

Pacific Journal of Mathematics

**THE SUBGROUPS OF A DIVISIBLE GROUP G WHICH CAN BE
REPRESENTED AS INTERSECTIONS OF DIVISIBLE
SUBGROUPS OF G**

SAMIR A. KHABBAZ

THE SUBGROUPS OF A DIVISIBLE GROUP G WHICH
CAN BE REPRESENTED AS INTERSECTIONS
OF DIVISIBLE SUBGROUPS OF G

SAMIR A. KHABBAZ

Introduction. In [1], page 70, L. Fuchs asks the following question: Which are those subgroups of a divisible group G that can be represented as intersections of divisible subgroups of G ?

The main purpose of this paper is to give an answer to this question.

NOTATION.

- N1: If H is a primary p -group, let $S(H)$ denote the subgroup of elements of H whose orders are 1 or p .
- N2: If G is Abelian, let $T(G)$ be the torsion subgroup of G ; let G_p denote the primary p -component of $T(G)$; and, in case G is divisible, let $F(G)$ denote a maximal torsion free subgroup of G .
- N3: Let the symbol \oplus denote a direct sum. Let the symbol $<$ denote "properly contained in." Let \subset denote "contained in." Let $N \setminus M$ denote "the set of elements in N and not in M ." Let \cong denote "is isomorphic to." Let \exists denote "there exists (exist)." Let \ni denote "such that." Let $(N_a)_{a \in A}$ denote a family of sets N_a indexed by members of the set A . Finally if Q is a subset of a group, let $\langle Q \rangle$ denote the subgroup of that group generated by the elements of Q .
- N4: Let R denote the additive group of rationals. Let P denote the set of primes. Let the group $C(p^\infty)$ be the indecomposable divisible primary p -group.
- N5: Let $C = C(2^\infty) \oplus C(3^\infty) \oplus C(5^\infty) \oplus \dots$; and if $S \subset P$, let $C_S = \bigoplus_{p \in S} C(p^\infty)$.
- N6: If G is a group, let $P(G)$ be the set of $p \in P$, such that $\exists x \in G$ with order $x = p$.
- N7: Finally, we recall the following convenient and succinct classification of the subgroups of R [see Kurosh I, page 208]. Let p_1, p_2, p_3, \dots be the sequence of primes in natural order. A characteristic is a sequence $a = (a_1, a_2, a_3, \dots)$, where $a_i = a$ non-negative integer or ∞ . A type is a class of equivalent characteristics, two characteristic $a = (a_1, a_2, a_3, \dots)$ and $b = (b_1, b_2, b_3, \dots)$ being equivalent if and only if $\sum_{i=1}^{\infty} |a_i - b_i| < \infty$, where $\infty - \infty = 0$.

$A \subset R$ has type a if and only if it is isomorphic to the subgroup

Received April 15, 1960. This paper constitutes the first part of the author's doctoral dissertation submitted to the University of Kansas, and was presented before the Amer. Math. Soc. An abstract of this paper was received by the Society on July 17, 1959.

of R consisting of those rationals whose denominators in the reduced form are divisible by no higher power of the prime p_i than the a_i th if $a_i < \infty$, and by every power of p_i if $a_i = \infty$.

Define $a \geq b$ if and only if $a_i \geq b_i$, for $i = 1, 2, 3, \dots$.

N8: Let $S \subset P$. We shall say that A above has type T_S if and only if $a_i = 0$ for $p_i \in S$ and $a_i = \infty$ otherwise. Then it is well known that $R/B \subset C_S$ if and only if B contains a subgroup A of type T_S , and that the intersection of two subgroups of R containing subgroups of type T_S again contains a subgroup of type T_S .

N9: Let the symbol $\bigcap_{\mathcal{G}}$ stand for the phrase "an intersection of divisible subgroups of G ."

LEMMA 1. (Kulikov):

- a. A divisible group M is a minimal divisible group containing the subgroup L if and only if $H \subset M$ and $H \cap L = 0$ imply $H = 0$, H being a subgroup.
- b. If M is a minimal divisible group containing L , then M/L is torsion and divisible.

LEMMA 2. Let G be divisible and L a subgroup of G . Let M be a minimal divisible subgroup of G containing L . Thus, using the notational in N2, we may write, $G = M \oplus E = M \oplus T(E) \oplus F(E)$. We have:

- a. If M is minimal divisible containing L , then $S(M_p) = S(L_p)$ for each $p \in P$, and $T(M)$ is minimal divisible containing $T(L)$.
- b. Kulikov: If L is torsion, then M is minimal divisible containing L if and only if $S(L_p) = S(M_p)$ for all $p \in P$.
- c. If L is $\bigcap_{\mathcal{G}}$ then $T(L)$ is $\bigcap_{T(G)}$ and hence $\bigcap_{\mathcal{G}}$.

Proof.

- a. Let $x \in S(M_p) \setminus S(L_p)$. Then $\{x\} \cap S(L_p) = 0$, and therefore $\{x\} \cap L = 0$. By Lemma 1a, $x = 0$. Next, $T(M)$ is divisible, and contains $T(L)$. If $N \subset T(M)$ is divisible and contains $T(L)$, $T(M)$ can be written as $T(M) = N \oplus K$, where $K \cap T(L) = 0$; and hence $K \cap L = 0$, so that by Lemma 1a, $K = 0$; and hence, $N = T(M)$.
- b. The "only if" part is contained in part a. For the "if" part, assume $N \subset M$ is divisible and contains L . Then we may write $M = N \oplus K$. Then, by hypothesis, K cannot have elements of prime order, and must therefore be 0.
- c. Assume $L = \bigcap_{\alpha \in \mathcal{A}} M_\alpha$, where each M_α is divisible and contains L . Then each $T(M_\alpha)$ is divisible and contains $T(L)$. Moreover, we have $\bigcap_{\alpha \in \mathcal{A}} T(M_\alpha) = T(\bigcap_{\alpha \in \mathcal{A}} M_\alpha) = T(L)$. Hence $T(L)$ is $\bigcap_{T(G)}$ and hence $\bigcap_{\mathcal{G}}$.

LEMMA 3. Let G be a minimal divisible group containing the

subgroup L , and having a representation of the form $G = \bigoplus_{a \in A} G_a$. Then G/L (which by Lemma 1b is divisible and torsion) contains a subgroup isomorphic to $C(p^\infty)$ if and only if for some $a \in A$, $G_a/G_a \cap L$ contains a subgroup of the same kind. In other words: $P(G/L) = \bigcup_{a \in A} P(G_a/G_a \cap L) = \bigcup_{a \in A} P(G_a + L/L)$.

Proof. Because of the divisibility of all the groups concerned, it suffices to check the existence of elements of order p . Suppose $x \in G_a/G_a \cap L$ has order p . Then $G/L \supset G_a + L/L \cong G_a/(G_a \cap L)$. Hence, G/L has an element of order p . Conversely, suppose that for no $a \in A$ does $G_a/(G_a \cap L)$ contain an element of order p . Then no $G_a + L/L$ contains an element of order p . Hence, the subgroup of G/L generated by the $(G_a + L)/L$ contains no elements of order p . But since the G_a 's generate G , the $G_a + L/L$'s generate all of G/L .

THEOREM 1. *Let G be a divisible group; let L be a subgroup of G ; let M be a minimal divisible subgroup of G containing L . Suppose G has a representation of the form $G = M \oplus E$, then $L = M \cap (\bigcap_{\omega \in \Omega} M_\omega)$, where M_ω is a divisible subgroup of G containing L for each $\omega \in \Omega$, if and only if there exists homomorphisms $h_\omega: M \rightarrow E$ for each $\omega \in \Omega$ such that $\bigcap_{\omega \in \Omega} \ker h_\omega = L$.*

Proof. To prove the "if" part, let I be the identity map of M ; and for each $\omega \in \Omega$ let $g_\omega: M \rightarrow G$ be defined by $g_\omega = I + h_\omega$. Let $M_\omega = g_\omega(M)$. Then $L \subset M_\omega$ since $h_\omega(L) = 0$, and therefore M_ω is divisible since it is a homomorphic image of M . Finally, $x \in M_\omega \cap M$ implies $x \in \ker h_\omega$ (since $x = y + h_\omega(y)$ implies $h_\omega(y) = x - y \in M \cap E = 0$); and hence, $L \subset \bigcap_{\omega \in \Omega} M_\omega \cap M = \bigcap_{\omega \in \Omega} \ker h_\omega = L$.

To prove the "only if" part, suppose $L = M \cap (\bigcap_{\omega} M_\omega)$. It can be assumed that each M_ω is minimal divisible containing L and, therefore, also minimal divisible containing $M \cap M_\omega$. Also, M is minimal divisible containing $M \cap M_\omega$. Also, M is minimal divisible containing $M \cap M_\omega$; so there is an isomorphism $i_\omega: M \rightarrow M_\omega$ which is the identity on $M \cap M_\omega$. Note that $i_\omega(x) \in M \Rightarrow i_\omega(i_\omega(x)) = i_\omega(x) \Rightarrow i_\omega(x) = x \Rightarrow x \in M \cap M_\omega$. Let $p: G \rightarrow E$ be the projection determined by the decomposition $G = M \oplus E$. Let h_ω be defined by $h_\omega = pi_\omega$. Then $h_\omega(x) = 0$, $i_\omega(x) \in M$, and $x \in M \cap M_\omega$ are equivalent. Thus $\bigcap_{\omega \in \Omega} \ker h_\omega = \bigcap_{\omega \in \Omega} M \cap M_\omega = L$.

REMARK. The underlined portion of Theorem 1 may be replaced by $h_\omega: M \rightarrow T(E)$.

COROLLARY 1. *Let G be a divisible group; let L be a subgroup of G ; and let M be a subgroup of G which is minimal with respect to being divisible and containing L . Thus, we may write $G = M \oplus E =$*

$M \oplus T(E) \oplus F(E)$, and $P(E) = P(T(E))$. Then L is \bigcap_G if and only if $P(M/L) \subset P(E)$.

Proof. The condition $P(M/L) \subset P(E)$ is easily seen to be equivalent to the existence of the family $\{h_\omega\}_{\omega \in \Omega}$ of homomorphisms in Theorem 1.

REMARK. Let G be divisible and torsion free, then $L \subset G$ is \bigcap_G if and only if L is divisible or, equivalently, is a direct summand of G .

COROLLARY 2. Let G be divisible, and let L be a torsion subgroup of G . Then L is \bigcap_G if and only if for each $p \in P$, $S(L_p) \subset S(G_p)$ whenever $S(L_p) \neq 0$, and L_p is not divisible.

Proof. If L is \bigcap_G then by Lemma 2c for each $p \in P$ obviously L_p is \bigcap_{G_p} ; and, hence, to prove that our condition is necessary, we may assume that G is primary, and L is not divisible, in which case the necessity becomes obvious in view of the fact that otherwise G would be the only minimal divisible subgroup of itself containing L ; and, consequently, $L = G$, since L is \bigcap_G , contrary to L being not divisible.

To Prove the "only if" part, note that $p \in P(M/L)$ implies $M_p/L_p = (M/L)_p \neq 0$, since L is torsion. Thus, by hypothesis, $p \in P(M/L) \Rightarrow S(L_p) \subset S(G_p) \Rightarrow S(M_p) \subset S(G_p) = S(M_p) \oplus S(E_p) \Rightarrow S(E_p) \neq 0 \Rightarrow p \in P(E)$.

COROLLARY 3. Let G be divisible and $L \subset G$ be torsion, reduced and \bigcap_G . Then every subgroup of L is \bigcap_G .

COROLLARY 4. Let G be divisible and L be \bigcap_G . Let M be a minimal divisible subgroup of G containing L . Let K be a subgroup of G such that $L \subset K \subset M$. Then K is \bigcap_G .

Proof. If $G = M \oplus E$, then $P(M/K) \subset P(M/L) \subset P(E)$.

COROLLARY 5. Let K be any Abelian group of arbitrary cardinal number A . Then K can be embedded in a divisible group G of power $A \aleph_0$ in such a way that any subgroup of K can be represented as an intersection of two divisible subgroups of G .

Proof. Let M be a minimal divisible group containing K and let E be a group isomorphic to M/K . Let G be the direct sum of M and E . The cardinality of G is clearly $A \aleph_0$ and the isomorphism of M/K into E induces a homomorphism $h: M \rightarrow E$ with $\ker h = K$. Thus, Theorem 1 gives the required conclusion.

REMARK. Let $L \subset G$, and let $L = L_1 \oplus L_2$, where L_2 is divisible and

L_1 is reduced. Then, also, $G = L_2 \oplus K$, where K may be chosen to contain L_1 . It is easy to see that L is \bigcap_{σ} if and only if L_1 is \bigcap_{κ} . Thus, in order to avoid excessive wording, we may in the following theorems assume without loss of generality that L is reduced.

THEOREM 2. *Assume $L \subset G$ is reduced, then L is \bigcap_{σ} if and only if $T(L)$ is \bigcap_{σ} and $P(G/L) \subset P(G)$, equality holding if L is \bigcap_{σ} .*

Proof. Let $G = M \oplus E$, where M and E are as in Theorem 1. Then $P(G) = P(T(M)) \cup P(E)$, and $P(G/L) = P(M/L) \cup P(E)$, because $G/L \cong (M/L) \oplus E$. Note that if $T(L)$ is \bigcap_{σ} , then $T(L)$ is $\bigcap_{T(G)}$ and hence $P(T(M)/T(L)) \subset P(T(E)) = P(E)$, by Corollary 1. But since $T(L)$ is reduced, $P(T(M)/T(L)) = P(T(M))$. Thus the assumption that $T(L)$ is \bigcap_{σ} implies $P(G) = P(E)$ and, therefore, that the conditions $P(G/L) = P(G)$, $P(G/L) \subset P(G)$, and $P(M/L) \subset P(E)$ are equivalent. This observation, together with Lemma 2c and Corollary 1, proves Theorem 2.

COROLLARY 6. *Let G be any divisible group. Let C be as defined in N5, and let $\bar{G} = C \oplus G$. Then any subgroup $K \subset G$ is $\bigcap_{\bar{\sigma}}$.*

REMARK. In Corollary 6, C may be replaced by any Abelian group containing it.

COROLLARY 7. *Any torsion free subgroup T of \bar{G} above is $\bigcap_{\bar{\sigma}}$.*

Proof. T is contained in a direct summand of \bar{G} whose complementary direct summand contains a subgroup isomorphic to C .

REMARK. The following example shows that if $L \subset G$ is \bigcap_{σ} and if $\bar{L} \subset G$ is isomorphic to L , \bar{L} need not be \bigcap_{σ} . Let $G = \bigoplus_{i=1}^{\infty} C_i$ where $C_i \cong C(p^{\infty})$ and where p is fixed. Then,

$$S\left(\bigoplus_{i=1}^{\infty} C_i\right) \cong S\left(\bigoplus_{i=2}^{\infty} C_i\right);$$

however, $S(\bigoplus_{i=1}^{\infty} C_i)$ is not \bigcap_{σ} , while $S(\bigoplus_{i=2}^{\infty} C_i)$ is \bigcap_{σ} .

In this connection we have:

COROLLARY 8. *Let $L \subset G$ be \bigcap_{σ} , and let $\bar{L} \subset G$ be isomorphic to L . Then if $T(\bar{L})$ is \bigcap_{σ} —this is in particular the case if $\bar{L} \subset L$ — \bar{L} is also \bigcap_{σ} . Thus, \bar{L} is \bigcap_{σ} if and only if $T(\bar{L})$ is \bigcap_{σ} .*

Proof. For the proof we may assume L is reduced. By Theorem 2, it suffices to show that $P(G/\bar{L}) \subset P(G)$. Let M and \bar{M} be minimal divisible subgroup of G containing L and \bar{L} , respectively, so that $G =$

$M \oplus N = \bar{M} \oplus \bar{N}$. Then we have $M/L \cong \bar{M}/\bar{L}$ [see Kurosh I, page 168]. Thus, $P(G/\bar{L}) = P(\bar{M}/\bar{L}) \cup P(\bar{N})$
 $= P(M/L) \cup P(\bar{N})$
 $\subset P(G/L) \cup P(G)$
 $\subset P(G)$, since L is \bigcap_{σ} .

THEOREM 3. *Let G be a divisible group and let $L \subset G$ be reduced. Then the following statements are equivalent:*

- (a) L is \bigcap_{σ} .
- (b) $T(L)$ is \bigcap_{σ} and for any subgroup \bar{R} of G isomorphic to R , either $L \cap \bar{R}$ is zero or of type $\geq T_{P(G)}$.
- (c) $S(L_p) < S(G_p)$ if $p \in P(G)$ and for any subgroup \bar{R} of G isomorphic to R , either $L \cap \bar{R}$ is zero or of type $\geq T_{P(G)}$.
- (d) $T(L)$ is \bigcap_{σ} and $L \subset (\bigoplus_{a \in A} R_a) \oplus T(G) \subset G$, where $R_a \cong R$ and each $L \cap R_a$ is either zero or of $\geq T_{P(G)}$.

Proof. By Theorem 2, (a) is equivalent to the conditions $T(L)$ is \bigcap_{σ} and $P(G/L) \subset P(G)$. These conditions imply (b) since G/L contains a subgroup isomorphic to $\bar{R}/\bar{R} \cap L$, so that $P(\bar{R}/\bar{R} \cap L) \subset P(G/L) \subset P(G)$ and therefore $\bar{R} \cap L$ is zero or of type $\geq T_{P(G)}$ by N8. Properties (b) and (c) are equivalent by Lemma 2c and Corollary 2. Also (b) implies (d). Finally, suppose (d) holds. Then, $P(T(G)/(T(G) \cap L)) = P(T(G)/T(L)) \subset P(T(G)) = P(G)$ by Theorem 2. Let $G = (\bigoplus_{b \in B} R_b) \oplus (\bigoplus_{a \in A} R_a) \oplus T(G)$. Then $R_b \cap L$ is 0 for all $b \in B$, and for each $a \in A$, $R_a \cap L$ is 0 or has type $\geq T_{P(G)}$ by hypothesis. Thus, by Lemma 3, $P(G/L) = \bigcap_{b \in B} P(R_b/(R_b \cap L)) \cup \bigcup_{a \in A} P(R_a/(R_a \cap L)) \cup P(T(G)/(T(G) \cap L)) \subset P(G)$. By Theorem 2, this implies (a).

DEFINITION. Define a subset $(x_a)_{a \in A}$ of elements of an Abelian group H to be independent if and only if

$$n_1 x_{a_1} + n_2 x_{a_2} + \dots + n_m x_{a_m} = 0$$

implies $n_1 = n_2 = \dots = n_m = 0$ where each $a_i \in A$ and the n_i 's are integers.

COROLLARY 10. Assume $L \subset G$ is reduced, then L is \bigcap_{σ} if and only if $T(L)$ is \bigcap_{σ} and L contains a subgroup H which contains a maximal independent subset of L and which has the form $\bigoplus_{a \in A} S_a$, where each S_a is isomorphic to a subgroup of R having type $T_{P(G)}$.

Proof. Assume L is \bigcap_{σ} , then by Lemma 2c $T(L)$ is \bigcap_{σ} . Let $M \subset G$ be minimal divisible containing L , and assume that $M = T(M) \oplus F(M) = T(M) \oplus (\bigoplus_{a \in A} R_a)$, where $R_a \cong R$ for all $a \in A$. Then, by Theorem 3c and Lemma 1a, each $L \cap R_a$ contains a subgroup S_a of

type $T_{P(G)}$. Then it is easy to see that $\bigoplus_{a \in A} S_a$ exists; and that, if $x_a \in S_a$, then $(x_a)_{a \in A}$ is a maximal independent subset of M and therefore, of L .

Next assume the condition holds and let M be as usual. For each S_a , let $R_a \supset S_a$ be a subgroup of M of type R . Since any two non-zero subgroups of R have a non-zero intersection, and since $\bigoplus_{a \in A} S_a$ exists, also $\bigoplus_{a \in A} R_a$ exists. Let $x_a \in S_a$; then $(x_a)_{a \in A}$ is a maximal independent subset of H ; and since H contains a maximal independent subset of L , $(x_a)_{a \in A}$ is also a maximal independent subset of L . Thus, we must have $M = T(M) \oplus (\bigoplus_{a \in A} R_a)$. Theorem 3d and the fact that $T(L)$ is $\bigcap_{\mathcal{G}}$ imply that L is $\bigcap_{\mathcal{G}}$.

REMARK. Concerning the last definition given above, it is well known that H contains a maximal independent subset and that if $(x_a)_{a \in A}$ is independent, then $H/\{(x_a)_{a \in A}\}$ is torsion if and only if $(x_a)_{a \in A}$ is also maximal independent. Thus, Corollary 10 may be worded as follows: Assume L is reduced; then L is $\bigcap_{\mathcal{G}}$ if and only if $T(L)$ is $\bigcap_{\mathcal{G}}$ and L contains a subgroup H which has the form $\bigoplus_{a \in A} S_a$ where S_a is isomorphic to a subgroup of R of type $T_{P(G)}$ and such that L/H is torsion.

REMARK. The author wishes to thank the referee, R. S. Pierce, for the present arrangement of the material in this paper, as well as for many changes in the proofs. The author also owes thanks to W. R. Scott.

BIBLIOGRAPHY

1. L. Fuchs, *Abelian Groups*, Publishing House of the Hungarian Academy of Sciences, Budapest, 1958.
2. Kaplansky, Irving, *Infinite Abelian Groups*, University of Michigan Press, Ann Arbor, 1954.
3. A. G. Kurosh, *The Theory of Groups*, Chelsea Publishing Company, New York, N. Y., 1955.

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

RALPH S. PHILLIPS
Stanford University
Stanford, California

F. H. BROWNELL
University of Washington
Seattle 5, Washington

A. L. WHITEMAN
University of Southern California
Los Angeles 7, California

L. J. PAIGE
University of California
Los Angeles 24, California

ASSOCIATE EDITORS

E. F. BECKENBACH
T. M. CHERRY

D. DERRY
M. OHTSUKA

H. L. ROYDEN
E. SPANIER

E. G. STRAUS
F. WOLF

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA
NEW MEXICO STATE UNIVERSITY
OREGON STATE COLLEGE
UNIVERSITY OF OREGON
OSAKA UNIVERSITY
UNIVERSITY OF SOUTHERN CALIFORNIA

STANFORD UNIVERSITY
UNIVERSITY OF TOKYO
UNIVERSITY OF UTAH
WASHINGTON STATE COLLEGE
UNIVERSITY OF WASHINGTON
* * *
AMERICAN MATHEMATICAL SOCIETY
CALIFORNIA RESEARCH CORPORATION
HUGHES AIRCRAFT COMPANY
SPACE TECHNOLOGY LABORATORIES
NAVAL ORDNANCE TEST STATION

Printed in Japan by International Academic Printing Co., Ltd., Tokyo, Japan

Reprinted 1966 in the United States of America

Pacific Journal of Mathematics

Vol. 11, No. 1

November, 1961

A. A. Albert, <i>Generalized twisted fields</i>	1
Richard Arens, <i>Operational calculus of linear relations</i>	9
John Herbert Barrett, <i>Disconjugacy of a self-adjoint differential equation of the fourth order</i>	25
Paul Richard Beesack, <i>Hardy's inequality and its extensions</i>	39
Julius Rubin Blum and David Lee Hanson, <i>On invariant probability measures. II</i>	63
Robert Allen Bonic, <i>Symmetry in group algebras of discrete groups</i>	73
R. Creighton Buck, <i>Multiplication operators</i>	95
Jack Gary Ceder, <i>Some generalizations of metric spaces</i>	105
Meyer Dwass, <i>Random crossings of cumulative distribution functions</i>	127
Albert Edrei, Wolfgang H. J. Fuchs and Simon Hellerstein, <i>Radial distribution and deficiencies of the values of a meromorphic function</i>	135
William Cassidy Fox, <i>Harmonic functions with arbitrary local singularities</i>	153
Theodore Thomas Frankel, <i>Manifolds with positive curvature</i>	165
Avner Friedman, <i>A strong maximum principle for weakly subparabolic functions</i>	175
Watson Bryan Fulks and J. O. Sather, <i>Asymptotics. II. Laplace's method for multiple integrals</i>	185
Adriano Mario Garsia and Eugene Richard Rodemich, <i>An embedding of Riemann surfaces of genus one</i>	193
Irving Leonard Glicksberg, <i>Weak compactness and separate continuity</i>	205
Branko Grünbaum, <i>On a conjecture of H. Hadwiger</i>	215
Frank J. Hahn, <i>On the action of a locally compact group on E_n</i>	221
Magnus R. Hestenes, <i>Relative hermitian matrices</i>	225
G. K. Kalisch, <i>On similarity invariants of certain operators in L_p</i>	247
Yitzhak Katznelson and Walter Rudin, <i>The Stone-Weierstrass property in Banach algebras</i>	253
Samir A. Khabbaz, <i>The subgroups of a divisible group G which can be represented as intersections of divisible subgroups of G</i>	267
Marvin Isadore Knopp, <i>Construction of a class of modular functions and forms</i>	275
Charles Alan McCarthy, <i>Commuting Boolean algebras of projections</i>	295
T. M. MacRobert, <i>Transformations of series of E-functions</i>	309
Heinz Renggli, <i>An inequality for logarithmic capacities</i>	313
M. S. Robertson, <i>Applications of the subordination principle to univalent functions</i>	315
David Sachs, <i>Partition and modulated lattices</i>	325
Frank S. Scalora, <i>Abstract martingale convergence theorems</i>	347
Elbert A. Walker, <i>Torsion endomorphic images of mixed Abelian groups</i>	375
Morgan Ward, <i>The prime divisors of Fibonacci numbers</i>	379
Charles R. B. Wright, <i>On the nilpotency class of a group of exponent four</i>	387