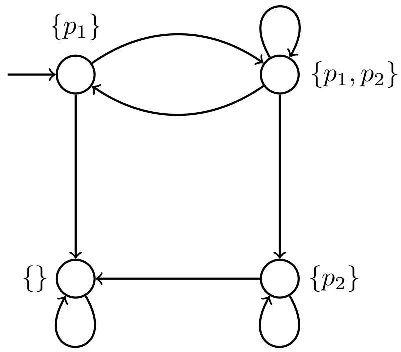
1. Suppose the set of atomic propositions is $\{p_1, p_2\}$. Consider the following transition system -



(Notation: For each state, the atoms written inside the curly braces next to the corresponding state are the atoms that are true at that state. For example, p_1 is true at the top left state (initial state), whereas p_2 is true at the bottom right state, and so on.)

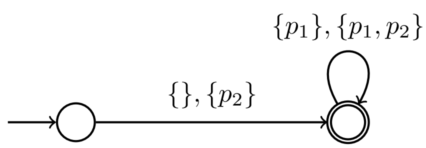
Which of the following LTL formulas does this transition system satisfy?

(a) Fp_2
(b) $G(p_1 \vee p_2)$
(c) $(p_1 U p_2) \vee G(\neg p_2)$
(d) $(p_1 \wedge p_2) \rightarrow Xp_2$
(e) $G((p_1 \wedge p_2) \rightarrow Xp_2)$

2. Suppose p, q, r are three propositional atoms.

(a) Are the two formulas $((p\ U\ q)\ U\ r)$ and $(p\ U\ (q\ U\ r))$ equivalent? That is, whichever (infinite) word satisfies the first formula would satisfy the second and vice versa?
(b) Is $(p\ U\ (q \vee r))$ equivalent to $((p\ U\ q) \vee (p\ U\ r))$?
(c) Is $((q \vee r)\ U\ p)$ equivalent to $((q\ U\ p) \vee (r\ U\ p))$?

3. Suppose the set of atomic propositions is $\{p_1, p_2\}$. Consider the following NBA -



Show that the language of the above NBA is exactly the set of words satisfying the LTL formula $F(\neg p_1) \wedge XGp_1$.

4. Draw the NBA corresponding to LTL formulas $p_1\, \boldsymbol{U}(\neg p_2)$, $(\neg p_1)\, \boldsymbol{U} p_2$.

5. Let ϕ, ψ and χ be LTL formulas. We say two formulas are *equivalent*, written as $\phi \equiv \psi$ if they define the same language. For each of the following, prove or disprove the equivalences:

(a) $\boldsymbol{G}(\phi \wedge \psi) \equiv (\boldsymbol{G}\phi) \wedge (\boldsymbol{G}\psi)$
(b) $\boldsymbol{GFG}\phi \equiv \boldsymbol{FGF}\phi$
(c) $\boldsymbol{X}(\phi\, \boldsymbol{U} \psi) \equiv (\boldsymbol{X}\phi)\, \boldsymbol{U}(\boldsymbol{X}\psi)$
(d) $(\phi\, \boldsymbol{U}\psi)\, \boldsymbol{U}\chi \equiv \phi\, \boldsymbol{U}(\psi\, \boldsymbol{U}\chi)$