{d\boldsymbol{j}}{dt} + \gamma \boldsymbol{j} = \left(\frac{Ne^2}{m}\right)\boldsymbol{E}$$

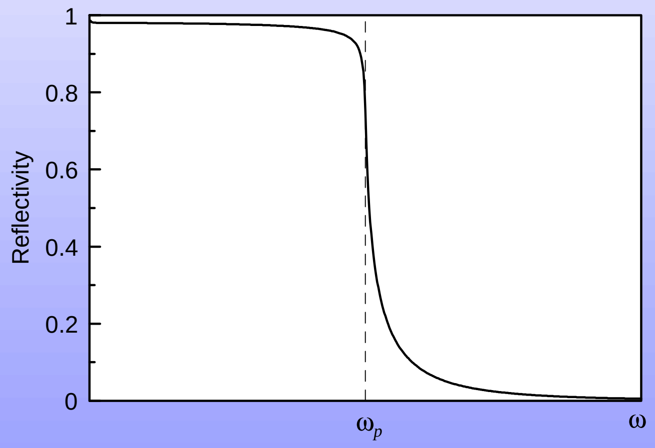
$$\sigma = \frac{Ne^2/m}{\gamma + i\omega} = \frac{\sigma_0}{1 + i\omega/\gamma}$$

$$\varepsilon = \varepsilon_0\left(1 + \frac{Ne^2}{m\varepsilon_0}\frac{1}{i\omega\gamma - \omega^2}\right) = \varepsilon_0\left(1 + \frac{\omega_p^2}{i\omega\gamma - \omega^2}\right)$$

ω_p … plasma frequency (longitudinal resonance of the free charges)

Dispersion: metallic resonance





Metallic mirrors