



# KG REDDY

College of Engineering  
& Technology

**Course File On**  
**NETWORK ANALYSIS AND TRANSMISSION LINES**

**By**  
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**2019-2020**

**HOD**  
**ECE**

**Principal**  
**KGR CET**



# **KG REDDY**

College of Engineering  
& Technology

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## **COURSE FILE**

**Subject Name** : Network Analysis and Transmission Lines

**Faculty Name** : A Deepika

**Designation** : Assistant Professor

**Regulation /Course Code** : R16/EC305ES

**Year / Semester** : II / I

**Department** : Electronics and Communication  
**Engineering**

**Academic Year** : 2018-19

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## **VISION, MISSION, PROGRAM EDUCATIONAL OBJECTIVES (PEOs), PROGRAM OUTCOMES (POs), PROGRAM SPECIFIC OUTCOMES (PSOs)**

### **1.**

#### **VISION**

To be recognized as a full-fledged center for learning and research in various fields of Electronics and Communication Engineering through industrial collaboration and provide consultancy for solving the real time problems.

#### **MISSION**

- To inculcate a spirit of research and teach the students about contemporary technologies in Electronics and Communication to meet the growing needs of the industry.
- To enhance the practical knowledge of students by implementing projects based on real time problems through industrial collaboration

## **PROGRAM EDUCATIONAL OBJECTIVES (PEOs)**

- PEO 1:** Apply knowledge and skills to provide solutions to Electrical and Electronics Engineering problems in industry and governmental organizations or to enhance student learning in educational institutions
- PEO 2:** Work as a team with a sense of ethics and professionalism, and communicate effectively to manage cross-cultural and multidisciplinary teams
- PEO 3:** Update their knowledge continuously through lifelong learning that contributes to personal, global and organizational growth

## **PROGRAM OUTCOMES**

- PO 1:Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.
- PO 2: Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural science and engineering sciences.
- PO 3: Design/development of solutions:** design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal and environmental considerations.
- PO 4:Conduct investigations of complex problems:** use research based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
- PO 5:Modern tool usage:** create, select and apply appropriate techniques, resources and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
- PO 6:The engineer and society:** apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
- PO 7:Environment sustainability:** understand the impact of the professional engineering solutions in the societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
- PO 8:Ethics:** apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

- PO 9:Individual and team work:** function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.
- PO 10:Communication:** communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- PO 11:Project management and finance:** demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
- PO 12:Lifelong learning:** recognize the need for, and have the preparation and ability to engage in independent and lifelong learning in the broader context of technological change.

## **PROGRAM SPECIFIC OUTCOMES**

- PSO 1: Problem Solving Skills** – Graduate will be able to apply latest electronics techniques and communications principles for designing of communications systems.
- PSO 2: Professional Skills** – Graduate will be able to develop efficient and effective Communications systems using modern Electronics and Communications engineering techniques.
- PSO 3: Successful Career** – To produce graduates with a solid foundation in Electronics and Communications engineering who will pursue lifelong learning and professional development including post graduation.
- PSO 4: The Engineer and Society**– Ability to apply the acquired knowledge for the advancement of society and self.

## 2. SYLLABUS (UNIVERSITY COPY)

R18 B.TECH ECE

### EC302PC: NETWORK ANALYSIS AND TRANSMISSION LINES

B.Tech. II Year I Sem.

L	T	P	C
3	0	0	3

Pre-Requisites: Nil

#### Course Objectives:

- To understand the basic concepts on RLC circuits.
- To know the behavior of the steady states and transients states in RLC circuits.
- To understand the two port network parameters.
- To study the propagation, reflection and transmission of plane waves in bounded and unbounded media.

Course Outcomes: Upon successful completion of the course, students will be able to:

- Gain the knowledge on basic RLC circuits behavior.
- Analyze the Steady state and transient analysis of RLC Circuits.
- Know the characteristics of two port network parameters.
- Analyze the transmission line parameters and configurations.

#### UNIT - I

Network Topology, Basic cutset and tie set matrices for planar networks, Magnetic Circuits, Self and Mutual inductances, dot convention, Impedance, reactance concept, Impedance transformation and coupled circuits, co-efficient of coupling, equivalent T for Magnetically coupled circuits, Ideal Transformer.



#### **UNIT - II**

Transient and Steady state analysis of RC, RL and RLC Circuits, Sinusoidal, Step and Square responses. RC Circuits as integrator and differentiators. 2<sup>nd</sup> order series and parallel RLC Circuits, Root locus, damping factor, over damped, under damped, critically damped cases, quality factor and bandwidth for series and parallel resonance, resonance curves.

#### **UNIT - III**

Two port network parameters, Z, Y, ABCD, h and g parameters, Characteristic impedance, Image transfer constant, image and iterative impedance, network function, driving point and transfer functions – using transformed (S) variables, Poles and Zeros. Standard T,  $\pi$ , L Sections. Characteristic impedance, image transfer constants, Design of Attenuators, impedance matching network.

#### **UNIT – IV**

Transmission Lines - I: Types, Parameters, Transmission Line Equations, Primary & Secondary Constants, Equivalent Circuit, Characteristic Impedance, Propagation Constant, Phase and Group Velocities, Infinite Line Concepts, Lossless / Low Loss Characterization, Types of Distortion, Condition for Distortion less line, Minimum Attenuation, Loading - Types of Loading.

#### **UNIT – V**

Transmission Lines – II: Input Impedance Relations, SC and OC Lines, Reflection Coefficient, VSWR,  $\lambda/4$ ,  $\lambda/2$ ,  $\lambda/8$  Lines – Impedance Transformations, Smith Chart – Configuration and Applications, Single Stub Matching.

#### **TEXT BOOKS:**

1. Network Analysis – Van Valkenburg, 3<sup>rd</sup> Ed., Pearson, 2016.
2. Networks, Lines and Fields - JD Ryder, PHI, 2<sup>nd</sup> Edition, 1999.

#### **REFERENCE BOOKS:**

1. Electric Circuits – J. Edminister and M. Nahvi – Schaum's Outlines, Mc Graw Hills Education, 1999.
2. Engineering Circuit Analysis – William Hayt and Jack E Kemmerly, MGH, 8<sup>th</sup> Edition, 1993.
3. Electromagnetics with Applications – JD. Kraus, 5<sup>th</sup> Ed., TMH
4. Transmission Lines and Networks – Umesh Sinha, Satya Prakashan, 2001, (Tech. India Publications), New Delhi.

### **3. COURSE OBJECTIVES AND COURSE OUTCOMES, TOPIC OUTCOMES**

#### **Course Objectives:**

- Define Magnetic circuits and demonstrate graph theory for circuits.
- Classify the behavior of the steady states and transients states in RLC circuits.
- Discuss the two port network parameters.
- Explain the propagation, reflection and transmission of plane waves in bounded and unbounded media.
- Demonstrate the smith chart-configuration

#### **Course Outcomes:**

Upon successful completion of the course, students will be able to:

- Compare circuit matrices of linear graphs and describe magnetic circuits.
- Examine the Steady state and transient analysis of RLC Circuits.
- Describe the characteristics of two port network parameters.
- Relate the transmission line parameters and configurations.
- Integrate the wave propagation through transmission lines and compute the smith chart and impedance matching the device.

**(c)TOPIC OUTCOMES**

L. No	TOPIC	TOPIC OUTCOMES
<i>At the end of the topic, the student will be able to</i>		
<b>UNIT – I</b>		
L1	Review of R,L,C,RL,RC,RLC Circuits	Recollect the Basic Parameters of electrical Circuits.
L2	Network Topology	Draw the Electrical Network Solving Technique in graphical Methods.
L3	Network Topology	Draw the Electrical Network Solving Technique in graphical Methods.
L4	Terminology	Define Electrical Network Terminologies
L5	Basic cutest Matrices for planar Networks	Define Electrical Network Basic cutest Matrices for planar Networks
L6	Basic Tieset Matrices for planar Networks	Define Electrical Network Basic Tieset Matrices for planar Networks
L7	Illustrative Problems	Applying the Learnt Technique for Solving Network Problems
L8	Magnetic circuits	List the Magnetic Circuits Basics
L9	Magnetic circuits	List the Magnetic Circuits Basics
<b>Gaps in the syllabus</b>		
L10	Gap : Faradays law of Electromagnetic Induction	Repeat Faradays law of Electromagnetic Induction
L11	Self Inductance	Describe Self Inductance
L12	Mutual Inductance	Describe Mutual Inductance
L13	Dot convention	Discuss Dot convention
L14	Impedance	Analyze the Circuit new Concepts
L15	Reactance Concept	Discuss Reactance Concept
L16	Impedance Transformation and coupled circuits	Describe Impedance Transformation and coupled circuits
L17	Co-efficient Of Coupling	State Co-efficient Of Coupling
L18	Equivalent T for Magnetically Coupled Circuits	Identify Equivalent T for Magnetically Coupled Circuits
L19	Ideal Transformer	Define Ideal Transformer
<b>UNIT - II</b>		
L20	Steady State And Transient Response	Analyze the different circuit Combinations and their Responses
L21	DC Response of an R-L circuit	Explain DC Response of an R-L circuit,

	DC Response of an R-C circuit	DC Response of an R-C circuit
L22	DC Response of an R-L-C circuit	Explain DC Response of an R-L-C circuit
L23	Sinusoidal Response of R-L circuit Sinusoidal Response of R-C circuit	Explain Sinusoidal Response of R-L circuit Sinusoidal Response of R-C circuit
L24	Sinusoidal Response of R-L-C circuit	Explain Sinusoidal Response of R-L-C circuit
L25	Step Response of R-C circuit, R-L circuit, R-L-C circuit	Explain Step Response of R-C circuit, R-L circuit, R-L-C circuit
L26	Square Response of R-L,R-C, R-L-C circuits.	Explain Square Response of R-L,R-C, R-L-C circuits
L27	RC Circuits as integrator and differentiators	Recognize RC Circuits as integrator and differentiators
L28	2 <sup>nd</sup> Order Series and Parallel RLC Circuits	Solve the different circuit Combinations of 2 <sup>nd</sup> Order Series and Parallel RLC Circuits
L29	Root Locus	Draw the Root Locus Path of the Circuit Parameters
L30	Damping Factor	Define the Importance of the Damping factor
L31	Over Damped, Under damped, Critically damped cases	Define the Importance of the Damping factor with different Values.
L32	Quality Factor And Bandwidth for Series And Parallel Resonance	Define the Quality Factor of the Coil, Electrical Resonance Condition in Circuits
L33	Quality Factor And Bandwidth for Series And Parallel Resonance	Define the Quality Factor of the Coil, Electrical Resonance Condition in Circuits
L34	Resonance Curves	Draw and Analyze Electrical Resonance Condition in Circuits
<b>Unit III</b>		
L35	Two port Network Parameters Z,Y Parameters	Discuss the Network Parameters Representation in terms of Z,Y
L36	Two port Network Parameters ABCD, h and g Parameters	Discuss the Network Parameters Representation in terms of ABCD, h and g
L37	Characteristic Impedance	Define the characteristic impedance and its importance
L38	Image Transfer Constant	Draw the Image Transfer Constant
L39	Image And Iterative Impedance	Calculate Image And Iterative Impedance and define its importance
L40	Network Function	Formulation of Network function
L41	Driving Points and Transfer functions –using Transformed (S)Variables	Define the Driving Points and Transfer functions –using Transformed (S)
L42	Poles and Zeros	Define the Concept of Poles and Zeroes and

		their importance in the electrical networks
L43	Standard T, $\pi$ , L Sections	Discuss Standard T, $\pi$ , L Sections
L44	Characteristic impedance	Classify Characteristic impedance
L45	Image transfer constants	Discuss Image transfer constants
L46	Design of Attenuators, impedance matching network.	Test Attenuators, Discuss impedance matching network
<b>UNIT-IV</b>	<b>Transmission Lines-I:</b>	
L47	Types, parameters	Define the transmission Lines types, parameters
L48	Transmission line equations, Primary and secondary constants	Formulate the transmission line equations, primary and secondary constants
L49	expressions for characteristics impedance	Derive the expressions for characteristics impedance the propagation constant phase and group velocities
L50	Propagation constant, Phase and group velocities	
L51	Infinite line concepts, lossless/low loss characterization	Apply the Infinite line concepts, lossless/low loss characterization
L52	Transmission Lines-I	
L53	Distortion-condition for distortion lessness and minimum attenuation	Describes the distortion-condition for distortion less and min attenuation
L54	Loading –types of loading	Apply the loading –types of loading
L55	Illustrative problems	Solve the illustrative problems
L56	Loading	Discuss Loading
<b>UNIT-V</b>	<b>Transmission Lines-II:</b>	
L57	Input impedance relations	Derive the input impedance relations
L58	SC and OC lines, Reflection coefficient & VSWR	Formulate the SC, OC lines reflection coefficient and VSWR
L59	, UHF lines as circuit elements: $\lambda/4$ , $\lambda/2$ , $\lambda/8$ lines	Explain the UHF lines as circuit elements: $\lambda/4$ , $\lambda/2$ , $\lambda/8$ lines
L60	Impedance transformation, Signification of $Z_{\min}$ and $Z_{\max}$	Describe the impedance transformation and formulate the Signification of $Z_{\min}$ and $Z_{\max}$
L61	Transmission Lines-II	
L62	Smith chart-configuration	Define the smith chart-configuration
L63	Smith chart application	Apply the Smith chart application
L64	Single stub matching	Compute the single matching
L65	Illustrative problems	Solve the illustrative problems
L66	Smith chart	Organize smith chart
<b>Topics beyond syllabus/advanced topics</b>		
L67	Micro waves	<b>Define</b> micro waves
L68	Electromagnetic interference	<b>Explain</b> electromagnetic interference

	and compatibility	and compatibility
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#### 4. COURSE PREREQUISITES

- Basic Electrical and Electronics engineering
- Vectors
- Co-ordinates systems
- Vector calculus

#### 5) CO'S, PO'S MAPPING:

CO&PO Mappings

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	1	3	-	-	-	-	-	-	-	-	-	-
CO2	1	2	-	-	-	3	-	-	-	-	-	-
CO3	1	3	2	-	-	-	-	-	-	-	-	-
CO4	-	2	-	-	-	-	-	-	-	-	-	1
CO5	3	1	2	-	-	-	-	-	-	-	-	-

1. Low 2. Moderate or medium 3. Substantial or high

## 6. COURSE INFORMATION SHEET (CIS)

### (a) Course description

PROGRAMME: <b>B. Tech.</b>  (Electronics and Communication Engineering)	DEGREE: <b>B.TECH</b>
COURSE: <b>NETWORK ANALYSIS AND TRANSMISSION LINES</b>	YEAR: <b>II</b> SEM: <b>I</b> CREDITS: <b>3</b>
COURSE CODE: <b>EC302PC</b>  REGULATION: <b>R18</b>	COURSE TYPE: <b>CORE</b>
COURSE AREA/DOMAIN: <b>Electrical</b>	CONTACT HOURS: 3 (L) hours/Week.

### (b) Syllabus

UNIT	DETAILS	CLASSES
<b>I</b>	Review of R, L, C, RC, RL, RLC circuits, Network Topology, Terminology, Basic cutset and tie set matrices for planar networks, Illustrative Problems, Magnetic Circuits, Self and Mutual inductances, dot convention, impedance, reactance concept, Impedance transformation and coupled circuits, co-efficient of coupling, equivalent T for Magnetically coupled circuits, Ideal Transformer.	18
<b>II</b>	Steady state and transient analysis of RC, RL and RLC Circuits, Circuits with switches, step response, 2 <sup>nd</sup> order series and parallel RLC Circuits, Root locus, damping factor, over damped, under damped, critically damped cases, quality factor and bandwidth for series and parallel resonance, resonance curves	15

<b>III</b>	Two port network parameters, Z, Y, ABCD, h and g parameters, Characteristic impedance, Image transfer constant, image and iterative impedance, network function, driving point and transfer functions – using transformed (S) variables, Poles and Zeros.	12
<b>IV</b>	<b>Transmission Lines - I:</b> Types, Parameters, Transmission Line Equations, Primary & Secondary Constants, Expressions for Characteristic Impedance, Propagation Constant, Phase and Group Velocities, Infinite Line Concepts, Loss lessness/Low Loss Characterization, Distortion – Condition for Distortion lessness and Minimum Attenuation, Loading - Types of Loading, Illustrative Problems.	10
<b>V</b>	<b>Transmission Lines – II:</b> Input Impedance Relations, SC and OC Lines, Reflection Coefficient, VSWR. UHF Lines as Circuit Elements; $\lambda/4$ , $\lambda/2$ , $\lambda/8$ Lines – Impedance Transformations, Significance of $Z_{min}$ and $Z_{max}$ , Smith Chart – Configuration and applications, Single Matching, Illustrative Problems.	10
<b>Contact classes for syllabus coverage</b>		<b>65</b>
<b>Lectures beyond syllabus</b>		<b>02</b>
<b>Classes for gaps &amp; Add-on classes</b>		<b>01</b>
<b>Total No. of classes</b>		<b>68</b>

**(c) Gaps in syllabus**

S.N	Topic	Propose Action	No. of classes
1	Faradays law of Electromagnetic Induction	PPT	1

**(d) Topics beyond Syllabus**

S.N.	Topic	Propose Action	No. of Classes
1	Micro waves	PPT	1 period
2	Electromagnetic interference and compatibility	NPTEL	1 periods



**(e) Web Source References**

S. No	Name of book/ website
1.	<a href="http://www.faadooengineers.com/threads/31262-Network-Analysis-pdf-download-ebook">http://www.faadooengineers.com/threads/31262-Network-Analysis-pdf-download-ebook</a>
2.	<a href="http://nptel.ac.in/downloads/108105053/">http://nptel.ac.in/downloads/108105053/</a>
3	<a href="https://onlinecourses.nptel.ac.in/noc18_ee04/preview">https://onlinecourses.nptel.ac.in/noc18_ee04/preview</a>
4	<a href="https://www.youtube.com/watch?v=pGdr9WLto4A&amp;list=PLl6m4jcR_DbOx6s2toprJQx1MORqPa9rG">https://www.youtube.com/watch?v=pGdr9WLto4A&amp;list=PLl6m4jcR_DbOx6s2toprJQx1MORqPa9rG</a>

**(f) Delivery / Instructional Methodologies:**

<input checked="" type="checkbox"/> CHALK & TALK	<input checked="" type="checkbox"/> STUD. ASSIGNMENT	<input checked="" type="checkbox"/> WEB RESOURCES
<input checked="" type="checkbox"/> LCD/SMART BOARDS	<input checked="" type="checkbox"/> STUD. SEMINARS	<input type="checkbox"/> ADD-ON COURSES

**(g) Assessment Methodologies - Direct**

<input checked="" type="checkbox"/> Assignments	<input checked="" type="checkbox"/> Stud. Seminars	<input checked="" type="checkbox"/> Tests/Model Exams	<input checked="" type="checkbox"/> Univ. Examination
<input checked="" type="checkbox"/> Stud. Lab Practices	<input checked="" type="checkbox"/> Stud. Viva	<input type="checkbox"/> Mini/Major Projects	<input type="checkbox"/> Certifications
<input type="checkbox"/> Add-On	<input type="checkbox"/> Others		

Courses			
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**(h) Assessment Methodologies - Indirect**

<input checked="" type="checkbox"/> Assessment Of Course Outcomes (By Feedback, Once)	<input checked="" type="checkbox"/> Student Feedback On Faculty (Twice)
<input type="checkbox"/> Assessment Of Mini/Major Projects By Ext. Experts	<input type="checkbox"/> Others

**(i) Text books and References**

<b>Text Books</b>	
1.	Network Analysis – ME Van Valkenburg, Prentice Hall of India, 3 <sup>rd</sup> Edition, 2000.
2.	Networks, Lines and Fields - JD Ryder, PHI, 2 <sup>nd</sup> Edition, 1999.
<b>Suggested / Reference Books</b>	
1.	Engineering Circuit Analysis – William Hayt and Jack E Kemmerly, MGH, 5 <sup>th</sup> Edition, 1993.
2.	Electric Circuits – J. Edminister and M.Nahvi – Schaum’s Outlines, MCGRAW HILL EDUCATION, 1999.
3.	Electromagnetics with Applications – JD. Kraus, 5th Ed., TMH

4.	Transmission Lines and Networks – Umesh Sinha, Satya Prakashan, 2001, (Tech. India Publications), New Delhi.
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## 7. MICRO LESSON PLAN

<b>UNIT-I</b>			
<b>L.No.</b>	<b>Topic</b>	<b>Scheduled date</b>	<b>Actual date</b>
<b>UNIT-I</b>			
1	Review of R,L,C,RL,RC,RLC Circuits		
2	Network Topology		
3	Network Topology		
4	Terminology		
5	Basic cutset Matrices for planar Networks		
6	Basic Tieset Matrices for planar Networks		
7	Illustrative Problems		

8	Magnetic circuits		
9	Magnetic circuits		
10	Gap : Faradays law of Electromagnetic Induction		
11	Self Inductance		
12	Mutual Inductance		
13	Dot convention		
14	Impedance		
15	Reactance Concept		
16	Impedance Transformation and coupled circuits		
17	Co-efficient Of Coupling		
18	Equivalent T for Magnetically Coupled Circuits		
19	Ideal Transformer		
<b>UNIT-II</b>			
20	Steady State And Transient Response		
21	DC Response of an R-L circuit DC Response of an R-C circuit		
22	DC Response of an R-L-C circuit		
23	Sinusoidal Response of R-L circuit Sinusoidal Response of R-C circuit		
24	Sinusoidal Response of R-L-C circuit		
25	Step Response of R-C circuit, R-L circuit, R-L-C circuit		
26	Square Response of R-L,R-C, R-L-C circuits.		
27	RC Circuits as integrator and differentiators		
28	2 <sup>nd</sup> Order Series and Parallel RLC Circuits		
29	Root Locus		
30	Damping Factor		
31	Over Damped, Under damped, Critically damped cases		
32	Quality Factor And Bandwidth for Series And Parallel Resonance		
33	Quality Factor And Bandwidth for Series And Parallel		

	Resonance		
34	Resonance Curves		
<b>UNIT-III</b>			
35	Two port Network Parameters Z,Y Parameters		
36	Two port Network Parameters ABCD, h and g Parameters		
37	Characteristic Impedance		
38	Image Transfer Constant		
39	Image And Iterative Impedance		
40	Network Function		
41	Driving Points and Transfer functions –using Transformed (S)Variables		
42	Poles and Zeros		
43	Standard T, $\pi$ , L Sections		
44	Characteristic impedance		
45	Image transfer constants		
46	Design of Attenuators, impedance matching network.		
<b>UNIT-IV</b>			
47	Types, parameters		
48	Transmission line equations, Primary and secondary constants		
49	expressions for characteristics impedance		
50	Propagation constant, Phase and group velocities		
51	Infinite line concepts, lossless/low loss characterization		
52	Transmission Lines-I		
53	Distortion-condition for distortion less and minimum attenuation		
54	Loading –types of loading		
55	Illustrative problems		
56	Loading		
<b>UNIT-V</b>			

57	Input impedance relations		
58	SC and OC lines, Reflection coefficient & VSWR		
59	UHF lines as circuit elements: $\lambda/4$ , $\lambda/2$ , $\lambda/8$ lines		
60	Impedance transformation, Signification of $Z_{\min}$ and $Z_{\max}$		
61	Transmission Lines-II		
62	Smith chart-configuration		
63	Smith chart application		
64	Single stub matching		
65	Illustrative problems		
66	Smith chart		
67	Micro waves		
68	Electromagnetic interference and compatibility		

## 8) Teaching Schedule

Subject	Network Analysis					
<b>Text Books (to be purchased by the Students)</b>						
<b>Book 1</b>	Network Analysis-ME Van Valkenburg, Prentice Hall of India, 3 <sup>rd</sup> Edition 2000					
<b>Book 2</b>	Networks, Lines and Fields-JD Ryder, PHI, 2 <sup>nd</sup> Edition, 1999					
<b>Reference Books</b>						
<b>Book 3</b>	Electric Circuits-J. Edminister and M. Nahvi-Schaum's Outlines, MCGRAW HILL EDUCATION, 1999					
<b>Book 4</b>	Network Theory-Sudarshan and Shyam Mohan, McGraw Hill Education					
<b>Book 5</b>	Engineering Electromagnetics – William H. Hayt Jr. and John A. Buck, 7th Ed., 2006, McGraw Hill Education					
Unit	Topic	Chapters Nos				No of
		Book 1	Book 2	Book 3	Book	

					<b>4</b>	<b>class es</b>
<b>I</b>	Circuit Elements and Energy Sources	1	1		1	3
	Introduction to Graph Theory	1	3	2		6
	Analysis of Coupled Circuits	1	2		1	4
<b>II</b>	Transient Response Of Passive Circuits	2	2	2	2	8
	Laplace Transformation and its Application In circuit Analysis	1	2		3	6
	Root Locus	2		1	2	5
<b>III</b>	Two Port Network Analysis	2	3		2	7
<b>IV</b>	<b>Transmission Lines - I:</b>	10	7	12	11	8
<b>V</b>	<b>Transmission Lines - II:</b>	10	7	13,14	11	8
<b>Contact classes for syllabus coverage</b>					<b>50</b>	
<b>Tutorial classes</b>					<b>10</b>	
<b>Classes for gaps, Add-on classes and Lectures beyond syllabus</b>					<b>02</b>	
<b>Total No. of classes</b>					<b>62</b>	

9. Unit-wise Hand written notes (Hard copy)



**10. OHP/LCD SHEETS /CDS/DVDS/PPT (SOFT/HARD COPIES)**

## 11. University Previous Question papers

R16

Code No: 133BJ

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**

**B.Tech II Year I Semester Examinations, November/December - 2017**

**NETWORK ANALYSIS**

(Electronics and Communication Engineering)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.  
Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A**

(25 Marks)

- 1.a) Define Graph, Tree, Basic Cut set and Basic Tie set. Illustrate with an example. [2]
- b) Explain Active elements in detail. [3]
- c) Derive the relation between voltage and current in a series connected RL Circuits. [2]
- d) Draw a power triangle in series connected RLC networks. [3]
- e) Derive the relation between RMS and maximum value. [2]
- f) Define form factor and peak factor. [3]
- g) Define characteristic impedance. [2]
- h) Define image and iterative impedance. [3]
- i) Draw and explain T section network. [2]
- j) Explain about LC Filters. [3]

**PART-B**

(50 Marks)

- 2.a) What is an electric circuit? What is a magnetic circuit? Make a comparison between electric circuit and magnetic circuit.
- b) Coil 1 of a pair of coupled coils has a continuous current of 5A, and the corresponding fluxes  $\phi_{11}$  and  $\phi_{12}$  are 0.2 and 0.4 mWb respectively. If the turns are  $N_1 = 500$  and  $N_2 = 1500$ , find  $L_1$ ,  $L_2$ ,  $M$  and  $k$ . [5+5]

OR

- 3.a) For the network shown in below Figure-1 find  $Z_{ab}$  and  $I_{\phi}$ .

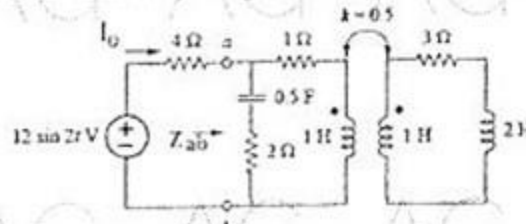
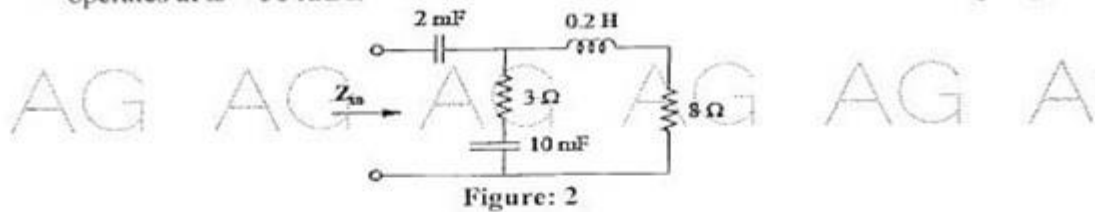


Figure: 1



- b) Find the input impedance of the circuit shown in Figure 2. Assume that the circuit operates at  $\omega = 50 \text{ rad/s}$ . [5+5]



- 4.a) Obtain the current locus of a fixed resistance and a variable capacitance.  
b) Given a series RLC circuit with  $R = 10 \text{ ohms}$ ,  $L = 1 \text{ mH}$  and  $C = 1 \text{ }\mu\text{F}$  is connected across a sinusoidal source of 20 V with variable frequency. Find: i) The resonant frequency ii) Q factor of the circuit at resonant frequency iii) Half power frequencies [5+5]
- OR**
- 5.a) Derive and draw the response of a series RLC circuit for step input.  
b) An impedance  $Z_1 = 10 + j10 \text{ }\Omega$  is connected in parallel with another impedance of resistance  $8.5 \text{ }\Omega$  and a variable capacitance connected in series. Find C such that the circuit is in resonance at 5 KHz. [5+5]
6. A series-connected RLC circuit has  $R = 4$  and  $L = 25 \text{ mH}$ :  
a) Calculate the value of C that will produce a quality factor of 50.  
b) Find  $\omega_1$ ,  $\omega_2$ , and B.  
c) Determine the average power dissipated at  $\omega = \omega_0$ ,  $\omega_1$ ,  $\omega_2$ . Take  $V_m = 100 \text{ V}$ . [3+3+4]
- OR**
- 7.a) Obtain the current locus of a series circuit having a fixed resistance and a variable inductance.  
b) Given a series RLC circuit with  $R = 100 \text{ ohms}$ ,  $L = 0.5 \text{ H}$  and  $C = 40 \text{ }\mu\text{F}$ , Calculate the resonant, lower and upper half – power frequencies. [5+5]
8. Explain clearly the terms:  
a) Characteristic Impedance and  
b) Image Transfer Constant. [5+5]
- OR**
- 9.a) Define Hybrid parameters of a Two Port network. Establish the relation between Hybrid Parameters and ABCD Parameters.  
b) A symmetrical T-section has an inductance of 0.47H in each series arm and a  $300 \text{ }\mu\text{F}$  capacitor in the shunt arm.  
i) Find the characteristic impedance at frequencies of 50 Hz and 100 Hz.  
ii) If the T-section is terminated in the characteristic impedance, find the ratio of load current to input current at both the frequencies. [5+5]



- 10.a) What is a high pass filter? In what respects it is different from a low pass filter?  
b) Derive the equations to find the inductances and capacitances of a constant K high pass filter. [5+5]

OR

- 11.a) What is an LC immittance function? State the properties of such functions.  
b) Design a constant 'K' T-section low pass filter having cutoff frequency of 2 kHz and nominal characteristic impedance of 600 ohms. [5+5]

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**R13**

Code No: 114CU

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year II Semester Examinations, May - 2016

ELECTROMAGNETIC THEORY AND TRANSMISSION LINES

(Common to ECE, ETM)

Time: 3 Hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART - A**

**(25 Marks)**

- 1.a) How can materials be classified in terms of their conductivity? [2]
- b) Give an expression for convection current density. Also state the point form of Ohm's Law. [3]
- c) State Maxwell's equations for a lossless or non conducting medium. [2]
- d) State the Ampere's Force Law. Give magnetic force for arbitrary geometrics. [3]
- e) Give an expression for intrinsic impedance in phasor form. What are its magnitude and phase components? [2]
- f) Explain in brief significance of loss tangent. [3]
- g) List any four types of transmission lines. [2]
- h) How does group velocity vary when compared to phase velocity? [3]
- i) What are the two families of circles that constitute the Smith Chart? [2]
- j) What are the advantages and disadvantages of a Single Stub? [3]

**PART - B**

**(50 Marks)**

- 2.a) State Coulomb's Law. Find the force on charge  $Q_1$ ,  $30 \mu\text{C}$  due to a charge  $Q_2$ ,  $-200 \mu\text{C}$ , where  $Q_1$  is at  $(0,0,2)$  m and  $Q_2$  is at  $(2,1,0)$  m.
- b) Derive the relation between electric field,  $E$  and Scalar potential,  $V$ . Find the electric field at  $(2,3,1)$  if the potential distribution is of the form  $3x^2y + y^2x + 3z$ . [5+5]

**OR**

- 3.a) Discuss the Maxwell's equations for electrostatic fields.
- b) Obtain the expression of Gauss's Law for infinite surface charge. Also state any two limitations of Gauss's Law. [5+5]
- 4.a) State the important properties of magnetic lines of forces.
- b) Show that the magnetic field due to a finite current element along z-axis at a point P "r" distance away from y-axis is given by  $\vec{H} = \frac{1}{4\pi r} (\sin \alpha_1 - \sin \alpha_2) a\phi$ , where "I" is the current through the conductor,  $\alpha_1, \alpha_2$  are the angles made by the tips of the conductor element at P. [5+5]

**OR**

- 5.a) What are boundary conditions? State the boundary conditions at the interface of dielectric and perfect conductor.  
b) A certain material has  $\sigma = 0$  and  $\epsilon_r = 1$ , if  $\vec{H} = 4 \sin(10^6 t - 0.01z) \vec{a}_y$  A/m. Use Maxwell's equations to find  $\mu_r$ . [5+5]
- 6.a) Derive the relation between E and H in a Uniform plane wave.  
b) What are the wave equations for a lossless medium and a conducting medium for sinusoidal variations? [5+5]
- OR**
- 7.a) Write short notes on normal incidence of a plane wave on a perfect dielectric.  
b) A plane wave travelling in air is normally incident on a material with  $\epsilon_r = 4$  and  $\mu_r = 1$ . Find the reflection and transmission coefficients. [5+5]
- 8.a) Derive the expression for voltage and current at any point on the transmission line in terms of characteristics impedance.  
b) Discuss the parameters that characterize a lossless and lowloss transmission line. [5+5]
- OR**
- 9.a) What is distortion? State the conditions that characterize a distortion less line.  
b) The propagation constant of a lossy transmission line is  $(1+j2)m^{-1}$  and its characteristic impedance is  $20 \Omega$  at  $\omega = 1M$  rad/s. Find L, C, R and G for the line. [5+5]
- 10.a) What are the applications of transmission lines?  
b) How can ultra high frequency transmission lines be used as circuit Elements? [5+5]
- OR**
- 11.a) What are the applications of Smit Chart.  
b) One end of a lossless transmission line having the characteristic impedance of  $75 \Omega$  and length of 1 cm is short circuited. At 3 GHz, What is the input impedance at the other end of the transmission line? [5+5]

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**R13**

Code No: 114CU

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**  
**B.Tech II Year II Semester Examinations, October/November - 2016**  
**ELECTROMAGNETIC THEORY AND TRANSMISSION LINES**  
(Common to ECE, ETM)

Time: 3 Hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.  
Part A is compulsory which carries 25 marks. Answer all questions in Part A.  
Part B consists of 5 Units. Answer any one full question from each unit.  
Each question carries 10 marks and may have a, b, c as sub questions.

**PART - A (25 Marks)**

- 1.a) State Divergence theorem and Stokes theorem. [2]
- b) Mention the differences between scalar and vector magnetic potentials. [3]
- c) If the flux flowing through closed surface is  $3nc$ . What is the total charge enclosed by that surface? [2]
- d) Find the input impedance of a section of a  $50\Omega$  lossless transmission line that of length  $0.1\lambda$  long and is terminated in a short circuit. [3]
- e) Define reflection coefficient and VSWR. [2]
- f) Derive expression for electrostatic energy of a capacitor. [3]
- g) State Maxwell's four laws in derivative form. [2]
- h) Find skin depth at 1GHz for copper having conductivity  $5.7 \times 10^7$  mho/m. [3]
- i) What is stub matching? Draw typical stub matching transmission line. [2]
- j) List the applications of smith chart. [3]

**PART - B (50 Marks)**

- 2.a) Derive Poisson's and Laplace's equations from fundamentals. List few of its applications concerned to electrostatic fields.
  - b) An infinitely long uniform line charge is located at  $y = 3, z = 5$ . If  $\rho_l = 30nc/m$ , find field  $\vec{E}$  intensity at (i) origin (ii) P(5,6,1) [5+5]
- OR**
- 3.a) Develop an expression for potential due to dipoles.
  - b) Evaluate the electric field intensity at a point P (-5, 7, -4) in free space due to a charge of 0.2 mille coulombs placed at point R (2,-1,-2). [5+5]
- 4.a) Distinguish between conduction and convection currents.
  - b) Find the polarization 'P' in a dielectric material with  $\epsilon_r = 2.8$  if  $D = 3.0 \times 10^{-7} \hat{a}_n \text{ C/m}^2$
  - c) Derive the boundary conditions at the interface between  
(i) Dielectric-Dielectric (ii) Dielectric-conductor [3+3+4]
- OR**
- 5.a) Derive Maxwell's equations in integral form. Based on this obtain the corresponding differential equation by applying Stoke's theorem.
  - b) Compare boundary conditions in Electrostatics and Magnetostatics. [5+5]



- 6.a) Evaluate the reflection and transmission coefficients for the case of an electromagnetic wave in air incident normally upon the copper sheet at frequency of 1 MHz. Given  $\mu_1 = \mu = \mu_0$ ,  $\epsilon_1 = \epsilon_2 = \epsilon_0$ ,  $\sigma_1 = 0$ ,  $\sigma_2 = 5.8 \times 10^7$  v/m.
- b) Find the energy stored in a standing wave incident normally on a perfect conductor over a distance  $-\lambda/4$  to 0 per unit in  $x, y$  coordinates. [5+5]

**OR**

- 7.a) State and prove Poynting theorem.
- b) Derive the equation in conducting medium. Discuss skin effect and find the skin depth at 1 GHz for copper having conductivity  $5.7 \times 10^7$  mho/m. [5+5]
- 8.a) Discuss in brief about inductance loading of telephone cables.
- b) A lossless transmission line of length  $0.434 \lambda$  and characteristic impedance  $100 \Omega$  is terminated in an impedance  $260 + j 180 \Omega$ . Find  
(i) Voltage reflection co-efficient  
(ii) Standing wave ratio  
(iii) Input Impedance [5+5]

**OR**

- 9.a) The attenuation constant on a 50 ohm distortionless transmission line is 0.01 dB/m. The line has a capacitance of 0.1 nF/m. Find the resistance, inductance and conductance per meter of the line.
- b) A loss less of 100 ohms is terminated by a load which produces SWR = 3. The first maximum is found to be occurring at 320 cm. If  $f = 300$  MHz determine the load matching. [5+5]
- 10.a) Write a short notes on reflection losses on unmatched transmission line.
- b) The input impedance of a short-circuited lossy transmission line of length 2m and characteristic impedance  $75 \Omega$  is  $45 + j 225 \Omega$ .  
(i) Find  $\alpha$  and  $\beta$  of the line.  
(ii) Determine the input impedance if the short-circuit is replaced by a  $Z_L = 67.5 - j4.5 \Omega$  [5+5]



**OR**

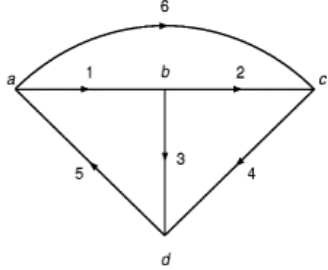
- 11.a) Explain the basis for construction of Smith chart. Illustrate as to how it can be used of an Admittance chart.
- b) A line having  $Z_0$  of 100 ohms is terminated into a load of  $50 - j50$  ohms. It is desired to provide matching between the line and the load by means of a short circuit stub. Determine the length of the stub if signal frequency is 10 MHz.. [5+5]



<b>Q.NO</b>	<b>Question</b>	<b>Bloom's Taxonomy Level</b>	<b>Course Outcome</b>  <b>Accredited by NAAC</b>
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**MID exam Descriptive Question Papers**

 <p><b>KG REDDY</b> College of Engineering &amp; Technology</p>	<p><b>KG Reddy College of Engineering &amp; Technology</b></p> <p>(Approved by AICTE, New Delhi, Affiliated to <b>JNTUH, Hyderabad</b>)</p> <p><b>Chilkur (Village), Moinabad (Mandal), R. R Dist, TS-501504</b></p>		 <p>Accredited by NAAC</p>	<p><b>College Code</b></p> <p><b>QM</b></p>
<p><b>Name of the Exam:</b></p>	<p><b>I Mid Examinations</b></p>		<p><b>Marks:</b></p>	<p><b>10</b></p>
<p><b>Year-Sem &amp; Branch:</b></p>	<p><b>II-I &amp; ECE</b></p>	<p><b>Duration:</b></p>	<p><b>60 Min</b></p>	
<p><b>Subject:</b></p>	<p><b>Network Analysis and Transmission Lines</b></p>	<p><b>Date &amp; Session</b></p>		
<p><b>Answer ANY TWO of the following Questions</b></p>			<p><b>2X5=10</b></p>	

1	<p><b>Define Network Topology. For the given oriented graph, According to the rules write the incidence matrix, tieset matrix and cut set matrix.</b></p> 	Apply	CO1
2	<p><b>Define a) Magnetic Flux Density b) Magneto Motive Force</b></p> <p><b>c) Reluctance d) Find the Relationship between mmf, Flux and Reluctance.</b></p>	Remember & Apply	CO1
3	<p><b>Obtain Equivalent T for the Magnetically Coupled Circuit.</b></p>	Analysis	CO1
4	<p><b>Derive Transient Analysis of RL Circuit for A.C Excitation</b></p>	Apply	CO2

 <p><b>KG Reddy College of Engineering &amp; Technology</b> (Approved by AICTE, New Delhi, Affiliated to JNTUH, Hyderabad)</p> <p>Chilkur (Village), Moinabad (Mandal), R. R Dist, TS-501504</p>		 <p>Accredited by NAAC</p>		<p><b>College Code</b></p>	
				<p><b>QM</b></p>	
<p><b>Name of the Exam:</b></p>		<p><b>II Mid Examinations</b></p>		<p><b>Marks: 10</b></p>	
<p><b>Year-Sem &amp; Branch:</b></p>		<p><b>II-I &amp; ECE</b></p>		<p><b>Duration: 60 Min</b></p>	
<p><b>Subject:</b></p>		<p><b>Network Analysis and Transmission Lines</b></p>		<p><b>Date &amp; Session</b></p>	
<p><b>Answer ANY TWO of the following Questions</b></p>				<p><b>2X5=10</b></p>	

Q.NO	Question	Bloom's Taxonomy Level	Course Outcome
1	Express Z-parameters in terms of h-parameters and ABCD parameters.	Apply	CO3
2	Derive the condition for a two port network to be symmetrical in terms of ABCD parameters.	Apply	CO3
3	Derive the expression for characteristic impedance and propagation constant	Apply	CO4
4	From the fundamental voltage and current equations of transmission line, derive expression for input impedance $Z_{in}$ of the line.	Apply	CO5

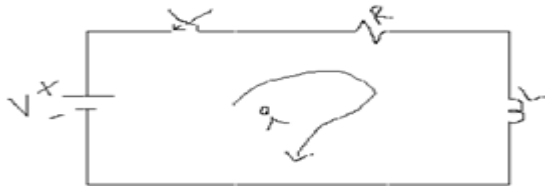


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### Mid Examination Answers

1. Draw the time response of inductor current in a series RL circuit excited by DC supply.

#### DC Response of an R-L Circuit:



Consider a circuit consisting of a resistance and inductance as shown in fig. the inductor in the circuit is initially uncharged and is in series with the resistor. When switch S is closed, we can find the complete solution for current. Application of Kirchhoff's law to the circuit results in following differential equations.

$$V = iR + L \frac{di}{dt} \text{ ----- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} I = \frac{V}{L} \text{ ----- (2)}$$

In the above equation, the current  $i$  is the solution to be found and  $V$  is the applied constant voltage. The voltage  $V$  is applied to the circuit only when the switch  $S$  is closed. The above equation is linear differential equation of the first order comparing with the non homogenous differential equation

$$\frac{dx}{dt} + P X = K \text{ whose solution is } x = e^{-pt} \int K e^{+pt} dt + c e^{-pt}$$

Where  $c$  is an arbitrary constant,

in similar way we can write the current equation as

$$i = c e^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \int \frac{V}{L} e^{\left(\frac{R}{L}\right)t} dt$$

$$i = c e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R} .$$

To determine the value of 'c', in equation (5) we use initial conditions. In the circuit shown in fig the switch  $S$  is closed at  $t=0$ . At  $t=0^-$ , i.e. just before closing the switch  $S$ , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at  $t=0^+$  just after the switch is closed, the current remains zero.

Substituting above conditions we get,

$$0 = c + (V/R)$$

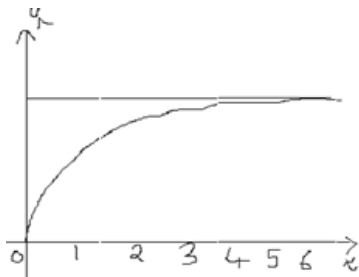
Therefore,  $c = -V/R$

Hence from equation

$$i = -\frac{V}{R} e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$$

$$i = \frac{V}{R} \left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$$

Above equation consists of two parts, the steady state part ( $V/R$ ) and other is transient part.



$$\tau = \frac{L}{R} \text{ seconds}$$

The transient part of solution is,  $i(\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}}$

At time constant is one,  $i(\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-1} = -0.368 \frac{V}{R}$

The transient response reaches 36.8 percent of its initial value.

Similarly,  $i(2\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-2} = -0.135 \frac{V}{R}$

$i(3\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-3} = -0.0498 \frac{V}{R}$

$i(5\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-5} = -0.0067 \frac{V}{R}$

After 5, the transient part reaches more than 99 percent of its final value. In fig we can find out the voltages and powers across each element by using the current.



Voltage across the resistor,  $V_R = R i = R * \frac{V}{R} (1 - e^{-\frac{R}{L}t})$

$$V_R = V (1 - e^{-\frac{R}{L}t})$$

Similarly, the voltage across the inductor,  $V_L = L \frac{di}{dt}$

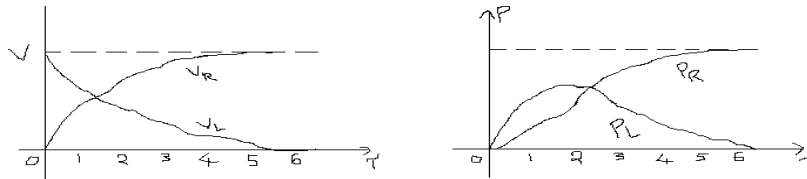
$$V_L = L * \frac{V}{R} e^{-\frac{R}{L}t} \frac{R}{L} = V e^{-\frac{R}{L}t}$$

Power in the resistor,  $P_R = V_R i = V (1 - e^{-\frac{R}{L}t}) * \frac{V}{R} (1 - e^{-\frac{R}{L}t})$

$$= \frac{V^2}{R} (1 - 2 e^{-\frac{R}{L}t}) + e^{-\frac{2R}{L}t}$$

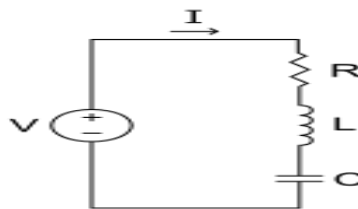
Power in the inductor,  $P_L = V_L i = V e^{-\frac{R}{L}t} * \frac{V}{R} (1 - e^{-\frac{R}{L}t})$

$$= \frac{V^2}{R} (e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t})$$



1. What is the condition for the response of a series RLC circuit excited by DC supply to have **over damped** response?

**Series RLC circuit:**



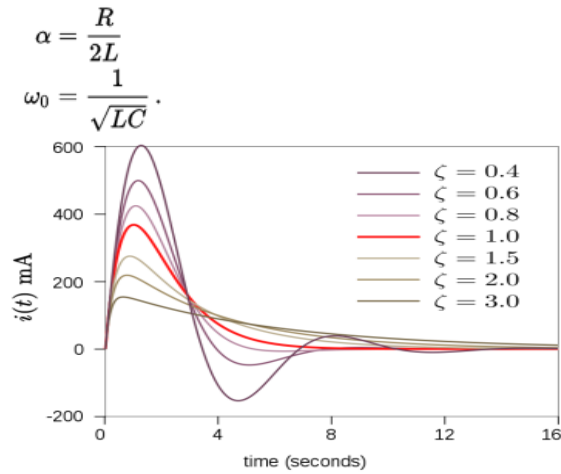
In this circuit, the three components are all in series with the voltage source. The governing differential equation can be found by substituting into Kirchhoff's voltage law (KVL) the constitutive equation for each of the three elements. From the KVL

$$V_R + V_L + V_C = V(t),$$

This can usefully be expressed in a more generally applicable form:

$$\frac{d^2}{dt^2} I(t) + 2\alpha \frac{d}{dt} I(t) + \omega_0^2 I(t) = 0.$$

For the case of the series RLC circuit these two parameters are given by



### Over damped response

The over damped response ( $\zeta > 1$ )

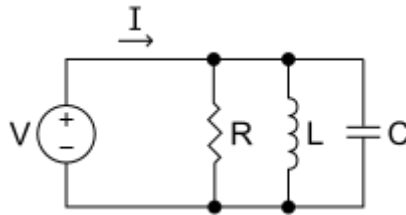
$$I(t) = A_1 e^{-\omega_0(\zeta + \sqrt{\zeta^2 - 1})t} + A_2 e^{-\omega_0(\zeta - \sqrt{\zeta^2 - 1})t}.$$

The under damped response ( $\zeta < 1$ ) is

$$I(t) = B_1 e^{-\alpha t} \cos(\omega_d t) + B_2 e^{-\alpha t} \sin(\omega_d t).$$

The critically damped response ( $\zeta = 1$ ) is

$$I(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}.$$



$$\alpha = \frac{1}{2RC}$$

$$\zeta = \frac{1}{2R} \sqrt{\frac{L}{C}}.$$

- Likewise, the other scaled parameters, fractional bandwidth and  $Q$  are also reciprocals of each other.

- This means that a wide-band, low- $Q$  circuit in one topology will become a narrow-band, high- $Q$  circuit in the other topology when constructed from components with identical values.
- The fractional bandwidth and  $Q$  of the parallel circuit are given by

$$B_f = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = R \sqrt{\frac{C}{L}}$$

3. For a unity feedback system,  $G(s) = K/[s(s+4)(s+2)]$ . Sketch the nature of root locus showing all details on it. Comment on the stability of the system

Solution:

Given system is unity feedback system. Therefore  $H(s) = 1$ .

Therefore  $G(s)H(s) = K/[s(s+4)(s+2)]$ .

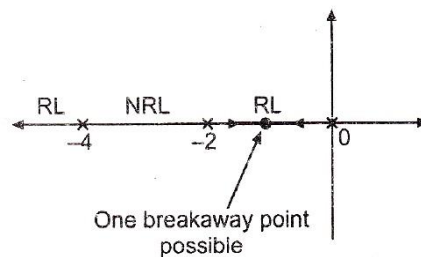
Step 1:

Poles = 0, -4, -2. Therefore  $P=3$ .

Zeros = there are no zeros.  $Z=0$ .

So all  $(P-Z=3)$  branches terminate at infinity.

Step 2: Pole-zero plot and sections of the real axis.



The pole-zero plot of the system is shown in the figure below. Here RL denotes Root Locus existence region and NRL denotes the non-existence region of root locus. These sections of real axis identified as a part of the root locus as to the right sum of poles and zeros is odd for those sections.

Step 3: Angle of asymptotes ‘A line to which root locus touches at infinity is called asymptotes.’

Number of asymptotes = P-Z = 3. Therefore 3 asymptotes are approaching to infinity.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2$$

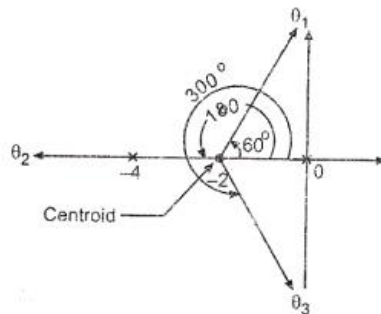
$$\therefore \theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{(2+1)180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{(2 \times 2 + 1)180^\circ}{3} = 300^\circ$$

Step 4: Centroid or Centre of asymptotes.

Asymptote touches real axis at a point called centroid.

Branches will approach infinity along these lines which are asymptotes.

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z} = \frac{0 - 2 - 4}{3} = -2$$



Step 5: To find breakaway point, we have characteristic equation as,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\therefore s^3 + 6s^2 + 8s + K = 0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$$

$$\text{Le } 3s^2 + 12s + 8 = 0$$

$$\text{Roots i.e. breakaway points} = \frac{-12 \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3} = -0.845, -3.15$$

As there is no root locus between -2 to -4, -3.15 cannot be a breakaway point. It also can be confirmed by calculating 'K' for  $s = -3.15$ . It will be negative that confirms  $s = -3.15$  is not a breakaway point.

For  $s = -3.15$ ,  $K = -3.079$  (Substituting in equation for K). But as there has to be a breakaway point between '0' and '-2',  $s = -0.845$  is a valid breakaway point.

For  $s = -0.845$ ,  $K = +3.079$ . As K is positive,  $s = -0.845$  is a valid breakaway point.

Step 6: Intersection point with the imaginary axis.

Characteristic equation

$$s^3 + 6s^2 + 8s + K = 0$$

Roth's array:

$s^3$	1	8
$s^2$	6	K
$s^1$	$\frac{48-K}{6}$	0
$s^0$	K	

$K_{\text{marginal}} = 48$  which makes row of  $s^1$  as row of zeros.

$$A(s) = 6s^2 + K = 0$$

$$K_{\text{max}} = 48$$

$$\therefore 6s^2 + 48 = 0$$

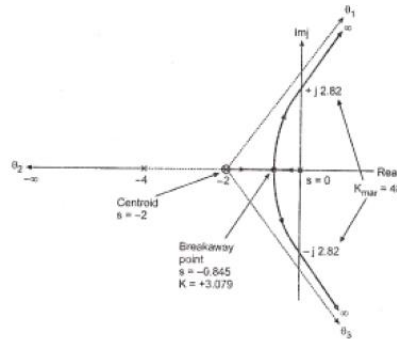
$$s^2 = -8$$

$$\therefore s = \pm j\sqrt{8} = \pm j2.828$$

Intersection of root locus with imaginary axis is at  $\pm j2.828$  and corresponding value of  $K(\text{marginal}) = 48$ .

Step 7 : As there are no complex conjugate poles or zeros, no angles of departures or arrivals are required to be calculated.

Step 8: The complete root locus is as shown in the figure below.



Step 9: Prediction about stability:

For  $0 < K < 48$ , all the roots are in left half of s-plane hence system is absolutely stable.

For  $K(\text{marginal}) = +48$ , a pair of dominant roots on imaginary axis with remaining root in left half. So the system is marginally stable oscillating at 2.82 rad/sec. For  $48 < K < \infty$ , dominant roots are located in right half of s-plane hence system is unstable.

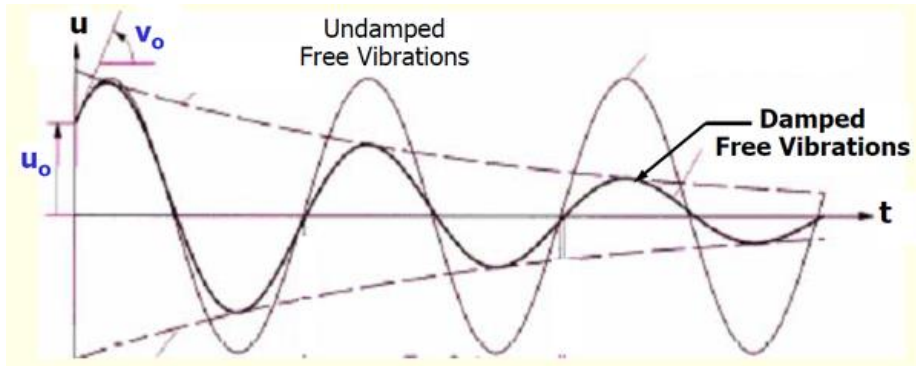
Stability is predicted by locations of dominant roots. Dominant roots are those which are located closest to the imaginary axis.

4. What is damping ratio?

In engineering, the damping ratio is a dimensionless measure describing how oscillations in a system decay after a disturbance. Why does the decay occur?

It is worth mentioning that the **Conservation of Energy** is “Energy can neither be created nor destroyed, it can only be transformed from one form to another”.

Initial energy inside system is dissipated in time by thermal effect of repeated or cyclic straining of material and internal friction when members are deformed this energy dissipation causes damping of vibration amplitude until eventually vibrations stop.

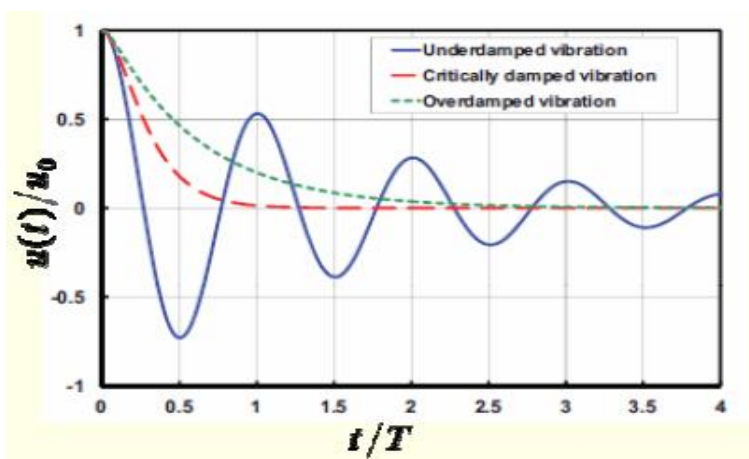


As aforementioned, the damping ratio is used to model the damping and it calculated based on the formula below.

$$\text{Damping Ratio} = \zeta = \frac{c}{c_{cr}}$$

Where critical damping coefficient  $C_{cr}$  is the smallest value of viscous damping coefficient that prevents occurrence of vibrations due to initial disturbance.

Consequently, it is used to describe how rapid the decay of amplitude occurs. There is three cases as shown in the figure below:



$\zeta = \frac{c}{c_{cr}} < 1.0$  **Underdamped**  
**(structure oscillates to reach equilibrium)**

$\zeta = \frac{c}{c_{cr}} = 1.0$  **Critically Damped**  
**(structure does not oscillate to reach equilibrium)**

$\zeta = \frac{c}{c_{cr}} > 1.0$  **Overdamped**  
**(no oscillations and slower response to reach equilibrium)**



## II mid Examination Answers

### 1. What is a two port network

A **two-port network** (a kind of **four-terminal network** or **quadruple**) is an electrical network (circuit) or device with two *pairs* of terminals to connect to external circuits. Two terminals constitute a port if the currents applied to them satisfy the essential requirement known as the port condition: the electric current entering one terminal must equal the current emerging from the other terminal on the same port. The ports constitute interfaces where the network connects to other networks, the points where signals are applied or outputs are taken. In a two-port network, often port 1 is considered the input port and port 2 is considered the output port. The two-port network model is used in mathematical circuit analysis techniques to isolate portions of larger circuits. A two-port network is regarded as a "black box" with its properties specified by a matrix of numbers. This allows the response of the network to signals applied to the ports to be calculated easily, without solving for all the internal voltages and currents in the network. It also allows similar circuits or devices to be compared easily. For example, transistors are often regarded as two-ports, characterized by their h-parameters (see below) which are listed by the manufacturer. Any linear circuit with four terminals can be regarded as a two-port network provided that it does not contain an independent source and satisfies the port conditions.

Examples of circuits analyzed as two-ports are filters, matching networks, transmission lines, transformers, and small-signal models for transistors (such as the hybrid-pi model). The analysis of passive two-port networks is an outgrowth of reciprocity theorems first derived by Lorentz. In two-port mathematical models, the network is described by a 2 by 2 square matrix of complex numbers. The common models that are used are referred to as *z-parameters*, *y-parameters*, *h-parameters*, *g-parameters*, and *ABCD-parameters*, each described individually below. These are all limited to linear networks since an underlying assumption of their derivation is that any given circuit condition is a linear superposition of various short-circuit and open circuit conditions. They are usually expressed in matrix notation, and they establish relations between the variables

$V_1$  , voltage across port 1

$I_1$  , current into port 1

$V_2$  , voltage across port 2

$I_2$  , current into port 2

Which are shown in figure. The difference between the various models lies in which of these variables are regarded as the independent variables. These current and voltage variables are most useful at low-to-moderate frequencies. At high frequencies (e.g., microwave frequencies), the use of power and energy variables is more appropriate, and the two-port current-voltage approach is replaced by an approach based upon scattering parameters. There are certain properties of two-ports that frequently occur in practical networks and can be used to greatly simplify the analysis. These include:

### **Reciprocal networks**

A network is said to be reciprocal if the voltage appearing at port 2 due to a current applied at port 1 is the same as the voltage appearing at port 1 when the same current is applied to port 2. Exchanging voltage and current results in an equivalent definition of reciprocity. A network that consists entirely of linear passive components (that is, resistors, capacitors and inductors) is usually reciprocal, a notable exception being passive circulators and isolators that contain magnetized materials. In general, it *will not* be reciprocal if it contains active components such as generators or transistors.

### **Symmetrical networks**

A network is symmetrical if its input impedance is equal to its output impedance. Most often, but not necessarily, symmetrical networks are also physically symmetrical. Sometimes also antimetrical networks are of interest. These are networks where the input and output impedances are the duals of each other.

### **Lossless network**

A lossless network is one which contains no resistors or other dissipative elements.

## 2. Root locus Procedure

### **Root Locus Method with step by step solution**

General steps for drawing the Root Locus of the given system:

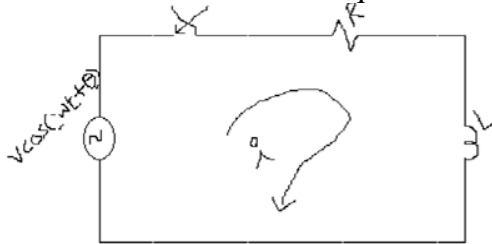
1. Determine the open loop poles, zeros and a number of branches from given  $G(s)H(s)$ .
2. Draw the pole-zero plot and determine the region of real axis for which the root locus exists. Also, determine the number of breakaway points (This will be explained while solving the problems).
3. Calculate the angle of asymptotes.
4. Determine the centroid.

5. Calculate the breakaway points (if any).
6. Calculate the intersection point of root locus with the imaginary axis.
7. Calculate the angle of departure or angle of arrivals if any.
8. From above steps draw the overall sketch of the root locus.
9. Predict the stability and performance of the given system by the root locus.

3. **Derive the expression for transient response of R-L series circuit for ac excitation**

**Sinusoidal Response of R-L Circuit:**

Consider a circuit consisting of resistance and inductance are connected as shown in fig. the switch S is closed at  $t=0$ . At  $t=0$ , sinusoidal voltage  $V\cos(\omega t + \theta)$  applied to RL circuit, where  $V$  is the amplitude of the wave and  $\theta$  is the phase angle. Application of Kirchhoff's laws we can determine the differential equations.



$$V\cos(\omega t + \theta) = iR + L \frac{di}{dt} \text{ ----- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \cos(\omega t + \theta) \text{ ----- (2)}$$

The corresponding characteristic equation is

$$(D + \frac{R}{L}) i = \frac{V}{L} \cos(\omega t + \theta) \text{ ----- (3)}$$

For the above equation, the solution consists of two parts.  
One is complementary function and other is particular integral.

The complementary function of the solution is

$$i_c = c e^{-t\left(\frac{R}{L}\right)} \text{ ----- (4)}$$

The particular solution can be obtained by using undetermined co-efficient.

By assuming,

$$i_p = A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \text{ ----- (5)}$$

$$i_p^1 = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) \text{ ----- (6)}$$

Substituting

equations 5 & 6 in 3 we get,

$$\{-A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) + \frac{R}{L}[A \cos(\omega t + \theta) + B \sin(\omega t + \theta)]\} = \frac{V}{L} \cos(\omega t + \theta)$$

$$\left(-A\omega + \frac{BR}{L}\right)\sin(\omega t + \theta) + \left(B\omega + \frac{AR}{L}\right)\cos(\omega t + \theta) = \frac{V}{L} \cos(\omega t + \theta)$$

#### 4. Foster's Reactance theorem,

For a positive real rational function  $Z(s)=1/Y(s)$  to be realizable as the driving point impedance of a lossless one-port, the necessary and sufficient condition is that it should be expressible in the form

$$Z(s) \text{ or } Y(s) = \frac{\left[ a_n \cdot (s^2 + \omega_1^2) \cdot (s^2 + \omega_3^2) \dots \right]}{\left[ b_m \cdot (s^2 + \omega_2^2) \cdot (s^2 + \omega_4^2) \dots \right]}$$

where  $a_n$  and  $b_m$  are constants and

2.  $0 < \omega < \omega_1 < \omega_2 < \omega_3$  (Interlacing poles and zeros, all on  $j\omega$  axis)
3. Foster's Theorem further restricts the degrees of the numerator,  $n$ , and denominator,  $m$ , by requiring that they must differ by unity. In other words, if the numerator is an even degree, the denominator is odd, and vice versa.

From these conditions, the following properties can be deduced:

1. Unity degree difference between numerator and denominator implies that  $Z(s)$  must have either a single pole or a single zero at both  $s=0$  and  $sj\omega$ . Therefore the function  $Z(s)$  or  $Y(s)$  will belong to one of the four types:
2.  $Z(j\omega)$  is purely reactive. Therefore it can be written as

$$Z(j\omega) = jX(\omega)$$

where  $X(\omega)$  is the input reactance with

$$X(\omega) \text{ or } \frac{1}{X(\omega)} = \frac{[a_n \cdot (\omega_1^2 - \omega^2) \cdot (\omega_3^2 - \omega^2) \dots]}{[b_m \cdot \omega \cdot (\omega_2^2 - \omega^2) \cdot (\omega_4^2 - \omega^2) \dots]}$$

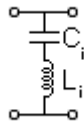
Alternation of poles and zeros leads to the property

$$0 < \frac{|X(\omega)|}{\omega} < \frac{d}{d\omega} X(\omega)$$

In other words, the reactance  $X(\omega)$  is always an increasing function of frequency. The rational functions satisfying these requirements are called Foster functions.

3. Since all poles of  $Z(s)$  and  $Y(s)$  are on the  $s=j\omega$  axis, they can always be expanded as

If  $Y(s)$  has a pole at  $s=j\omega_i$ , it can be extracted as a series resonator to ground:



$$L_i = \frac{1}{2 \cdot h_i} = \frac{s}{(s^2 + \omega_i^2) \cdot Y(s)} \Big|_{s=j\omega_i}, \quad C_i = \frac{1}{\omega_i^2 \cdot L_i}$$

- Note that if a pole of  $Z(s)$  at  $s=0$  or  $s=\mp j\omega_i$  is extracted, a zero appears at that frequency automatically in the remaining impedance function, which acts as a pole of the remaining admittance function.
- Hence, given  $Z(s)$ , one can synthesize a variety of circuits all having the same input impedance but with different structures by extracting elements in different orders from impedance or admittance functions.

### 13. MID exam Objective Question papers

#### I Mid Examination

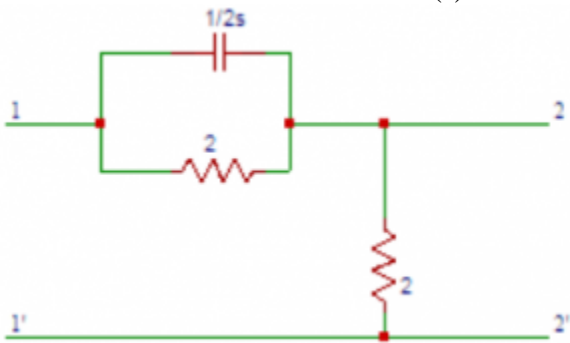
1. The current in a closed path in a loop is called?
  - a) Loop current
  - b) branch current
  - c) link current
  - d) twig current
2. Tie-set is also called?
  - a) f loop
  - b) g loop
  - c) d loop
  - d) e loop
3. In the expression of current in the R-L circuit the transient part is?
  - a)  $R/V$
  - b)  $(V/R)(-\exp^{-t/(R/L)})$
  - c)  $(V/R)(\exp^{-t/(R/L)})$
  - d)  $V/R$
4. The value of the time constant in the R-L circuit is?
  - a)  $L/R$
  - b)  $R/L$
  - c)  $R$
  - d)  $L$
5. In case of purely inductive circuit, average power = \_\_\_\_\_ and  $\theta =$  \_\_\_\_\_
  - a) 0,  $90^\circ$
  - b) 1,  $90^\circ$
  - c) 1,  $0^\circ$
  - d) 0,  $0^\circ$
6. If a circuit has complex impedance, the average power is \_\_\_\_\_
  - a) power stored in inductor only

- b) power stored in capacitor only
- c) power dissipated in resistor only
- d) power stored in inductor and power dissipated in resistor

7. Find the driving point impedance  $Z_{11}$  (S) in the circuit shown in question 5.

- a)  $2(s+2)$
- b)  $(s+2)$
- c)  $2(s+1)$
- d)  $(s+1)$

8. Obtain the transfer function  $G_{21}$  (s) in the circuit shown below.



- a)  $(8S+2)/(8S+1)$
- b)  $(8S+2)/(8S+2)$
- c)  $(8S+2)/(8S+3)$
- d)  $(8S+2)/(8S+4)$

9. The value of  $L_2$  in the question 6 is?

- a) 4
- b) 1
- c) 2
- d) 3

10. The value of  $R_\infty$  in the question 6 is?

- a) 3
- b) 1
- c) 2
- d) 4

Answer Key: 1.c 2.a 3.b 4.a 5.a 6.c 7..c 8.d 9.b 10.c

## II Mid Examination

1. The value of one decibel is equal to.....
2. A filter which passes without attenuation all frequencies up to the cut-off frequency  $f_c$  and attenuates all other frequencies greater than  $f_c$  is called.....
3. A filter which attenuates all frequencies below a designated cut-off frequency  $f_c$  and passes all other frequencies greater than  $f_c$  is called.....
4. A filter that passes frequencies between two designated cut-off frequencies and attenuates all other frequencies is called?
5. The zeros of driving point impedance are those frequencies corresponding to \_\_\_\_\_ conditions?
6. In the driving point admittance function, a zero of  $Y(s)$  means a \_\_\_\_\_ of  $I(S)$ .
7. The value of  $R_1$  in the question 6 is?
  - a) 4/3
  - b) 3/3
  - c)
  - d) 1/3
8. The value of  $R_2$  in the question 6 is?
  - a) 1
  - b)
  - c) 3
  - d) 4
9. Determine the average power delivered to the circuit.
  - a) 620
  - b) 630
  - c) 640
  - d)
10. Determine the average power delivered to the circuit consisting of an impedance  $Z = 5+j8$  when the current flowing through the circuit is  $I = 5\angle 30^\circ$ .
  - a) 61.5
  - b)
  - c) 63.5
  - d) 64.5

Answer Key: 1. 0.115 N 2. low pass filter 3. high pass filter 4. short circuit 5. The zeros of driving point impedance are those frequencies corresponding to short circuit conditions as pole of  $Z(s)$  is a zero of  $I(s)$  and zero of  $N(s)$  is a zero of  $V(s)$ , it signifies a short circuit. 6. Zero 7. 2/3



8. 2 9. 650 10. 62.5

## 14. ASSIGNMENT TOPICS WITH SOLUTIONS

### UNIT -I

- **Topic 1: Discuss dot convention and Coefficient of coupling.**

#### Coefficient of Coupling:

It is the factor which indicates the degree of coupling between the couple coils given by

$$K = \sqrt{\frac{\Phi_{21}}{\Phi_1} * \frac{\Phi_{12}}{\Phi_2}} \text{ ----- (1)}$$

Expressing (1) in terms of self and mutual inductances,

$$L_1 = \frac{N_1 \Phi_1}{i_1} \text{ ----- (2)}$$

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1} \text{ ----- (3)}$$

$$L_2 = \frac{N_2 \Phi_{12}}{i_2} \text{ ----- (4)}$$

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2} \text{ ----- (5)}$$

$$K = \sqrt{\left( \frac{M_{21} i_1 N_1}{N_2 L_1 i_1} \frac{M_{12} i_2 N_2}{N_1 L_2 i_2} \right)} = \sqrt{\frac{M_{21} N_1 M_{12} N_2}{L_1 N_2 L_2 N_1}}$$

If  $M_{21} = M_{12} = M$  then we get,

$$K = \sqrt{\frac{M^2}{L_1 L_2}} \text{ ----- (7)}$$

This will be equal to 1 if coils are coupled tightly

$$M = K \sqrt{L_1 L_2} \text{ ----- (8)}$$

Coefficient of coupling is also defined as the ratio of mutual flux to total flux. It is always less than one ( $K \leq 1$ ) (this is the principle used in transformer).

### Dot Notation

The Polarity or Dot Notation for a device with mutual inductance designates the relative instantaneous current directions of such device's winding leads.

The physical criteria of this notation are to preserve a proper magnetic flow circuit direction inside the material. Keeping all current entering a transformer or coupled inductances, in phase, through dotted leads or ports will keep the magnetic flow circulating without canceling each other, and the device will work under their rated efficiency.

Entering current on different dotted ports -one in a dotted port and the other with an un-dotted port- will cause the magnetic flow components to subtract each other, making the device working inefficiently. This criterion holds for devices with several coils, in order to have each of them aggregating in an additive way by entering dotted ports.

Leads of primary and secondary windings are said to be of the same polarity when instantaneous current entering the primary winding lead results in instantaneous current leaving the secondary winding lead as though the two leads were a continuous circuit. In the case of two windings wound around the same core in parallel, for example, the polarity will be the same on the same ends: A sudden (instantaneous) current in the first coil will induce a voltage opposing the sudden increase (Lenz's law) in the first and also in the second coil, because the inductive magnetic field produced by the current in the first coil traverses the two coils in the same manner. The second coil will, therefore, show an induced current opposite in direction to the inducing current in the first coil. Both leads behave like a continuous circuit, one current entering into the first lead and another current leaving the second lead.

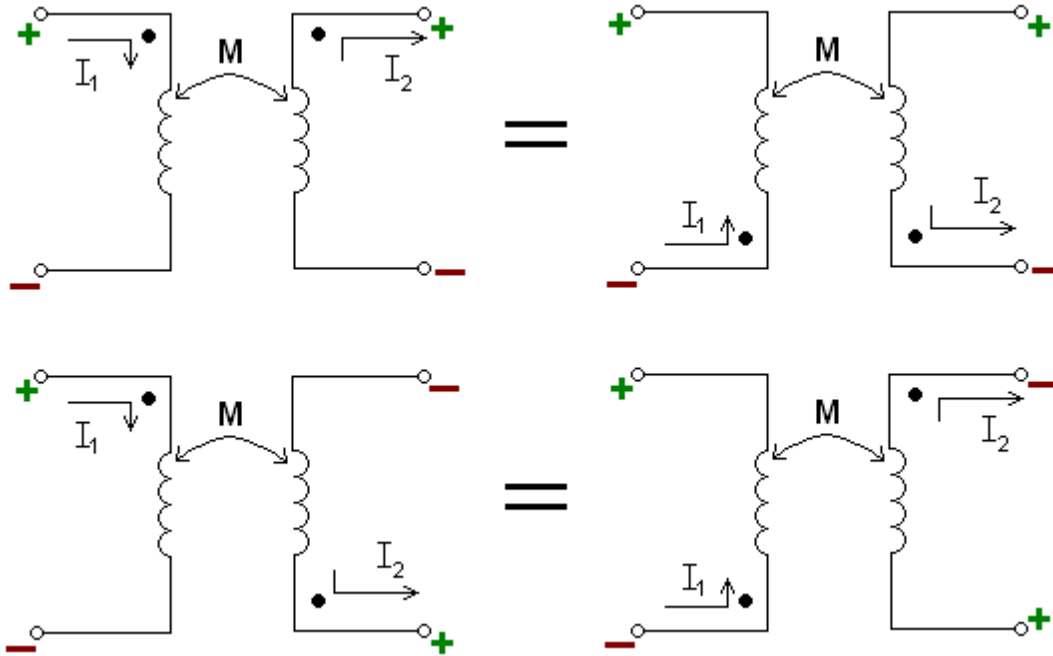
Referring to the circuit diagrams below:[dubious – discuss] The circuit polarity signs '+' and '-' indicate the relative polarities of the induced voltages in both coils, i.e. how an instantaneous (sudden) magnetic field traversing the primary and secondary coils induces a voltage in both coils.

The instantaneous polarities of the voltages across each inductor with respect to the dotted terminals are the same.

The circuit arrows indicate example applied and resultant relative current directions. The '+' and '-' polarities in the diagram are not the voltages driving the currents.

The instantaneous directions of the current entering the primary inductor at its dotted end and the current leaving of the secondary inductor at its dotted end are the same.

Subtractive polarity transformer designs are shown in the upper circuit diagrams. Additive polarity transformer designs are shown in the lower circuit diagrams.



## Topic 2

- What self and mutual inductances and Evaluate self and mutual inductances for the given magnetic circuits?

### Self Induction:

Inductance is the property of the circuit element which will oppose any change of current through it. By Faraday's laws of electromagnetic induction, it follows that whenever there is change of flux linking with a coil with time, and then there will be an induced emf in the coil. The induced emf is proportional to the rate of change flux linkages of the coil.

$$e \propto \frac{d\phi}{dx} \propto N \frac{d\Phi}{dt} \text{ ----- (1)}$$

Where  $N$  is the number of turns in the coil and  $\Phi$  is the flux in Weber in the coil.

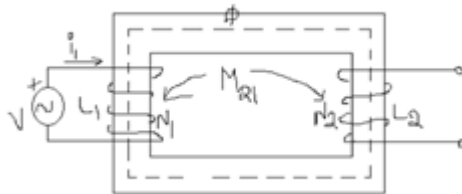
$$e = - N \frac{d\Phi}{dt}$$

The negative sign indicates that the direction of induced emf is such that it opposes the every cause which is producing it, also known as LENZ'S law. Since the flux in the coil is directly proportional to current flowing in it, the emf induced is proportional to the rate of change of current.

If the current  $I$  and flux linkages refer to the same physical system, than the parameter  $L$  is called self inductance. It is measured in HENRYS.

### Mutual Inductance:

Let us consider that there are two coils which are placed on the same magnetic core such that the flux produced by current flowing through one coil completely links with the other coils also. Let the coil1 is connected to AC supply and coil2 is open circuit.

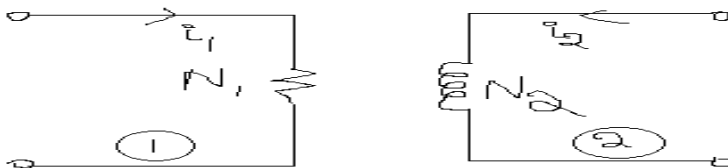


A current flowing in the first coil produces a flux as shown in fig. The direction of time varying flux is given by right hand thumb rule. The flux produced by current not only links with the coil1 but also links with coil2. The emf induced in coil1 is called self induced emf.

$$e_2 = N_2 \frac{d\Phi_1}{dt} = M_{21} \frac{di_1}{dt}$$

The proportionality constant  $M_{21}$  between induced emf in the second coil and rate of change of current in the first coil is called mutual inductance. Any two such coils are said to be magnetically coupled.

The unit of mutual inductance is HENRY. The mutual inductance between two coils is said to be 1 Henry when a change of current of 1 Amp/Sec in one coil produces a mutual induced emf of 1 volt in the other coil.



Self induced emf in coil1,

$$e_1 = L_{11} \frac{di_1}{dt}$$

Mutual induced emf in the coil2,

$$e_2 = M_{21} \frac{di_1}{dt}$$

Let us assume that second coil also carries a current of  $i_2$  as shown in fig, which in turn produces a self induced emf in coil2 and a mutual induced emf in coil1.

Self induced emf in coil2

$$L_{22} \frac{di_2}{dt}$$

Mutual induced emf in the coil1=

$$M_{21} \frac{di_2}{dt}$$

In practice all the flux produced by current in one coil may not completely link with the other coil. Depending on the position and orientation of the two coils, only a fraction of the flux may be linking with the other coil. Then the two circuits are said to be loosely coupled and if all the flux is linking with the other coil, then they are said to be tightly coupled.

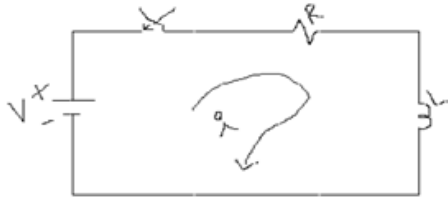
If  $\Phi_1$  is the total flux produced by  $i_1$  and only  $\Phi_{21}$  is common and linking with second coil, then the fraction of the flux linking with coil2 is  $\frac{\Phi_{21}}{\Phi_1}$ . Similarly  $\Phi_2$  is the total flux produced by  $i_2$  and only  $\Phi_{12}$  is common and linking with first coil, then the fraction of the flux linking with coil1 is  $\frac{\Phi_{12}}{\Phi_2}$ . These fractions indicate the degree of coupling between the two coils. If the two coils are very close to one another and properly oriented then these fractions approach to unity.

## UNIT -II

### Topic 1:

Derive the transient response of R-L, R-C and R-L-C for DC excitation with differential equation approach

### DC Response of an R-L Circuit:



Consider a circuit consisting of a resistance and inductance as shown in fig. the inductor in the circuit is initially uncharged and is in series with the resistor. When switch S is closed, we can find the complete solution for current. Application of Kirchoff's law to the circuit results in following differential equations.

$$V = iR + L \frac{di}{dt} \text{ ----- (1)}$$

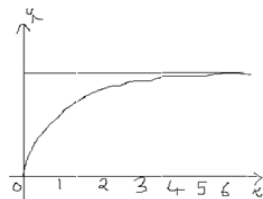
$$\frac{di}{dt} + \frac{R}{L} I = \frac{V}{L} \text{ ----- (2)}$$

In the above equation, the current  $i$  is the solution to be found and  $V$  is the applied constant voltage. The voltage  $V$  is applied to the circuit only when the switch  $S$  is closed. The above equation is linear differential equation of the first order comparing with the non homogenous differential equation

$$i = -\frac{V}{R} e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$$

$$i = \frac{V}{R} \left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$$

Above equation consists of two parts, the steady state part ( $V/R$ ) and other is transient part



$$\tau = \frac{L}{R} \text{ seconds}$$

The transient part of solution is,  $i(\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}}$

At time constant is one,  $i(\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-1} = -0.368 \frac{V}{R}$

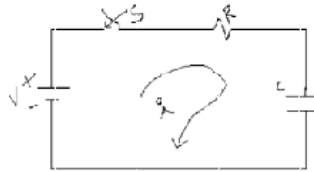
The transient response reaches 36.8 percent of its initial value.

Similarly,  $i(2\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-2} = -0.135 \frac{V}{R}$

$i(3\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-3} = -0.0498 \frac{V}{R}$

$i(5\tau) = -\frac{V}{R} e^{-\frac{t}{\tau}} = -\frac{V}{R} e^{-5} = -0.0067 \frac{V}{R}$

### DC Response of an R-C Circuit



Application of Kirchhoff's laws we can determine the differential equations.

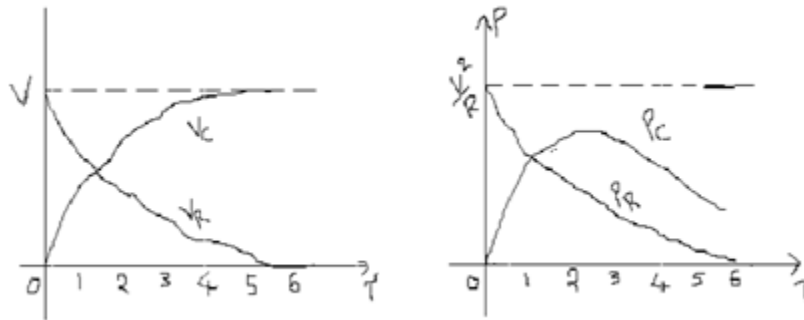
$$V = R i + \frac{1}{C} \int i dt$$

By differentiating the above equation we get,

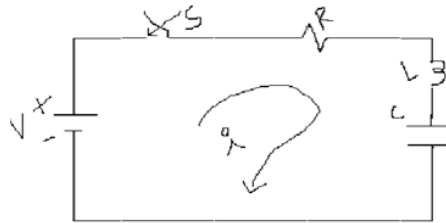
$$0 = R \frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

The responses are shown in fig.



### DC Response of an R-L-C Circuit



$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt \text{ ----- (1)}$$

$$0 = R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{1}{C} i \text{ ----- (2)}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0 \text{ ----- (3)}$$

$$D^2 + \frac{R}{L} D + \frac{1}{LC} = 0 \text{ ----- (4)}$$

The roots above equation are,

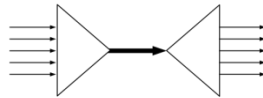
$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

➤ **Topic2:** Analyze the circuit switching  
**Circuit switching:**

- Circuit switching is a method of implementing a telecommunications network in which two network nodes establish a dedicated communications channel (circuit) through the network before the nodes may communicate.
- The circuit guarantees the full bandwidth of the channel and remains connected for the duration of the communication session. The circuit functions as if the nodes were physically connected as with an electrical circuit.



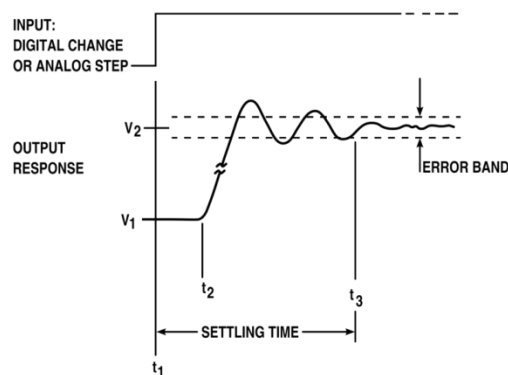
- The defining example of a circuit-switched network is the early analog telephone network. When a call is made from one telephone to another, switches within the telephone exchanges create a continuous wire circuit between the two telephones, for as long as the call lasts.



Multiplexing

- Circuit switching contrasts with packet switching which divides the data to be transmitted into packets transmitted through the network independently.
- In packet switching, instead of being dedicated to one communication session at a time, network links are shared by packets from multiple competing communication sessions, resulting in the loss of the quality of service guarantees that are provided by circuit switching.
- In circuit switching, the bit delay is constant during a connection, as opposed to packet switching, where packet queues may cause varying and potentially indefinitely long packet transfer delays.
- No circuit can be degraded by competing users because it is protected from use by other callers until the circuit is released and a new connection is set up. Even if no actual communication is taking place, the channel remains reserved and protected from competing users.
- Virtual circuit switching is a packet switching technology that emulates circuit switching, in the sense that the connection is established before any packets are transferred, and packets are delivered in order.
- While circuit switching is commonly used for connecting voice circuits, the concept of a dedicated path persisting between two communicating parties or nodes can be extended to signal content other than voice.
- Its advantage is that it provides for continuous transfer without the overhead associated with packets making maximal use of available bandwidth for that communication. Its disadvantage is that it can be relatively inefficient because unused capacity guaranteed to a connection cannot be used by other connections on the same network.
- The step response of a system in a given initial state consists of the time evolution of its outputs when its control inputs are Heaviside step functions.
- In electronic engineering and control theory, step response is the time behavior of the outputs of a general system when its inputs change from zero to one in a very short time.
- The concept can be extended to the abstract mathematical notion of a dynamical system using an evolution parameter.

- From a practical standpoint, knowing how the system responds to a sudden input is important because large and possibly fast deviations from the long term steady state may have extreme effects on the component itself and on other portions of the overall system dependent on this component.
- In addition, the overall system cannot act until the component's output settles down to some vicinity of its final state, delaying the overall system response.
- Formally, knowing the step response of a dynamical system gives information on the stability of such a system, and on its ability to reach one stationary state when starting from another.



### Topic 3:

Analyze and determine the resonant curve frequencies, the quality factor and band width of the series RLC circuits.

#### Band width:

- The resonance effect can be used for filtering, the rapid change in impedance near resonance can be used to pass or block signals close to the resonance frequency. Both band-pass and band-stop filters can be constructed and some filter circuits are shown later in the article. A key parameter in filter design is bandwidth.

$$B_f = \frac{\Delta\omega}{\omega_0} .$$

#### Q factor

- The Q factor is a widespread measure used to characterize resonators. It is defined as the peak energy stored in the circuit divided by the average energy dissipated in it per radian at resonance.

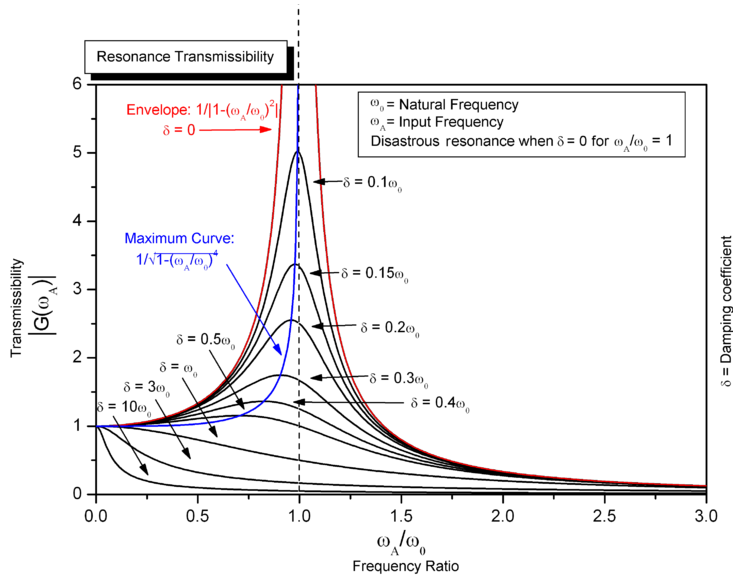
- Low-Q circuits are therefore damped and lossy and high-Q circuits are under damped. Q is related to bandwidth; low-Q circuits are wide-band and high-Q circuits are narrow-band. In fact, it happens that Q is the inverse of fractional bandwidth.

$$Q = \frac{1}{\omega_0 RC} = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

**Resonance curve definition:**

A curve whose abscissas are frequencies lying near to and on both sides of the natural frequency of a vibrating system and whose ordinates are the corresponding amplitudes of the near-resonant vibrations

- Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a simple pendulum).
- However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.
- Some systems have multiple, distinct, resonant frequencies.
- Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions.
- Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters).



## Topic 4: Root Locus

### Root Locus Method with step by step solution

General steps for drawing the Root Locus of the given system:

- 1. Determine the open loop poles, zeros and a number of branches from given  $G(s)H(s)$ .
- 2. Draw the pole-zero plot and determine the region of real axis for which the root locus exists. Also, determine the number of breakaway points (This will be explained while solving the problems).
- 3. Calculate the angle of asymptotes.
- 4. Determine the centroid.
- 5. Calculate the breakaway points (if any).
- 6. Calculate the intersection point of root locus with the imaginary axis.
- 7. Calculate the angle of departure or angle of arrivals if any.
- 8. From above steps draw the overall sketch of the root locus.
- 9. Predict the stability and performance of the given system by the root locus.

EX:

Question: For a unity feedback system,  $G(s) = K/[s(s+4)(s+2)]$ . Sketch the nature of root locus showing all details on it. Comment on the stability of the system

Solution:

Given system is unity feedback system. Therefore  $H(s) = 1$ .

Therefore  $G(s)H(s) = K/[s(s+4)(s+2)]$ .

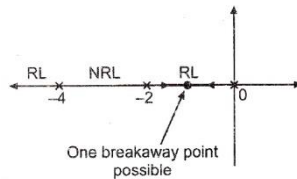
Step 1:

Poles = 0, -4, -2. Therefore  $P=3$ .

Zeros = there are no zeros.  $Z=0$ .

So all ( $P-Z=3$ ) branches terminate at infinity.

Step 2: Pole-zero plot and sections of the real axis.



The pole-zero plot of the system is shown in the figure below. Here RL denotes Root Locus existence region and NRL denotes the non-existence region of root locus. These sections of real axis identified as a part of the root locus as to the right sum of poles and zeros is odd for those sections.

Step 3: Angle of asymptotes 'A line to which root locus touches at infinity is called asymptotes.'

Number of asymptotes =  $P-Z = 3$ . Therefore 3 asymptotes are approaching to infinity.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2$$

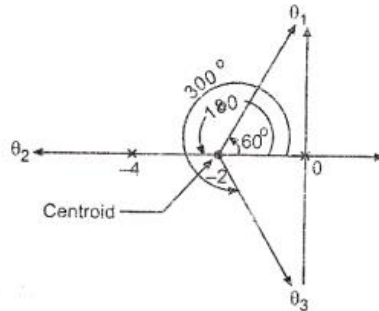
$$\therefore \theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{(2+1)180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{(2 \times 2+1)180^\circ}{3} = 300^\circ$$

Step 4: Centroid or Centre of asymptotes.

Asymptote touches real axis at a point called centroid.

Branches will approach infinity along these lines which are asymptotes.

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P - Z} = \frac{0 - 2 - 4}{3} = -2$$



Step 5: To find breakaway point, we have characteristic equation as,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\therefore s^3 + 6s^2 + 8s + K = 0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$$

$$\text{I.e. } 3s^2 + 12s + 8 = 0$$

$$\text{Roots i.e. breakaway points} = \frac{-12 \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3} = -0.845, -3.15$$

As there is no root locus between -2 to -4, -3.15 cannot be a breakaway point. It also can be confirmed by calculating 'K' for  $s = -3.15$ . It will be negative that confirms  $s = -3.15$  is not a breakaway point.

For  $s = -0.845$ ,  $K = +3.079$  (Substituting in equation for K). But as there has to be breakaway point between '0' and '-2',  $s = -0.845$  is a valid breakaway point.

For  $s = -0.845$   $K = +3.079$ . As K is positive  $s = -0.845$  is valid breakaway point.

Step 6: Intersection point with the imaginary axis.

Characteristic equation

$$S^3 + 6S^2 + 8s + K = 0$$

Roth's array:

$$\begin{array}{c|cc}
 s^3 & 1 & 8 \\
 s^2 & 6 & K \\
 s^1 & \frac{48-K}{6} & 0 \\
 s^0 & K & 
 \end{array}$$

$K_{\text{marginal}} = 48$  which makes row of  $s^1$  as row of zeros.

$$A(s) = 6s^2 + K = 0$$

$$K_{\text{mar}} = 48$$

$$\therefore 6s^2 + 48 = 0$$

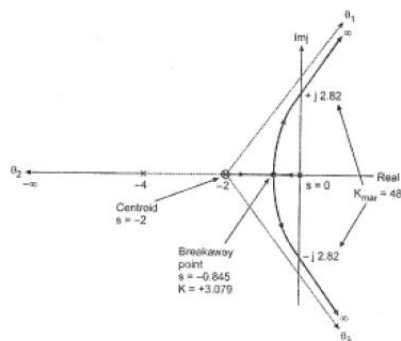
$$s^2 = -8$$

$$\therefore s = \pm j\sqrt{8} = \pm j2.828$$

Intersection of root locus with imaginary axis is at  $\pm j2.828$  and corresponding value of  $K(\text{marginal}) = 48$ .

Step 7 : As there are no complex conjugate poles or zeros, no angles of departures or arrivals are required to be calculated.

Step 8: The complete root locus is as shown in the figure below.



Step 9: Prediction about stability:

For  $0 < K < 48$ , all the roots are in left half of  $s$ -plane hence system is absolutely stable.

For  $K(\text{marginal}) = +48$ , a pair of dominant roots on imaginary axis with remaining root in left half. So the system is marginally stable oscillating at 2.82 rad/sec. For  $48 < K < 8$ , dominant roots are located in right half of  $s$ -plane hence system is unstable.

Stability is predicted by locations of dominant roots. Dominant roots are those which are located closest to the imaginary axis.

### UNIT -III

#### Topic 1:

- Derive the two port network parameters Z, Y, ABCD, h and g parameters.
- The Z-parameter matrix for the two-port network is probably the most common. In this case the relationship between the port currents, port voltages and the Z-parameter matrix is given by:

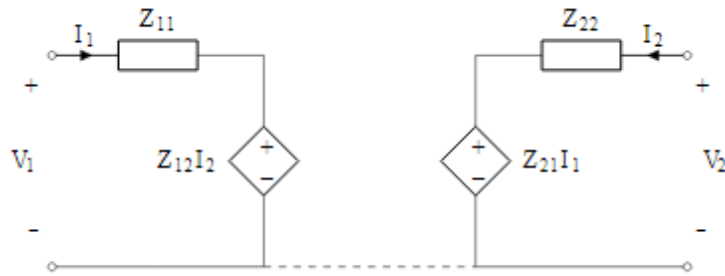
$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}.$$

Where

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$
$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

- The equivalent circuit for Z-parameters of a two port network is





- The Y-parameter matrix for the two-port network is probably the most common. In this case the relationship between the port voltages, port currents and the Y-parameter matrix is given by:

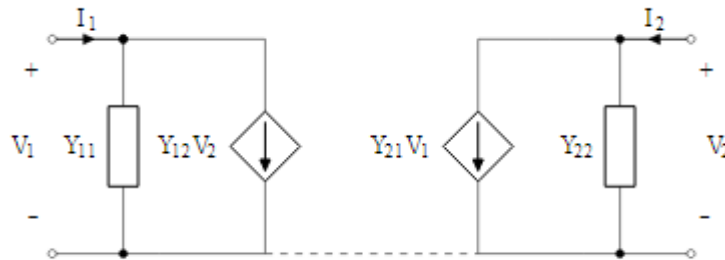
$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Where

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \quad Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

- Equivalent circuit for an arbitrary two-port admittance matrix. The circuit uses Norton sources with voltage-controlled current sources.



- The ABCD-parameters are known variously as chain, cascade, or transmission parameters. There are a number of definitions given for ABCD parameters, the most common is

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

- For reciprocal networks  $AD-BC=1$ . For symmetrical networks  $A=D$ . For networks which are reciprocal and lossless, A and D are purely real while B and C are purely imaginary.

- This representation is preferred because when the parameters are used to represent a cascade of two-ports, the matrices are written in the same order that a network diagram would be drawn, that is, left to right. However, a variant definition is also in use,

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

Where,

$$\begin{aligned} A' &\stackrel{\text{def}}{=} \frac{V_2}{V_1} \Big|_{I_1=0} & B' &\stackrel{\text{def}}{=} \frac{V_2}{I_1} \Big|_{V_1=0} \\ C' &\stackrel{\text{def}}{=} -\frac{I_2}{V_1} \Big|_{I_1=0} & D' &\stackrel{\text{def}}{=} -\frac{I_2}{I_1} \Big|_{V_1=0} \end{aligned}$$

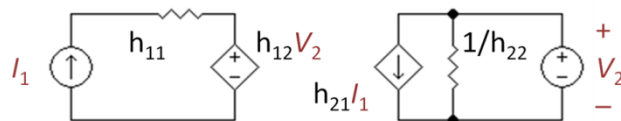
- The negative sign of  $-I_2$  arises to make the output current of one cascaded stage (as it appears in the matrix) equal to the input current of the next. Without the minus sign the two currents would have opposite senses because the positive direction of current, by convention, is taken as the current entering the port.
- Consequently, the input voltage/current matrix vector can be directly replaced with the matrix equation of the preceding cascaded stage to form a combined  $A'B'C'D'$  matrix.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Where

$$\begin{aligned} h_{11} &\stackrel{\text{def}}{=} \frac{V_1}{I_1} \Big|_{V_2=0} & h_{12} &\stackrel{\text{def}}{=} \frac{V_1}{V_2} \Big|_{I_1=0} \\ h_{21} &\stackrel{\text{def}}{=} \frac{I_2}{I_1} \Big|_{V_2=0} & h_{22} &\stackrel{\text{def}}{=} \frac{I_2}{V_2} \Big|_{I_1=0} \end{aligned}$$

- This circuit is often selected when a current amplifier is desired at the output. The resistors shown in the diagram can be general impedances instead.
- Off-diagonal h-parameters are dimensionless, while diagonal members have dimensions the reciprocal of one another.
- H-equivalent two-port showing independent variables  $I_1$  and  $V_2$ ;  $h_{22}$  is reciprocated to make a resistor



- If  $V_1$  and  $I_2$  are chosen as independent variables, the two-port network equations may be written as

$$(10.9) I_1 = g_{11} V_1 + g_{12} I_2$$

$$(10.10) V_2 = g_{21} V_1 + g_{22} I_2.$$

In matrix form, these equations are written as

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}.$$

- The constants  $g_{11}$ ,  $g_{12}$ ,  $g_{21}$ , and  $g_{22}$  are known as inverse hybrid parameters or g-parameters. The g-parameters are defined as follows by using Equations (10.9) and (10.10).

If  $I_2 = 0$  the output port is open circuit.

$$g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0} \text{ open circuit input admittance.}$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0} \text{ open circuit forward voltage gain.}$$

If

$V_1 = 0$  the input port is short circuit.

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0} \text{ Short circuit reverse current gain.}$$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0} \text{ Short circuit output impedance.}$$

- From the definitions of the g-parameters, it is seen that  $g_{11}$  has the dimensions of admittance,  $g_{21}$  and  $g_{12}$  are dimensionless, and  $g_{22}$  has the dimensions of impedance.
- By Equations (10.9) and (10.10), the equivalent circuit of a two-port network is as shown in the figure.

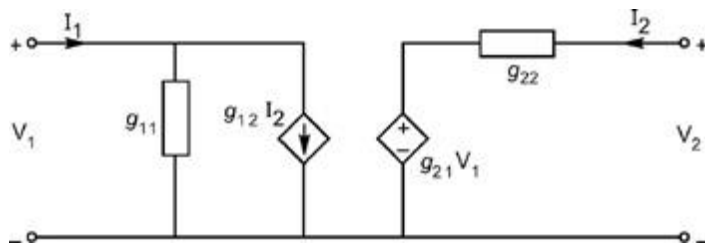
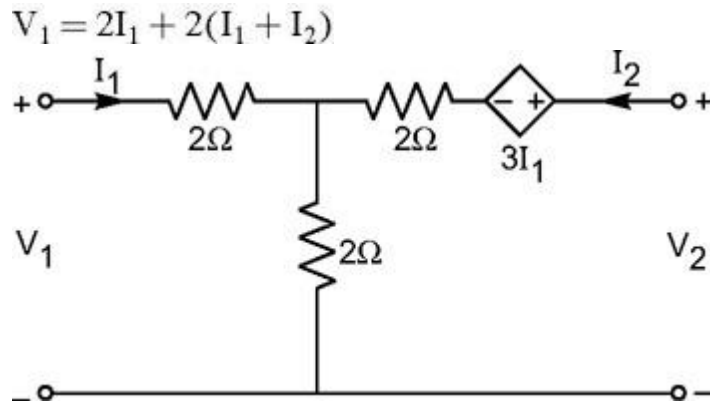


Figure 10.13: Equivalent circuit of a two-port network in terms of *g*-parameters

Example 10.6

Determine the Z- and g-parameters of the given network as shown in Figure 10.14

**Solution:** Two loop equations are



$$(1) \quad V_1 = 4I_1 + 2I_2$$

$$V_2 = 3I_1 + 2I_2 + 2(I_1 + I_2)$$

$$(2) \quad V_2 = 5I_1 + 4I_2$$

Comparing Equations (1) and (2) with the standard equation of the Z-parameters (10.1) and (10.2), we get

$$Z_{11} = 4, \quad Z_{12} = 2, \quad Z_{21} = 5, \quad Z_{22} = 4.$$

Now the standard equations of the g-parameters,

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad (\text{from 10.9})$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad (\text{from 10.9})$$

Rearranging Equation (1) into the form of Equation (10.9),

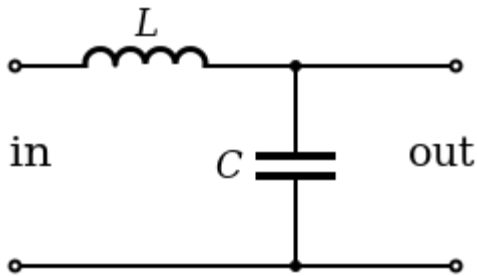
$$(1a) \quad I_1 = \frac{V_1}{4} - \frac{I_2}{2}.$$

### Topic 2:

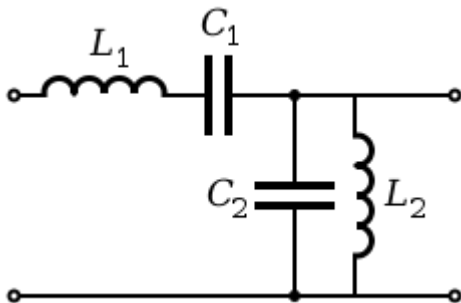
- Understand the image transfer constant

One-half the natural logarithm of the complex ratio of the steady-state apparent power entering and leaving a network terminated in its image impedance.

- The building block of constant k filters is the half-section "L" network, composed of a series impedance  $Z$ , and a shunt admittance  $Y$ . The "k" in "constant k" is the value given by



Constant k low-pass filter half section. Here inductance  $L$  is equal  $Ck^2$



Constant k band-pass filter half section.  
 $L_1 = C_2k^2$  and  $L_2 = C_1k^2$

$$k^2 = \frac{Z}{Y}$$

- Thus,  $k$  will have units of impedance, that is, ohms. It is readily apparent that in order for  $k$  to be constant,  $Y$  must be the dual impedance of  $Z$ .
- A physical interpretation of  $k$  can be given by observing that  $k$  is the limiting value of  $Z_i$  as the size of the section (in terms of values of its components, such as inductances, capacitances, etc.) approaches zero, while keeping  $k$  at its initial value. Thus,  $k$  is the characteristic impedance,  $Z_0$ , of the transmission line that would be formed by these infinitesimally small sections.
- It is also the image impedance of the section at resonance, in the case of band-pass filters, or at  $\omega = 0$  in the case of low-pass filters.<sup>[7]</sup> For example, the pictured low-pass half-section has

$$k = \sqrt{\frac{i\omega L}{i\omega C}} = \sqrt{\frac{L}{C}}$$

Elements L and C can be made arbitrarily small while retaining the same value of k. Z and Y however, are both approaching zero, and from the formulae (below) for image impedances,

$$\lim_{Z, Y \rightarrow 0} Z_i = k.$$

### Topic 3:

#### Driving Points and Transfer functions –using Transformed (S) Variables

Properties of Driving Point (Positive Real) Functions:

These conditions are required to satisfy to be positive realness Y(s) must be a rational function in s with real coefficients, i.e., the

- coefficients of the numerator and denominator polynomials is real and positive. The poles and zeros of Y(s) have either negative or zero real parts, i.e., Y(s) not have poles or zeros in the right half s plane. Poles of Y(s) on the imaginary axis must be simple and their residues must be real and positive, i.e., Y(s) not has multiple poles or zeros on the jω axis. The same statement applies to the poles of 1/Y(s).

The degrees of the numerator and denominator polynomials in Y(s) differ at most by 1. Thus the number of finite poles and finite zeros of Y(s) differ at most by 1.

The terms of lowest degree in the numerator and denominator polynomials of Y(s) differ in degree at most by 1. So Y(s) has neither multiple poles nor zeros at the origin. There be no missing terms in numerator and denominator polynomials unless all even or all odd terms are missing. Test for necessary and sufficient conditions:

Y(s) must be real when s is real. ♣ If Y(s) = p(s)/q(s), then p(s) + q(s) must be Hurwitz. This requires that: ♣i. the continued fraction expansion of the Hurwitz test gives only real and positive coefficients, and ii. the continued fraction expansion not end prematurely. In order that Re [Y(jω)] ♣ >= 0 for all ω, it is necessary and sufficient that

$$A(\omega^2) = m_1 m_2 - n_1 n_2 \Big|_{s=j\omega}$$

Have no real positive roots of odd multiplicity. This may be determined by factoring A(ω<sup>2</sup>) or by the use of Sturm's theorem.

EX: the function

### Topic 4: Poles and Zeros

The transfer function provides a basis for determining important system response characteristics without solving the complete differential equation. As defined, the transfer function is a rational function in the complex variable s = σ + jω, that is

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} \quad [1]$$

It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors

$$H(s) = \frac{N(s)}{D(s)} = K \frac{(s-z_1)(s-z_2)\dots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_{n-1})(s-p_n)} \quad [2]$$

Where the numerator and denominator polynomials,  $N(s)$  and  $D(s)$ , have real coefficients defined by the system's differential equation and  $K = b_m/a_n$ . As written in Eq. (2) the  $z_i$ 's are the roots of the equation

$$N(s) = 0$$

and are defined to be the system zeros, and the  $p_i$ 's are the roots of the equation

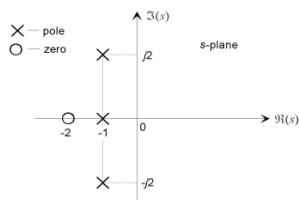
$$D(s) = 0$$

and are defined to be the system poles. In Eq. (2) the factors in the numerator and denominator are written so that when  $s = z_i$  the numerator  $N(s) = 0$  and the transfer function vanishes, that is

$$\lim_{s \rightarrow z_i} H(s) = 0$$

and similarly when  $s = p_i$  the denominator polynomial  $D(s) = 0$  and the value of the transfer function becomes unbounded,

$$\lim_{s \rightarrow p_i} H(s) = \infty$$



### Topic 5: Formulate image impedance expression.

- Image impedance is a similar concept to the characteristic impedance used in the analysis of transmission lines. In fact, in the limiting case of a chain of cascaded networks where the size of each single network is approaching an infinitesimally small element, the mathematical limit of the image impedance expression is the characteristic impedance of the chain. That is,

$$Z_i^2 \rightarrow \frac{Z}{Y}$$

- The connection between the two can further be seen by noting an alternative, but equivalent, definition of image impedance. In this definition, the image impedance of a network is the input impedance of an infinitely long chain of cascaded identical networks (with the ports arranged so that like impedance faces like). This is directly analogous to the definition of characteristic impedance as the input impedance of an infinitely long line.
- Conversely, it is possible to analyze a transmission line with lumped components, such as one utilizing loading coils, in terms of an image impedance filter

### Topic 6:

- Analyze the image transfer constant, designing of attenuator.
- The third parameter needed to complete the description is obtained by determining the ratios and when the second port is terminated in  $Z_{im2}$  and  $v_1$  is applied at the first port. The geometric mean of these two ratios is expressed as the exponential of a number  $\gamma$  and that  $\gamma$  is called the Image Transfer Constant. Image transfer constant defined.

$$e^\gamma = \sqrt{\frac{v_1}{v_2} \times \frac{i_1}{(-i_2)}} \text{ with } v_2 \text{ developed across } Z_{im2}.$$

- Image transfer constant can be expressed in terms of the ABCD parameters.

$$v_1 = Av_2 - Bi_2$$

$$i_1 = Cv_2 - Di_2$$

But, with  $Z_{im2}$  termination,  $-i_2 = \frac{v_2}{Z_{im2}}$ . Substituting this in the first ABCD equation,

we get,  $\frac{v_1}{v_2} = (A + \frac{B}{Z_{im2}})$ . Substituting  $v_2 = -Z_{im2}i_2$  in the second ABCD equation, we get,

$$\frac{i_1}{(-i_2)} = (D + CZ_{im2}). \text{ But } Z_{im2} = \sqrt{\frac{DB}{CA}}.$$

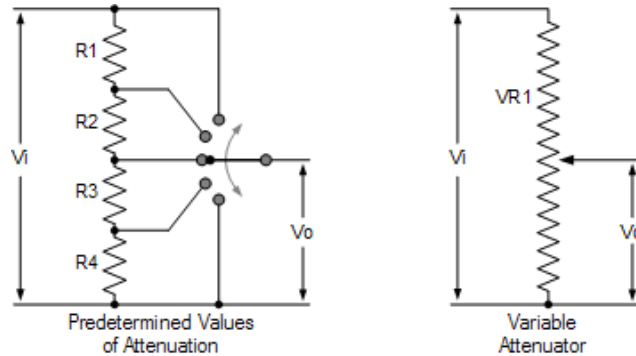
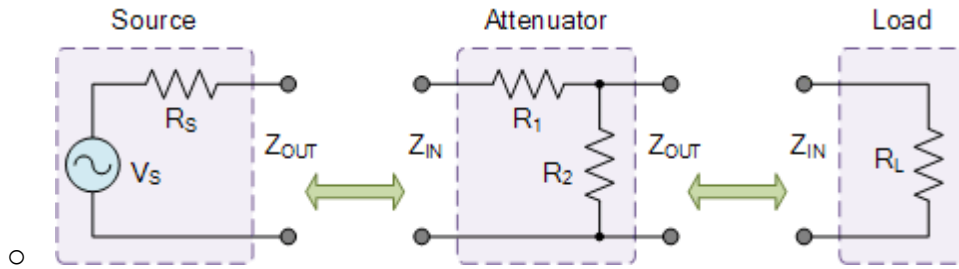
$$\therefore \frac{v_1}{v_2} = A + \frac{B}{Z_{im2}} = A + \frac{\sqrt{ABCD}}{D} \text{ and } \frac{i_1}{(-i_2)} = D + CZ_{im2} = D + \frac{\sqrt{ABCD}}{A}$$

$$\therefore e^\gamma = \sqrt{\frac{v_1}{v_2} \times \frac{i_1}{(-i_2)}} = \sqrt{AD + BC + 2\sqrt{ABCD}} = \sqrt{(\sqrt{AD} + \sqrt{BC})^2}$$

$$= \sqrt{AD} + \sqrt{BC} = \sqrt{AD} + \sqrt{AD - 1}$$

- $AD - BC = 1$
- A passive attenuator reduces the amount of power being delivered to the connected load by either a single fixed amount, a variable amount or in a series of known switchable steps. Attenuators are generally used in radio, communication and transmission line applications to weaken a stronger signal.





$$dB_v = 20 \log_{10} \frac{V_{out}}{V_{in}} \text{ (dB)}$$

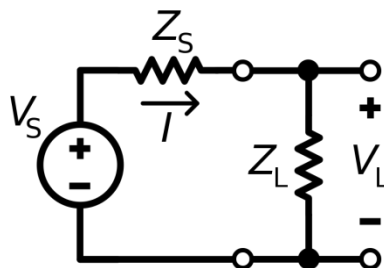
**Topic 7:**  
**Impedance Matching Networks**

In electronics, impedance matching is the practice of designing the input impedance of an electrical load or the output impedance of its corresponding signal source to maximize the power transfer or minimize signal reflection from the load.

In the case of a complex source impedance  $Z_S$  and load impedance  $Z_L$ , maximum power transfer is obtained when

$$Z_S = Z_L^*$$

where the asterisk indicates the complex conjugate of the variable. Where  $Z_S$  represents the characteristic impedance of a transmission line, minimum reflection is obtained when



$$Z_S = Z_L$$

- The concept of impedance matching found first applications in electrical engineering, but is relevant in other applications in which a form of energy, not necessarily electrical, is transferred between a source and a load.
- An alternative to impedance matching is impedance bridging, in which the load impedance is chosen to be much larger than the source impedance and maximizing voltage transfer, rather than power, is the goal.

### Topic 8: Foster's Reactance Theorem

For a positive real rational function  $Z(s)=1/Y(s)$  to be realizable as the driving point impedance of a lossless one-port, the necessary and sufficient condition is that it should be expressible in the form

$$Z(s) \text{ or } Y(s) = \frac{[a_n \cdot (s^2 + \omega_1^2) \cdot (s^2 + \omega_3^2) \dots]}{[b_m \cdot (s^2 + \omega_2^2) \cdot (s^2 + \omega_4^2) \dots]}$$

where  $a_n$  and  $b_m$  are constants and

4.  $0 < \omega < \omega_1 < \omega_2 < \omega_3$  (Interlacing poles and zeros, all on  $j\omega$  axis)
5. Foster's Theorem further restricts the degrees of the numerator,  $n$ , and denominator,  $m$ , by requiring that they must differ by unity. In other words, if the numerator is an even degree, the denominator is odd, and vice versa.

From these conditions, the following properties can be deduced:

1. Unity degree difference between numerator and denominator implies that  $Z(s)$  must have either a single pole or a single zero at both  $s=0$  and  $s=j\omega$ . Therefore the function  $Z(s)$  or  $Y(s)$  will belong to one of the four types:
2.  $Z(j\omega)$  is purely reactive. Therefore it can be written as

$$Z(j\omega) = jX(\omega)$$

where  $X(\omega)$  is the input reactance with

$$X(\omega) \text{ or } \frac{1}{X(\omega)} = \frac{[a_n \cdot (\omega_1^2 - \omega^2) \cdot (\omega_3^2 - \omega^2) \dots]}{[b_m \cdot \omega \cdot (\omega_2^2 - \omega^2) \cdot (\omega_4^2 - \omega^2) \dots]}$$

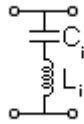
Alternation of poles and zeros leads to the property

$$0 < \frac{|X(\omega)|}{\omega} < \frac{d}{d\omega} X(\omega)$$

In other words, the reactance  $X(\omega)$  is always an increasing function of frequency. The rational functions satisfying these requirements are called Foster functions.

3. Since all poles of  $Z(s)$  and  $Y(s)$  are on the  $s=j\omega$  axis, they can always be expanded as

If  $Y(s)$  has a pole at  $s=j\omega_i$ , it can be extracted as a series resonator to ground:



$$L_i = \frac{1}{2 \cdot h_i} = \left. \frac{s}{(s^2 + \omega_i^2) \cdot Y(s)} \right|_{s=j\omega_i}, \quad C_i = \frac{1}{\omega_i^2 \cdot L_i}$$

- Note that if a pole of  $Z(s)$  at  $s=0$  or  $s=\infty$  is extracted, a zero appears at that frequency automatically in the remaining impedance function, which acts as a pole of the remaining admittance function.
- Hence, given  $Z(s)$ , one can synthesize a variety of circuits all having the same input impedance but with different structures by extracting elements in different orders from impedance or admittance functions.

### Topic 9 Design of Constant K, LP, HP, and BP filters

Prototype Low-pass Filter Design:

The design specifications are the values of cut-off frequency  $f_c$  and the values of load resistance  $R_L$ . We make the filter experience characteristic Impedance termination at  $\omega = 0$ . Both T-section and  $\Pi$ -Section have characteristic impedance of  $R_o$  at this frequency

$$\therefore R_o = \sqrt{\frac{L}{C}} = R_L$$

$$f_c = \frac{1}{\pi \sqrt{LC}}$$

Solving these two equations for  $L$  and  $C$ , we get,

$$L = \frac{R_L}{\pi f_c} \text{ H and } C = \frac{1}{\pi R_L f_c} \text{ F}$$

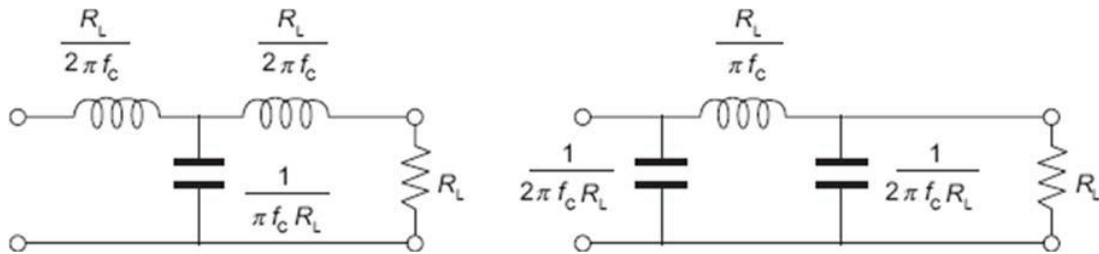
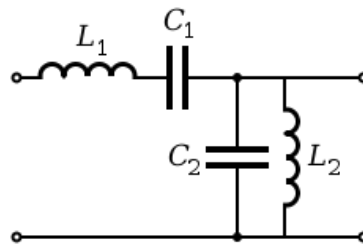


Fig: Prototype Low-Pass Filter Designs

Now, we address the issue of unsatisfactory rise in attenuation in the stop band in the prototype filter and arrive at a solution to this problem in the next section.

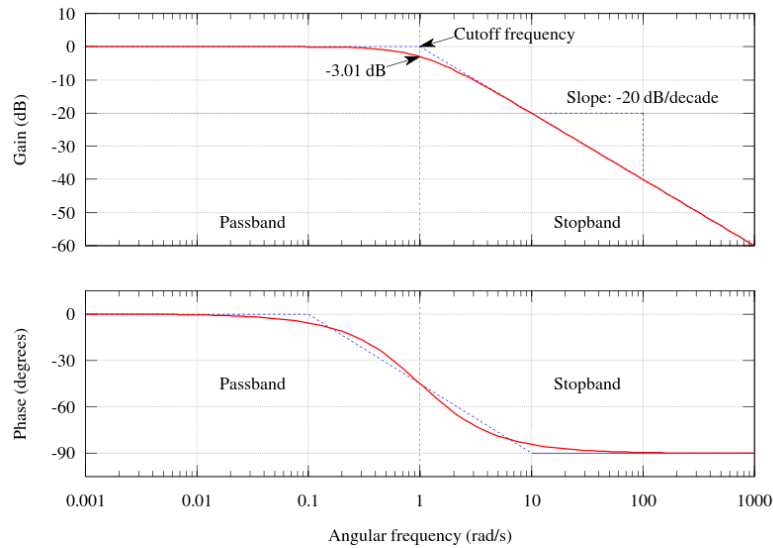
- **BP filter design:**



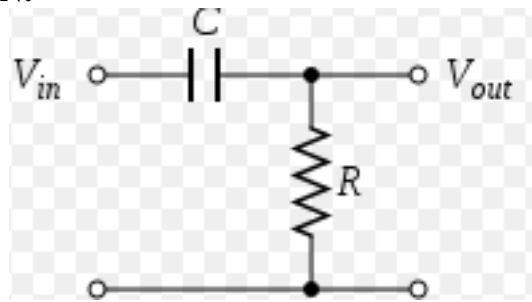
Constant k band-pass filters half section.

$$L_1 = C_2 k^2 \text{ and } L_2 = C_1 k^2$$

The Butterworth filter is a type of signal processing filter designed to have as flat a frequency response as possible in the pass band. It is also referred to as a maximally flat magnitude filter. It was first described in 1930 by the British engineer and physicist Stephen in his paper entitled "On the Theory of Filter Amplifiers".



### HP FILTER DESIGN:



The simple first-order electronic high-pass filter shown in Figure 1 is implemented by placing an input voltage across the series combination of a capacitor and a resistor and using the voltage across the resistor as an output. The product of the resistance and capacitance ( $R \times C$ ) is the time constant ( $\tau$ ); it is inversely proportional to the cutoff frequency  $f_c$ , that is,

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC},$$

## UNIT-IV

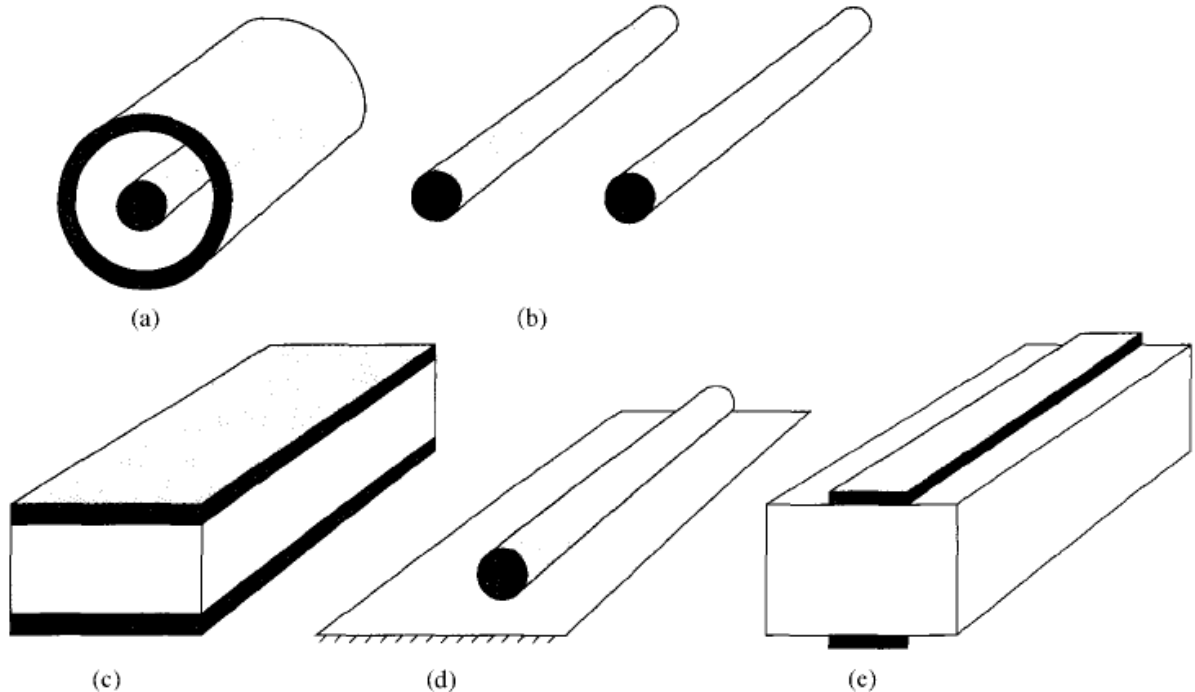
### 1. Various types of TRANSMISSION LINES

**Ans:** Our discussion in the previous chapter was essentially on wave propagation in unbounded media, media of infinite extent. Such wave propagation is said to be unguided in that the uniform plane wave exists throughout all space and EM energy associated with the wave spreads over a wide area. Wave propagation in unbounded media is used in radio or TV broadcasting, where the information being transmitted is meant for everyone who may be interested. Such means of wave propagation will not help in a situation like telephone conversation, where the information is received privately by one person.

Another means of transmitting power or information is by guided structures. Guided structures serve to guide (or direct) the propagation of energy from the source to the load. Typical examples of such structures are transmission lines and waveguides. Wave guides are discussed in the next chapter; transmission lines are considered in this chapter. Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). Various kinds of transmission lines such as the twisted-pair and coaxial cables (thin net and thick net) are used in computer networks such as the Ethernet and internet.

A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be a hydroelectric generator, a transmitter, or an oscillator; the load may be a factory, an antenna, or an oscilloscope, respectively.

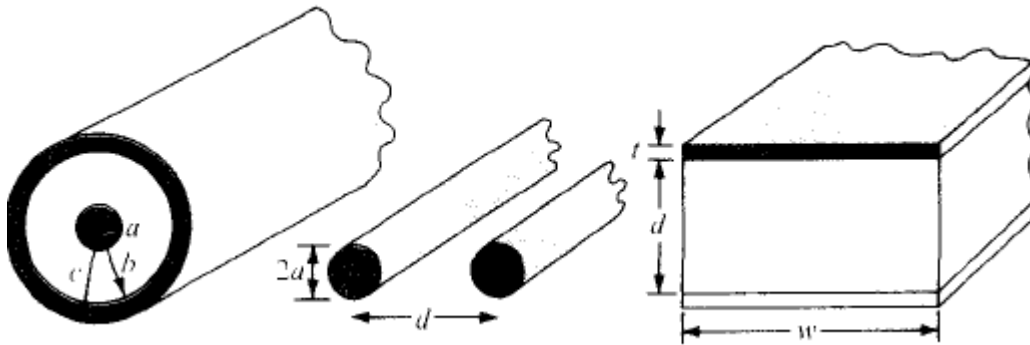
Typical transmission lines include coaxial cable, a two-wire line, a parallel-plate or planar line, a wire above the conducting plane, and a micro strip line. These lines are portrayed in Figure. Notice that each of these lines consists of two conductors in parallel. Coaxial cables are routinely used in electrical laboratories and in connecting TV sets to TV antennas. Micro strip lines are particularly important in integrated circuits where metallic strips connecting electronic elements are deposited on dielectric substrates. Transmission line problems are usually solved using EM field theory and electric circuit theory, the two major theories on which electrical engineering is based. In this chapter, we use circuit theory because it is easier to deal with mathematically. The basic concepts



of wave propagation (such as propagation constant, reflection coefficient, and standing wave ratio) covered in the previous chapter apply here. Our analysis of transmission lines will include the derivation of the transmission-line equations and characteristic quantities, the use of the Smith chart, various practical applications of transmission lines, and transients on transmission lines.

## 2. TRANSMISSION LINE PARAMETERS

**Ans:** It is customary and convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length  $R$ , inductance per unit length  $L$ , conductance per unit length  $G$ , and capacitance per unit length  $C$ . Each of the lines shown in Figure has specific formulas for finding  $R$ ,  $L$ ,  $G$ , and  $C$ . For coaxial, two-wire, and planar lines, the formulas for calculating the values of  $R$ ,  $L$ ,  $G$ , and  $C$  of the lines are as shown in Figure



The line parameters  $R$ ,  $L$ ,  $G$ , and  $C$  are not discrete or lumped but distributed as shown in Figure. By this we mean that the parameters are uniformly distributed along the entire length of the line.

For each line, the conductors are characterized by  $\sigma$ ,  $\epsilon$  and  $\mu$  the homogeneous dielectric separating the conductors is characterized by  $\sigma$ ,  $\epsilon$  and  $\mu$

$G = 1/R$ ;  $R$  is the ac resistance per unit length of the conductors comprising the line and  $G$  is the conductance per unit length due to the dielectric medium separating the conductors.

The value of  $L$  is the external inductance per unit length; that is,  $L = L_{ext}$ . The effects of internal inductance  $L_m (= R/\omega)$  are negligible as high frequencies at which most communication systems operate.

For each line,

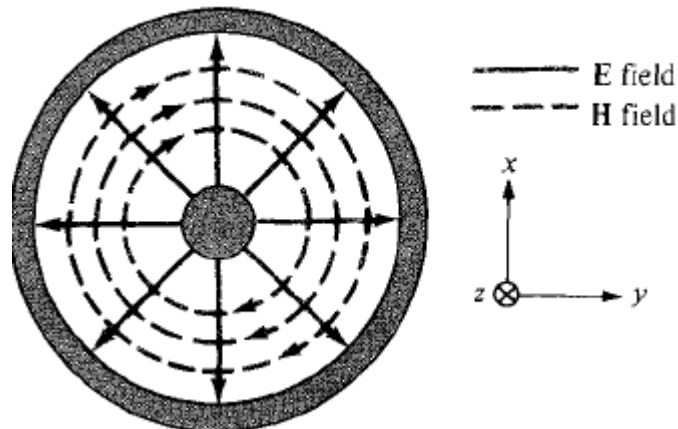
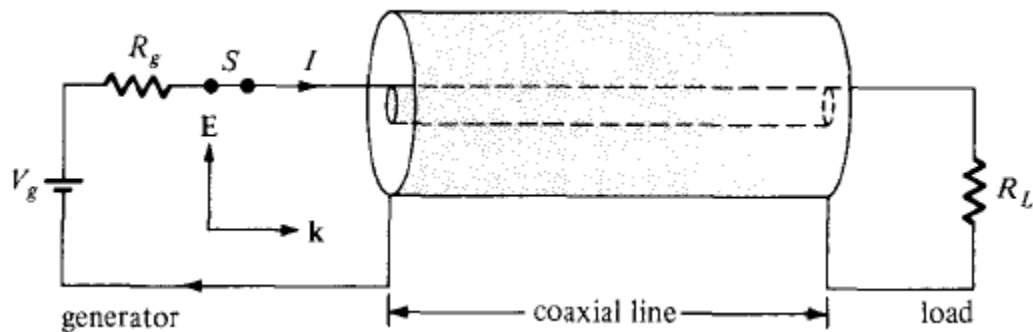
$$LC = \mu\epsilon \text{ and } G/C = \sigma/\epsilon$$

### 3. transverse electromagnetic (TEM) wave propagating along the line

**Ans:** let us consider how an EM wave propagates through a two-conductor transmission line. For example, consider the coaxial line connecting the generator or source to the load



as in Figure. When switch  $S$  is closed, the inner conductor is made positive with respect to the outer one so that the  $E$  field is radially outward as in Figure b. According to Ampere's law, the  $H$  field encircles the current carrying conductor as in Figure. The Poynting vector ( $E \times H$ ) points along the transmission line. Thus, closing the switch simply establishes a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line. This wave is a non-uniform plane wave and by means of it power is transmitted through the line.



#### 4. Lossless Line ( $R = 0 = G$ ) and Distortion less Line ( $R/L = G/C$ )

**Ans:** A transmission line is said to be **lossless** if the conductors of the line are perfect ( $\sigma \sim \alpha$ ) and the dielectric medium separating them is lossless ( $\sigma=0$ ). For such a line, it is evident  $R=0=G$

This is a necessary condition for a line to be lossless. Thus for such a line

$$\alpha = 0, \quad \gamma = j\beta = j\omega \sqrt{LC}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

$$X_o = 0, \quad Z_o = R_o = \sqrt{\frac{L}{C}}$$

### **Distortion less Line {R/L = G/C}**

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as  $\alpha$  is frequency dependent. This results in distortion.

A **distortion less line** is one in which the attenuation constant  $\alpha$  is frequency independent while the phase constant  $\beta$  is linearly dependent on frequency.

From the general expression for  $\alpha$  and  $\beta$

$$R/L=G/C$$

$$\alpha = \sqrt{RG}, \quad \beta = \omega \sqrt{LC}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

### **Note that**

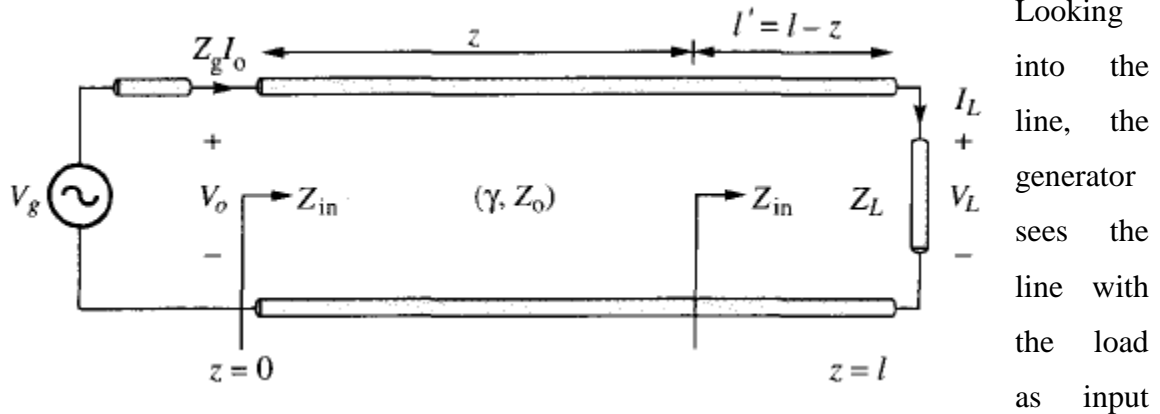
The phase velocity is independent of frequency because the phase constant  $\beta$  linearly depends on frequency. We have shape distortion of signals unless  $\alpha$  and  $u$  are independent of frequency.

$U$  and  $Z_o$  remain the same as for lossless lines.

A lossless line is also a distortion less line, but a distortion less line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortion less.

## **5. INPUT IMPEDANCE of transmission line**

**Ans:** Consider a transmission line of length  $\ell$ , characterized by  $\gamma$  and  $Z_0$ , connected to a load  $Z_L$  as shown in Figure.



Looking into the line, the generator sees the line with the load as input

impedance  $Z_{in}$ . It is our intention in this section to determine the input impedance, the standing wave ratio (SWR), and the power flow on the line.

Let the transmission line extend from  $z = 0$  at the generator to  $z = l$  at the load. First of all, we need the voltage and current waves

$$Z_{in} = Z_0 \left[ \frac{Z_L + Z_0 \tanh \gamma \ell}{Z_0 + Z_L \tanh \gamma \ell} \right]$$

Although above equation has been derived for the input impedance  $Z_{in}$  at the generation end, it is a general expression for finding  $Z_{in}$  at any point on the line. To find  $Z_{in}$  at a distance  $l$  from the load we replace  $l$  by  $l'$ . A formula for calculating the hyperbolic tangent of a complex number, required in above equation is found in

For a lossless line,  $\gamma = j\beta$ ,  $\tanh j\beta l = j \tan \beta l$ , and  $Z_0 = R_0$ , so above equation

becomes

$$Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell} \right]$$

Showing that the input impedance varies periodically with distance  $\ell$  from the load. The quantity  $\beta l$  in above equation is usually referred to as the electrical length of the line and can be expressed in degrees or radians.

## **UNIT-V**

- 1. voltage reflection coefficient current reflection coefficient and SWR**

**Ans:** We now define  $\Gamma_L$  as the voltage reflection coefficient (at the load).  $\Gamma_L$  is the ratio of the voltage reflection wave to the incident wave at the load, that is, from the known equations we can define as

$$V_o^+ = \frac{1}{2} (V_L + Z_o I_L) e^{\gamma \ell} \quad \text{_____ 1}$$

$$V_o^- = \frac{1}{2} (V_L - Z_o I_L) e^{-\gamma \ell} \quad \text{_____ 2}$$

The voltage reflection coefficient (at the load)

$$\Gamma_L = \frac{V_o^- e^{\gamma \ell}}{V_o^+ e^{-\gamma \ell}} \quad \text{_____ 3}$$

Substitute the equation 1 and 2 into the equation 3

Hence we can get

$$\Gamma_L = (Z_L - Z_o) / (Z_L + Z_o)$$

The **voltage reflection coefficient** at any point on the line is the ratio of the magnitude of the reflected voltage wave to that of the incident wave.

That is,

$$\Gamma(z) = \frac{V_o^- e^{\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{2\gamma z}$$

But  $z = l - l'$ . Substituting and combining with equation 1. We get

$$\Gamma(z) = \frac{V_o^-}{V_o^+} e^{2\gamma \ell} e^{-2\gamma l'} = \Gamma_L e^{-2\gamma l'}$$

The **current reflection coefficient** at any point on the line is negative of the voltage reflection coefficient at that point.

Just as we did for plane waves, we define the standing wave ratio  $s$  (otherwise denoted by SWR) as

$$S = V_{\max} / V_{\min} = I_{\max} / I_{\min}$$

## 2. Maxima and minima has the input impedance $Z_{in}$ of the transmission lines

**Ans:** It is easy to show that  $I_{\max} = V_{\max} / Z_o$  and  $I_{\min} = V_{\min} / Z_o$ . The input impedance  $Z_{in}$  in equation

$$Z_{in} = Z_o \left[ \frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \right]$$

Have maxima and minima that occur, respectively, at the maxima and minima of the voltage and current standing wave. It can also be shown that

$$Z_{in \max} = V_{\max} / I_{\min} = SZ_0$$

$$Z_{in \min} = V_{\min} / I_{\max} = Z_0 / S$$

### 3. Shorted Line ( $Z_L = 0$ ), Open-Circuited Line and Matched Line ( $Z_L = Z_0$ )

**Ans:** From known equation

$$Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Substitute shorted Line ( $Z_L = 0$ ) in the above equation we get

$$Z_{sc} = jZ_0 \tan \beta l$$

Also

$$\Gamma_L = -1 \text{ and } S = \infty$$

We notice from above  $Z_{in}$  equation that  $Z_{in}$  is a pure reactance, which could be capacitive or inductive depending on the value of  $l$ .

#### Open-Circuited Line ( $Z_L = \infty$ )

In this case, above  $Z_{in}$  equation becomes

$$Z_{oc} = -jZ_0 \cot \beta l$$

Also

$$\Gamma_L = 1 \text{ and } S = \infty$$

We notice that

$$Z_{sc} Z_{oc} = Z_0^2$$

#### Matched Line ( $Z_L = Z_0$ )

This is the most desired case from the practical point of view. For this case,  $Z_{in}$  equation reduces to  $Z_{in} = Z_0$

And

$$\Gamma_L = 0 \text{ and } S = 1$$

That is,  $V_{\text{ref}} = 0$ , the whole wave is transmitted and there is no reflection. The incident power is fully absorbed by the load. Thus maximum power transfer is possible when a transmission line is matched to the load.

#### 4. What are points should be noted about the Smith chart:

- At point Psc on the chart  $r = 0$ ,  $x = 0$ ; that is,  $ZL = 0 + j0$  showing that Psc represents a short circuit on the transmission line. At point Poc,  $r = \alpha$  and  $x = -\alpha$ , or  $ZL = -\alpha + j\alpha$ , which implies that Poc corresponds to an open circuit on the line. Also at Poc,  $r = 0$  and  $x = 0$ , showing that Poc is another location of a short circuit on the line.
- A complete revolution ( $360^\circ$ ) around the Smith chart represents a distance of  $\lambda/2$  on the line. Clockwise movement on the chart is regarded as moving toward the generator (or away from the load) as shown by the arrow G in figure a and b similarly, counter clockwise movement on the chart corresponds to moving toward the load (or away from the generator) as indicated by the arrow L in Figure. Notice from Figure (b) that at the load, moving toward the load does not make sense (because we are already at the load). The same can be said of the case when we are at the generator end.
- There are three scales around the periphery of the Smith chart as illustrated in Figure (a). The three scales are included for the sake of convenience but they are actually meant to serve the same purpose; one scale should be sufficient. The scales are used in determining the distance from the load or generator in degrees or wavelengths. The outermost scale is used to determine the distance on the line from the generator end in terms of wavelengths, and the next scale determines the distance from the load end in terms of wavelengths. The innermost scale is a

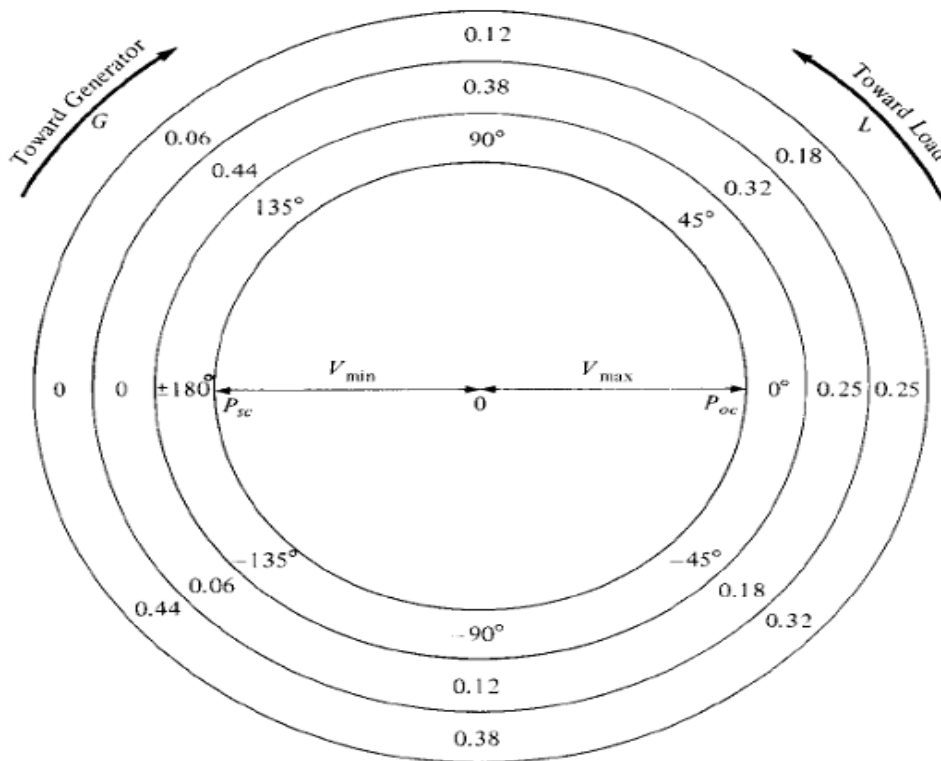


figure a

protractor

(in degrees) and is primarily used in determining  $\Gamma$ ; it can also be used to determine the distance from the load or generator. Since a  $\lambda/2$  distance on the line corresponds to a movement of  $360^\circ$  on the chart, a distance  $\lambda$  on the line corresponds to a  $720^\circ$  movement on the chart.  $720^\circ$  (11.55)

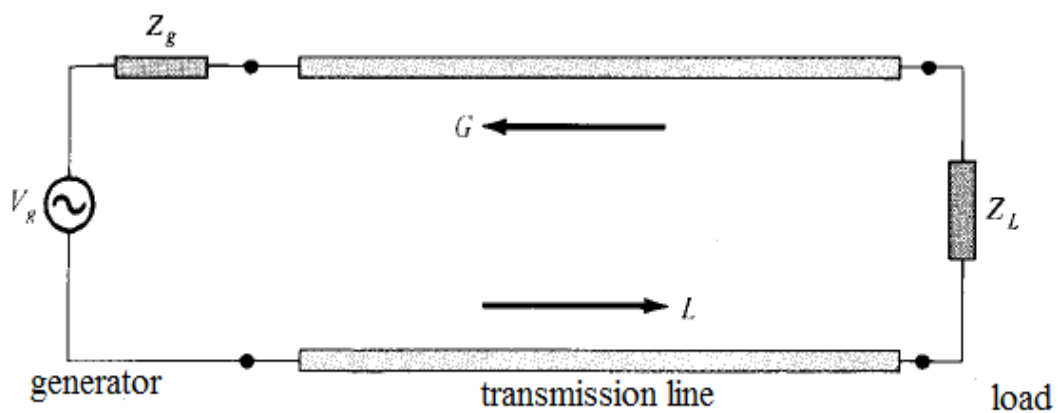


figure b

- Thus we may ignore the other outer scales and use the protractor (the innermost scale) for all our  $\Gamma$  and distance calculations.  $\lambda = 720^\circ$



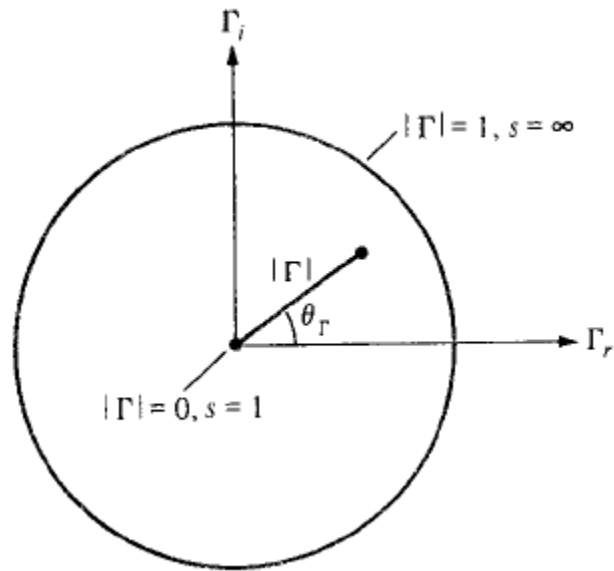
- $V_{max}$  occurs where  $Z_{in \ max}$  is located on the chart and that is on the positive  $T_r$  axis or on OPOC in Figure (a).  $V_{min}$  is located at the same point where we have  $Z_{in \ min}$  on the chart; that is, on the negative  $T_r$  axis or on OP sc in Figure (a). Notice that  $V_{max}$  and  $V_{min}$  (or  $Z_{in \ max}$  and  $Z_{in \ min}$ ) are  $\lambda/4$  (or  $180^\circ$ ) apart.
- The Smith chart is used both as impedance chart and admittance chart ( $Y = 1/Z$ ). As admittance chart (normalized impedance  $y = Y/Y_0 = g + jb$ ), the  $g$ - and  $b$  circles correspond to  $r$ - and  $x$ -circles, respectively.

**5. THE SMITH CHART**

**Ans:** Prior to the advent of digital computers and calculators, engineers developed all sorts of aids (tables, charts, graphs, etc.) to facilitate their calculations for design and analysis. To reduce the tedious manipulations involved in calculating the characteristics of transmission lines, graphical means have been developed. The Smith chart is the most commonly used of the graphical techniques. It is basically a graphical indication of the impedance of a transmission line as one moves along the line. It becomes easy to use after a small amount of experience.

We will first examine how the Smith chart is constructed and later employ it in our calculations of transmission line characteristics such as TL,  $s$ , and  $Z_{in}$ . We will assume that the transmission line to which the Smith chart will be applied is lossless ( $Z_0 = R_0$ ) although this is not fundamentally required.

The Smith chart is constructed within a circle of unit radius ( $|\Gamma| < 1$ ) as



shown in Figure. The construction of the chart is based on the relation in equation; that is

$$\Gamma_L = (Z_L - Z_0) / (Z_L + Z_0)$$

Where,  $\Gamma_r$  and  $\Gamma_i$  are the real and imaginary parts of the reflection coefficient  $F$ . Instead of having separate Smith charts for transmission lines with different characteristic impedances such as  $Z_0 = 60, 100, \text{ and } 120 \text{ ohm}$ , we prefer to have just one that can be used for any line. We achieve this by using a normalized chart in which all impedances are normalized with respect to the characteristic impedance  $Z_0$  of the particular line under consideration.

For the load impedance  $Z_L$ , for example, the normalized impedance  $Z_L$  is given by

$$Z_L = Z_L / Z_0 = r + jx$$

Normalizing and equating components, we obtain

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$x = \frac{2 \Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Rearranging terms in eq

$$\left[ \Gamma_r - \frac{r}{1+r} \right]^2 + \Gamma_i^2 = \left[ \frac{1}{1+r} \right]^2$$

$$[\Gamma_r - 1]^2 + \left[ \Gamma_i - \frac{1}{x} \right]^2 = \left[ \frac{1}{x} \right]^2$$

For typical values of the normalized resistance  $r$ , the corresponding centres and radii of the  $r$ -circles are presented in Table. Typical examples of the  $r$ -circles based on the data in TABLE Radii and Centres of  $r$ -Circles for Typical Values of  $r$

Normalized Resistance ( $r$ )	Radius $\left(\frac{1}{1+r}\right)$	Center $\left(\frac{r}{1+r}, 0\right)$
0	1	(0, 0)
1/2	2/3	(1/3, 0)
1	1/2	(1/2, 0)
2	1/3	(2/3, 0)
5	1/6	(5/6, 0)
$\infty$	0	(1, 0)

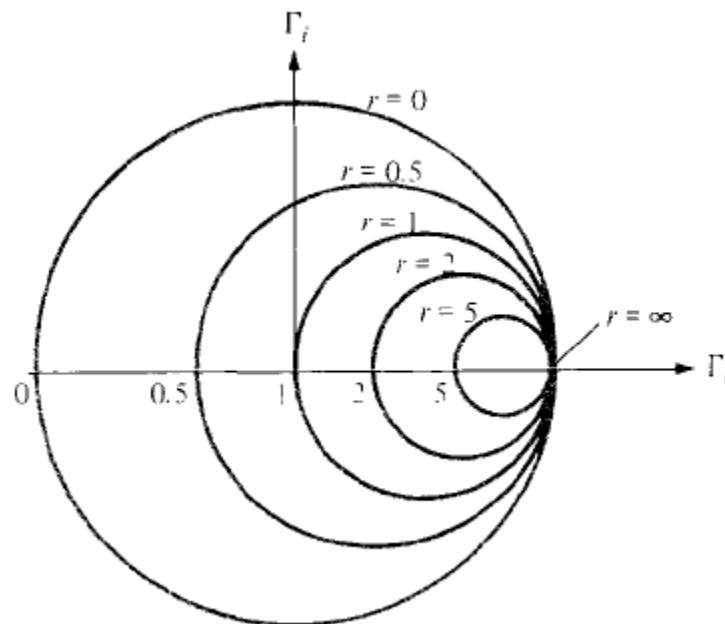


Table are shown in above figure. Similarly

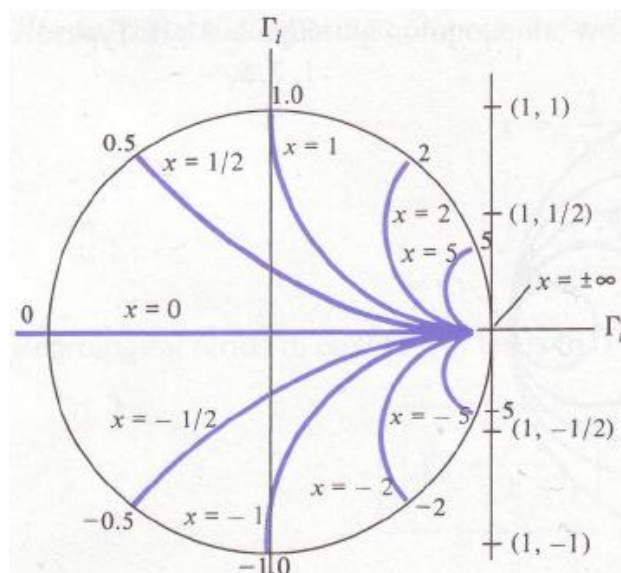
Below Table presents centres and radii of the x-circles for typical values of x, and below Figure shows the corresponding plots. Notice that while r is always positive, x can be positive (for inductive impedance) or negative (for capacitive impedance).

If we superpose the r-circles and x-circles, what we have is the Smith chart shown in Figure On the chart, we locate a normalized impedance  $z = 2 + j$ , for example, as the point of intersection of the  $r = 2$  circle and the  $x = 1$  circle. This is point P1 in Figure Similarly,  $z = 1 - j 0.5$  is located at P2, where the  $r = 1$  circle and the  $x = -0.5$  circle intersect. Apart from the r- and x-circles (shown on the Smith chart), we can draw the s-

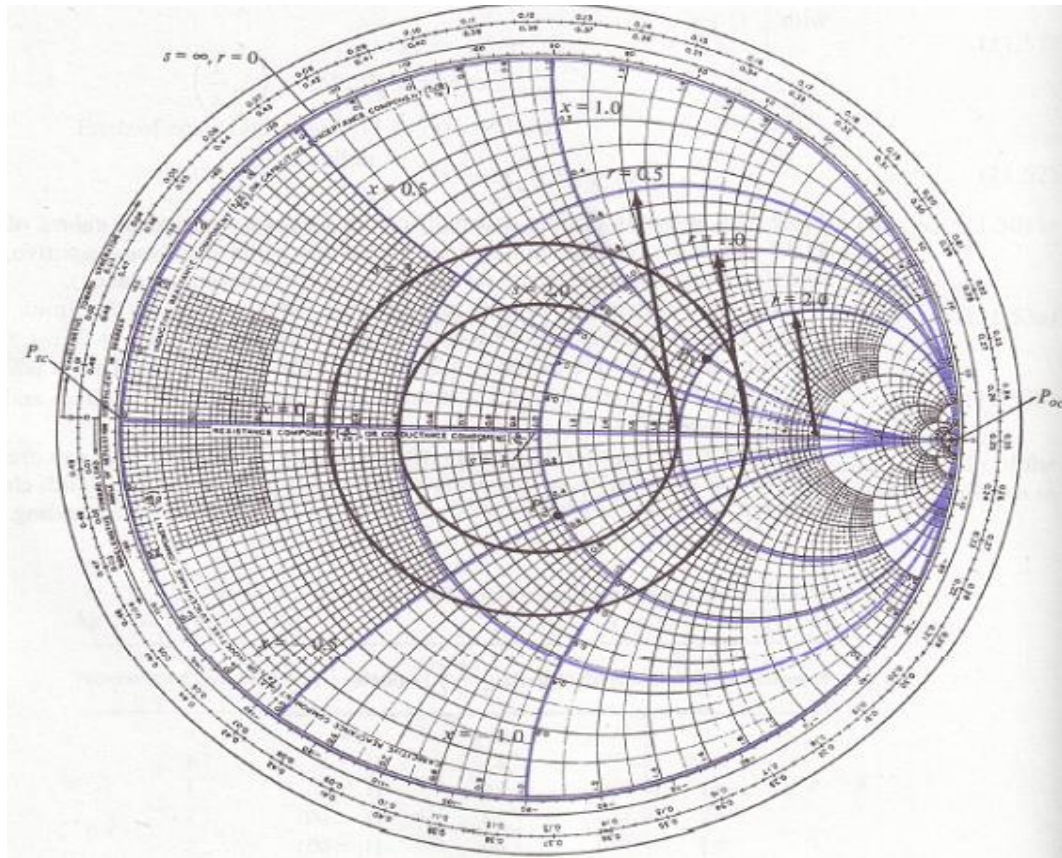
circles or constant standing-wave-ratio circles (always not shown on the Smith chart), which are centered at the origin with  $s$  varying from 1 to  $\infty$ . The value of the standing wave ratio  $s$  is

Below TABLE Radii and Centres of  $x$ -Circles for Typical Value of  $x$

Normalized Reactance ( $x$ )	Radius $\left(\frac{1}{x}\right)$	Center $\left(1, \frac{1}{x}\right)$
0	$\infty$	$(1, \infty)$
$\pm 1/2$	2	$(1, \pm 2)$
$\pm 1$	1	$(1, \pm 1)$
$\pm 2$	$1/2$	$(1, \pm 1/2)$
$\pm 5$	$1/5$	$(1, \pm 1/5)$
$\pm \infty$	0	$(1, 0)$



Determined by locating where an  $s$ -circle crosses the  $\Gamma_r$  axis. Typical examples of  $s$ -circles for  $s = 1, 2, 3$ , and  $\infty$  are shown in Figure



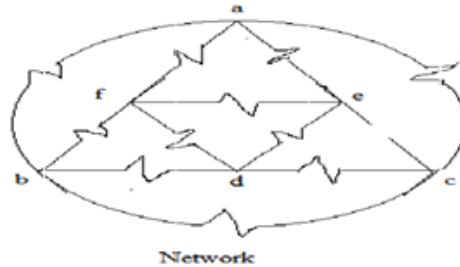
Since  $|\Gamma|$  and  $s$  are related according to equation the  $s$  -circles are sometimes referred to as  $|\Gamma|$ -circles with  $|\Gamma|$  varying linearly from 0 to 1 as we move away from the center  $O$  toward the periphery of the chart while  $s$  varies nonlinearly from 1 to  $=\alpha$

## 15. Tutorial topics and Questions

Network Topology, Basic cutset and tie set matrices for planar networks,

### Illustrative Problems

- For the given network draw a graph and a tree. Select suitable tree branch voltage and write the cut-set schedule. Write equation for the branch voltages in terms of tree branch?

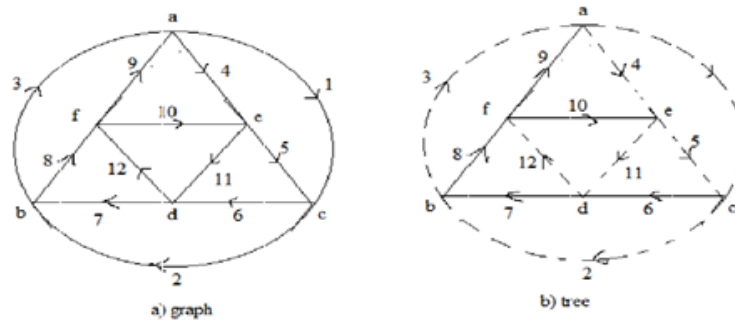


### Solution:

No of nodes of a tree,  $n_t = 6$

No of tree branches,  $n = 6 - 1 = 5$

Cut-set schedule:



Treebranch voltages	Branch voltages (v)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	-1	0	1	0	-1	0	0	1	0	0	-1	1
2	-1	0	1	-1	0	0	0	0	1	0	0	0
3	0	0	0	1	-1	0	0	0	0	1	-1	0
4	-1	1	0	0	-1	0	1	0	0	0	-1	1
5	-1	1	0	0	-1	1	0	0	0	0	0	0

Tree Branch	Basic Cut-Set
e1	1, 3, 5, 8, 11, 12
e2	1, 4, 3, 9
e3	4, 5, 10, 11
e4	1, 2, 5, 7, 11, 12
e5	1, 2, 5, 6

The equations are,  $r_1 = -e_1 - e_2 - e_4 - e_5$

$$r_2 = e_4 + e_5$$

$$r_3 = e_1 + e_2$$

$$r_4 = -e_2 + e_3$$

$$r_5 = -e_1 - e_3 - e_4 - e_5$$

$$r_6 = e_5$$

$$r_7 = e_4$$

$$r_8 = e_1$$

$$r_9 = e_2$$

$$r_{10} = e_3$$

$$r_{11} = -e_1 - e_3 - e_4$$

$$r_{12} = e_1 + e_4$$

Steady state and transient analysis of RC, RL and RLC Circuits

- 1) An input voltage  $v(t) = 10\sqrt{2}\cos(t+10^\circ) + 10\sqrt{5}\cos(2t+10^\circ)$  V is applied to a series combination of  $R = 1 \Omega$  and an inductance  $L = 1$  H. Find out the resulting steady state current  $i(t)$  in ampere?

Sol.  $V(t) = 10\sqrt{2}\cos(t + 10^\circ) + 10\sqrt{5}\cos(2t + 10^\circ)$

$$V(t) = 10\sqrt{2}\cos(t + 10^\circ) + 10\sqrt{5}\cos(2t + 10^\circ)$$

$$\omega_1 = 1, \omega_2 = 2$$

Steady state current  $i(t) = i_1(t) + i_2(t)$

$$= [10\sqrt{2}\cos(t + 10^\circ)/(R + j\omega_1 L)] + [10\sqrt{5}\cos(2t + 10^\circ)/(R + j\omega_2 L)]$$

$$= [10\sqrt{2}\cos(t + 10^\circ)/(1 + j)] + [10\sqrt{5}\cos(2t + 10^\circ)/(1 + 2j)]$$

$$= [10\sqrt{2}\cos(t + 10^\circ)/\sqrt{2}\angle 45^\circ + 10\sqrt{5}\cos(2t + 10^\circ)/\sqrt{5}\angle \tan^{-1}2$$

$$= 10\cos(t - 35^\circ) + 10\cos(2t + 10^\circ - \tan^{-1}2)$$

- 2) The R-L-C Series circuit with  $R = 13 \Omega$ ,  $L = 14 H$ ,  $C = 3 F$  has input voltage  $V(t) = \sin 2t$ . Determine the resulting current  $i(t)$ ?

Sol. Admittance  $Y = 1/R + 1/j\omega L + j\omega C = 3 + 4/j2 + j2 \times 3$   
 $= 3 - j2 + j6$   
 $= 3 + j4$

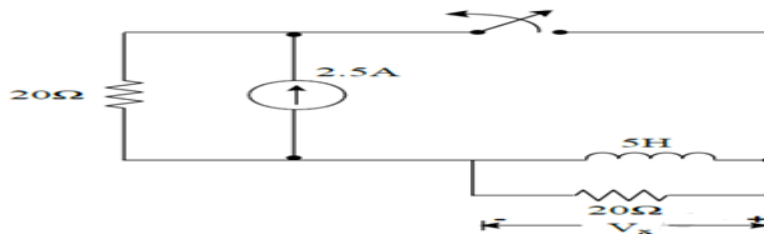
$$i(t) = V(t) \cdot Y$$

$$= (3 + j4) \sin 2t$$

$$= 5 \sin 2t \angle \tan^{-1}(4/3)$$

$$= 5 \sin(2t + 53.1^\circ)$$

- 3) In the figure, the switch was closed for a long time before opening at  $t = 0$ . Determine the voltage  $V_x$  at  $t = 0^+$ ?



Soln. When the switch was closed for a long time, the steady state is reached and inductor is short circuit.

$$IL(0^-) = 2.5 A$$

The current cannot change instantaneously in an inductor  $(0^-) = (0^+) = 2.5 A$

At  $t = 0^+$  inductor can be replaced by a current source of 2.5A. The equivalent circuit for the same is drawn.

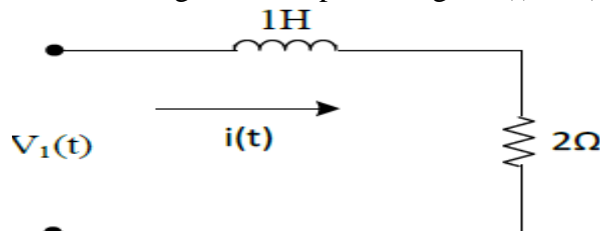
$$V = 20 \times 2.5$$

$$= 50V$$

$V_X = V$  is of opposite polarity

$$V_X = -50V$$

- 4) For the R-L circuit shown in the figure, the input voltage  $V_i(t) = u(t)$ . Plot the current  $i(t)$ ?



Sol.  $I(S) = V_1(S)/(LS+2) = 1/S(S+2)$   
 $= (1/2)[1/S - 1/(S+2)]$

$$i(t) = (1/2) - (1/2)e^{-2t}$$

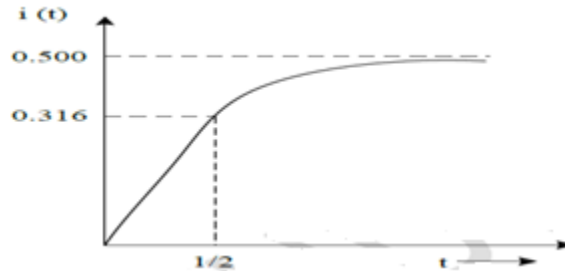
$$\text{At } t=0, i(0) = (1/2) - (1/2) = 0$$



$$Att=\infty, (\infty)= (1/2)$$

$$Att=1/2, (1/2)=(1/2)(1- e^{-1})$$

$$=(1/2)*(1-0.368)= 0.316$$



Design of constant K, LP, HP and BP Filters, Composite filter design

2. Draw the resonant curve frequencies.

**Solution: Resonance curve definition:**

A curve whose abscissas are frequencies lying near to and on both sides of the natural frequency of a vibrating system and whose ordinates are the corresponding amplitudes of the near-resonant vibrations

- Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a simple pendulum).
  - However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.
  - Some systems have multiple, distinct, resonant frequencies.
  - Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions.
  - Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters).
3. For a unity feedback system,  $G(s) = K/[s(s+4)(s+2)]$ . Sketch the nature of root locus showing all details on it. Comment on the stability of the system

Solution:

Given system is unity feedback system. Therefore  $H(s) = 1$ .

Therefore  $G(s)H(s) = K/[s(s+4)(s+2)]$ .

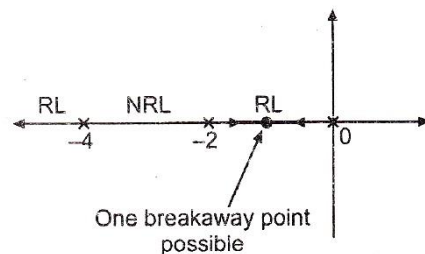
Step 1:

Poles = 0, -4, -2. Therefore  $P=3$ .

Zeros = there are no zeros.  $Z=0$ .

So all  $(P-Z=3)$  branches terminate at infinity.

Step 2: Pole-zero plot and sections of the real axis.



The pole-zero plot of the system is shown in the figure below. Here RL denotes Root Locus existence region and NRL denotes the non-existence region of root locus. These sections of real axis identified as a part of the root locus as to the right sum of poles and zeros is odd for those sections.

Step 3: Angle of asymptotes 'A line to which root locus touches at infinity is called asymptotes.'

Number of asymptotes =  $P-Z = 3$ . Therefore 3 asymptotes are approaching to infinity.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2$$

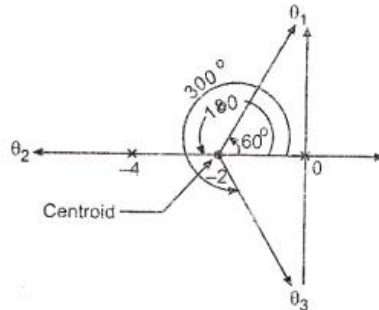
$$\therefore \theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{(2+1)180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{(2 \times 2+1)180^\circ}{3} = 300^\circ$$

Step 4: Centroid or Centre of asymptotes.

Asymptote touches real axis at a point called centroid.

Branches will approach infinity along these lines which are asymptotes.

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P - Z} = \frac{0 - 2 - 4}{3} = -2$$



Step 5: To find breakaway point, we have characteristic equation as,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\therefore s^3 + 6s^2 + 8s + K = 0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$$

Let  $3s^2 + 12s + 8 = 0$

$$\text{Roots i.e. breakaway points} = \frac{-12 \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3} = -0.845, -3.15$$

- As there is no root locus between -2 to -4, -3.15 cannot be a breakaway point. It also can be confirmed by calculating 'K' for s = -3.15. It will be negative that confirms s = -3.15 is not a breakaway point.

## UNIT-IV

### TOPIC 1: Transmission Lines-I

#### Questions:

1. Define transmission line?
2. What are the types of transmission lines?
3. What are the primary parameters of transmission lines?
4. What are the secondary parameters of transmission lines?
5. What is the generalized equation of transmission lines?

### TOPIC2: Loading

#### Questions:

1. Give the condition for maximum attenuation in transmission lines?
2. What is mean by line distortion in transmission lines?
3. What are various loads in transmission lines?
4. What is use of generalize transmission line equations?
5. What is maximum range of frequency transmission lines?

## UNIT-V

### TOPIC 1: Transmission Lines-II

#### Questions:

1. Specify the relation between VSWR and reflection coefficient in transmission line?
2. Give the expression for input impedance inters of reflection coefficient?
3. What is the use of impedance transformation?
4. What is mean by stub matching?
5. What are the advantages of stub matching?

### TOPIC 2: Smith chart

#### Questions:

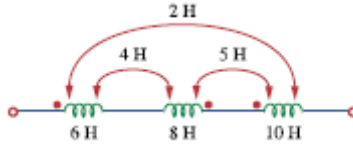
1. What is the use of smith chart?
2. List the application of smith chart?
3. Explain how an open circuit line acts as a circuit element?
4. Explain how a short circuit line acts as a circuit element?
5. Explain the construction of smith chart?

**16. Unit wise-Question bank with answers**

## UNIT -I

### 16. 1 Two marks of questions with answers

- 1) For the three coupled coils in Fig., calculate the total inductance.



Sol:

$$\text{For coil 1, } L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$$

$$\text{For coil 2, } L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$$

$$\text{For coil 3, } L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$$

$$LT = 4 - 1 + 7 = 10H$$

$$\text{or } LT = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$LT = 6 + 8 + 10 = \mathbf{10H}$$

- 2) Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150 mH. If the inductance of one coil ( $L_1$ ) is three times the other, find  $L_1$ ,  $L_2$ , and  $M$ . What is the coupling coefficient?

Sol:

$$L_1 + L_2 + 2M = 250 \text{ mH (1)}$$

$$L_1 + L_2 - 2M = 150 \text{ mH (2)}$$

Adding (1) and (2),

$$2L_1 + 2L_2 = 400 \text{ mH}$$

$$\text{But, } L_1 = 3L_2, \text{ or } 8L_2 + 400, \text{ and } L_2 = \mathbf{50 \text{ mH}}$$

$$L_1 = 3L_2 = \mathbf{150 \text{ mH}}$$

$$\text{From (2), } 150 + 50 - 2M = 150 \text{ leads to } M = \mathbf{25 \text{ mH}}$$

$$k = M / \sqrt{L_1 L_2} = 25 / \sqrt{150 \times 50} = \mathbf{0.2887}$$

- 3) Two coils are mutually coupled, with  $L_1 = 25 \text{ mH}$ ,  $L_2 = 60 \text{ mH}$ , and  $k = 0.5$ . Calculate the maximum possible equivalent inductance if: (a) the two coils are connected in series (b) the coils are connected in parallel

Sol: (a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 25 + 60 + 2(0.5) \sqrt{25 \times 60} = \mathbf{123.7 \text{ mH}}$$

(b) If they are connected in parallel,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{25 \times 60 - 19.36^2}{25 + 60 - 2 \times 19.36} \text{ mH} = \mathbf{24.31 \text{ mH}}$$

- 4) In the series circuit shown in figure, for series resonance, determine the value of the coupling coefficient  $k$ ?



Sol. The coils are connected in series additive manner as the current is entering the dot in both coils

$$L_{eq} = L_1 + L_2 + 2M$$

$$M = K\sqrt{(L_1 L_2)} = K\sqrt{(j2 \cdot j8)} = K/j4$$

$$\text{At resonance } |X_L| = |X_C|$$

$$X_L = X_{L1} + X_{L2} + X_M$$

$$= j2 + j8 + 2K/j4 = j10 + 2K/j4$$

$$\text{At resonance } X_L = X_C$$

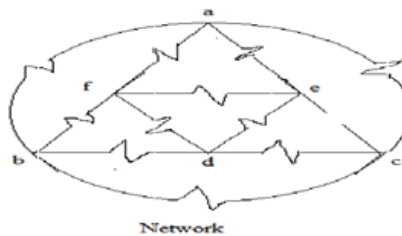
$$j10 + (2K/j4) = j12$$

$$2Kj \cdot 4 = j \cdot 2$$

$$jK \cdot 8 = j \cdot 2$$

$$K = 2/8 = 1/4 = 0.25$$

5. For the given network draw a graph and a tree. Select suitable tree branch voltage and write the cut-set schedule. Write equation for the branch voltages in terms of tree branch?

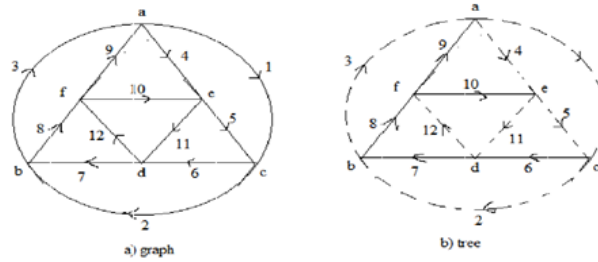


**Solution:**

No of nodes of a tree,  $n_t = 6$

No of tree branches,  $n = 6 - 1 = 5$

Cut-set schedule:



Treebranch voltages	Branch voltages (v)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	-1	0	1	0	-1	0	0	1	0	0	-1	1
2	-1	0	1	-1	0	0	0	0	1	0	0	0
3	0	0	0	1	-1	0	0	0	0	1	-1	0
4	-1	1	0	0	-1	0	1	0	0	0	-1	1
5	-1	1	0	0	-1	1	0	0	0	0	0	0

Tree Branch	Basic Cut-Set
e1	1, 3, 5, 8, 11, 12
e2	1, 4, 3, 9
e3	4, 5, 10, 11
e4	1, 2, 5, 7, 11, 12
e5	1, 2, 5, 6

The equations are,  $r_1 = -e_1 - e_2 - e_4 - e_5$

$$r_2 = e_4 + e_5$$

$$r_3 = e_1 + e_2$$

$$r_4 = -e_2 + e_3$$

$$r_5 = -e_1 - e_3 - e_4 - e_5$$

$$r_6 = e_5$$

$$r_7 = e_4$$

$$r_8 = e_1$$

$$r_9 = e_2$$

$$r_{10} = e_3$$

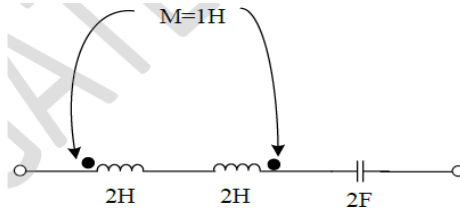
$$r_{11} = -e_1 - e_3 - e_4$$



$$r_{12} = e_1 + e_4$$

### 16.2 Three marks of questions with answers

- 5) The resonant frequency of the series circuit shown in figure is



Sol. The coils are connected in a series opposing way. As the current is entering the dot of coil L1 and leaving the dot of coil L2

$$\begin{aligned} Leq &= L1 + L2 - 2M \\ &= 2 + 2 - 2 \\ &= 2H \end{aligned}$$

At resonance  $X_L = X_C$

$$\omega Leq = 1/\omega C$$

$$\omega = 1/\sqrt{LeqC} = 1/\sqrt{(2 \times 2)} = 1/2 \text{ rad/sec}$$

$$f = 1/4\pi \text{ Hz}$$

- 6) In the series circuit shown in figure, for series resonance, determine the value of the coupling coefficient k?



Sol. The coils are connected in series additive manner as the current is entering the dot in both coils

$$Leq = L1 + L2 + 2M$$

$$M = K/\sqrt{(L1L2)} = K/\sqrt{(j2 \cdot j8)} = K/j4$$

At resonance  $|X_L| = |X_C|$

$$X_L = X_{L1} + X_{L2} + X_M$$

$$= j2 + j8 + 2K/j4 = j10 + 2K/j4$$

At resonance  $X_L = X_C$

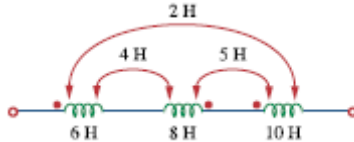
$$j10 + (2K/j4) = j/12$$

$$2Kj \cdot 4 = j \cdot 2$$

$$jK \cdot 8 = j \cdot 2$$

$$K = 2/8 = 1/4 = 0.25$$

- 7) For the three coupled coils in Fig., calculate the total inductance.



Sol:

$$\text{For coil 1, } L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$$

$$\text{For coil 2, } L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$$

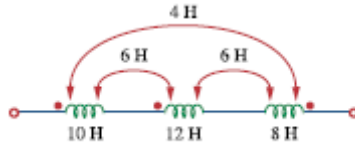
$$\text{For coil 3, } L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$$

$$LT = 4 - 1 + 7 = 10\text{H}$$

$$\text{or } LT = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$LT = 6 + 8 + 10 = \mathbf{10\text{H}}$$

- 8) Determine the inductance of the three series-connected inductors of Fig. 13.73.



Sol:

$$L = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$$

$$= 10 + 12 + 8 + 2 \times 6 - 2 \times 6 - 2 \times 4 = \mathbf{22\text{H}}$$

- 9) Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150 mH. If the inductance of one coil ( $L_1$ ) is three times the other, find  $L_1$ ,  $L_2$ , and  $M$ . What is the coupling coefficient?

Sol:

$$L_1 + L_2 + 2M = 250 \text{ mH (1)}$$

$$L_1 + L_2 - 2M = 150 \text{ mH (2)}$$

Adding (1) and (2),

$$2L_1 + 2L_2 = 400 \text{ mH}$$

$$\text{But, } L_1 = 3L_2, \text{ or } 8L_2 + 400, \text{ and } L_2 = \mathbf{50 \text{ mH}}$$

$$L_1 = 3L_2 = \mathbf{150 \text{ mH}}$$

$$\text{From (2), } 150 + 50 - 2M = 150 \text{ leads to } M = \mathbf{25 \text{ mH}}$$

$$k = M / \sqrt{L_1 L_2} = 25 / \sqrt{150 \times 50} = \mathbf{0.2887}$$

- 10) Two coils are mutually coupled, with  $L_1 = 25 \text{ mH}$ ,  $L_2 = 60 \text{ mH}$ , and  $k = 0.5$ . Calculate the maximum possible equivalent inductance if: (a) the two coils are connected in series (b) the coils are connected in parallel

Sol: (a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 25 + 60 + 2(0.5) \sqrt{25 \times 60} = \mathbf{123.7 \text{ mH}}$$

(b) If they are connected in parallel,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{25 \times 60 - 19.36^2}{25 + 60 - 2 \times 19.36} \text{ mH} = \mathbf{24.31 \text{ mH}}$$

### 16.3 Five marks of questions with answers

1. Explain the following terms?

- a) What is coefficient of coupling? Derive the formula for K?
- b) Discuss dot notation?

**Solution:**

**Coefficient of Coupling:**

It is the factor which indicates the degree of coupling between the coupled coils given by

$$K = \sqrt{\frac{\Phi_{21}}{\Phi_1} * \frac{\Phi_{12}}{\Phi_2}} \text{ ----- (1)}$$

Expressing (1) in terms of self and mutual inductances,

$$L_1 = \frac{N_1 \Phi_1}{i_1} \text{ ----- (2)}$$

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1} \text{ -----(3)}$$

$$L_2 = \frac{N_2 \Phi_{12}}{i_2} \text{ ----- (4)}$$

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2} \text{ -----(5)}$$

$$K = \sqrt{\left( \frac{M_{21} i_1 N_1}{N_2 L_1 i_1} \frac{M_{12} i_2 N_2}{N_1 L_2 i_2} \right)} = \sqrt{\frac{M_{21} N_1 M_{12} N_2}{L_1 N_2 L_2 N_1}}$$

If  $M_{21} = M_{12} = M$  then we get,

$$K = \sqrt{\frac{M^2}{L_1 L_2}} \text{ ----- (7)}$$

This will be equal to 1 if coils are coupled tightly

$$M = K \sqrt{L_1 L_2} \text{ ----- (8)}$$

Coefficient of coupling is also defined as the ratio of mutual flux to total flux. It is always less than one ( $K \leq 1$ ) (this is the principle used in transformer).

### **Dot Notation**

The Polarity or Dot Notation for a device with mutual inductance designates the relative instantaneous current directions of such device's winding leads.

The physical criteria of this notation are to preserve a proper magnetic flow circuit direction inside the material. Keeping all current entering a transformer or coupled inductances, in phase, through dotted leads or ports will keep the magnetic flow circulating without canceling each other, and the device will work under their rated efficiency.

Entering current on different dotted ports -one in a dotted port and the other with an un-dotted port- will cause the magnetic flow components to subtract each other, making the device working inefficiently. This criterion holds for devices with several coils, in order to have each of them aggregating in an additive way by entering dotted ports.

Leads of primary and secondary windings are said to be of the same polarity when instantaneous current entering the primary winding lead results in instantaneous current leaving the secondary winding lead as though the two leads were a continuous circuit. In the case of two windings wound around the same core in parallel, for example, the polarity will be the same on the same ends: A sudden (instantaneous) current in the first coil will induce a voltage opposing the sudden increase (Lenz's law) in the first and also in the second coil, because the inductive magnetic field produced by the current in the first coil traverses the two coils in the same manner. The second coil will, therefore, show an induced current opposite in direction to the inducing current in the first coil. Both leads behave like a continuous circuit, one current entering into the first lead and another current leaving the second lead.

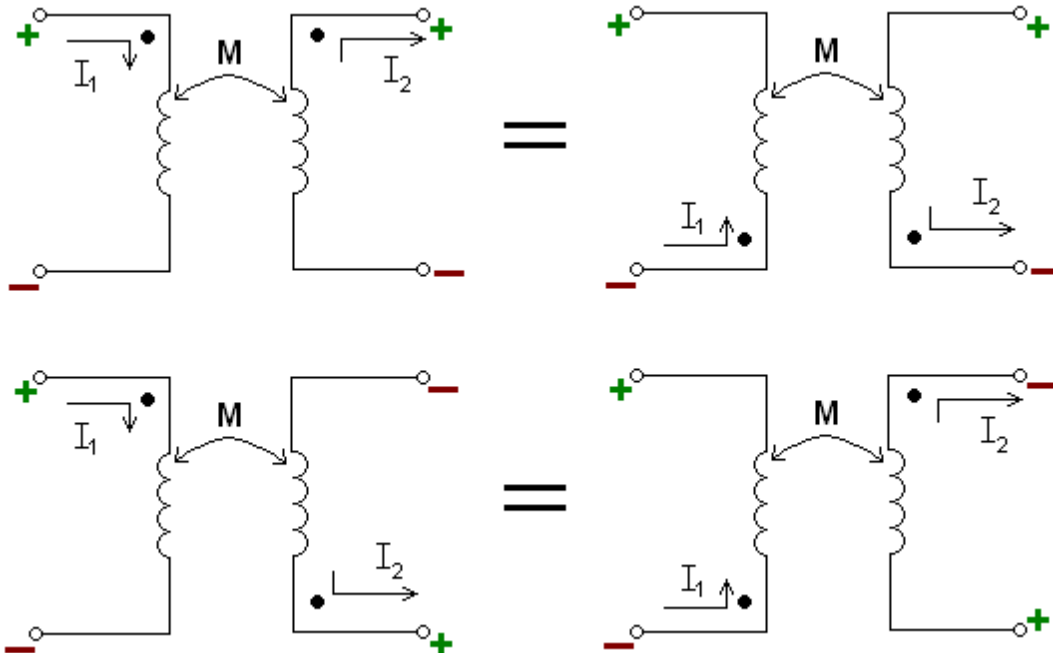
Referring to the circuit diagrams below:[dubious – discuss] The circuit polarity signs '+' and '-' indicate the relative polarities of the induced voltages in both coils, i.e. how an instantaneous (sudden) magnetic field traversing the primary and secondary coils induces a voltage in both coils.

The instantaneous polarities of the voltages across each inductor with respect to the dotted terminals are the same.

The circuit arrows indicate example applied and resultant relative current directions. The '+' and '-' polarities in the diagram are not the voltages driving the currents.

The instantaneous directions of the current entering the primary inductor at its dotted end and the current leaving of the secondary inductor at its dotted end are the same.

Subtractive polarity transformer designs are shown in the upper circuit diagrams. Additive polarity transformer designs are shown in the lower circuit diagrams.



## 2. Explain the concept of self and mutual inductance?

**Solution:**

**Self Induction:**

Inductance is the property of the circuit element which will oppose any change of current through it. By Faraday's laws of electromagnetic induction, it follows that whenever there is change of flux linking with a coil with time, and then there will be an induced emf in the coil. The induced emf is proportional to the rate of change flux linkages of the coil.

$$e \propto \frac{d\phi}{dx} \propto N \frac{d\Phi}{dt} \text{ ----- (1)}$$

Where N is the number of turns in the coil and  $\Phi$  is the flux in weber in the coil.

$$e = - N \frac{d\Phi}{dt}$$

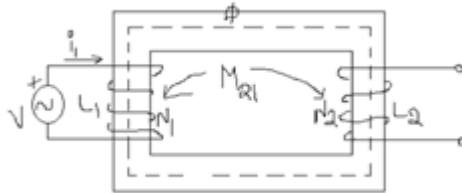
The negative sign indicates that the direction of induced emf is such that it opposes the every cause which is producing it, also known as LENZ'S law. Since the flux in the coil is directly

proportional to current flowing in it, the emf induced is proportional to the rate of change of current.

If the current  $I$  and flux linkages refer to the same physical system, then the parameter  $L$  is called self inductance. It is measured in HENRYS.

### Mutual Inductance:

Let us consider that there are two coils which are placed on the same magnetic core such that the flux produced by current flowing through one coil completely links with the other coils also. Let the coil1 is connected to AC supply and coil2 is open circuit.

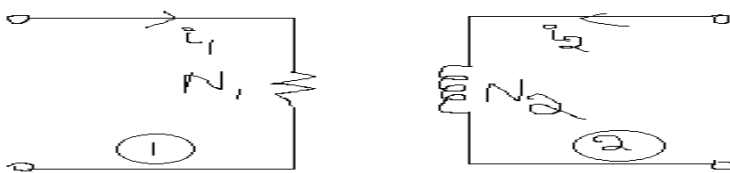


A current flowing in the first coil produces a flux as shown in fig. The direction of time varying flux is given by right hand thumb rule. The flux produced by current not only links with the coil1 but also links with coil2. The emf induced in coil1 is called self induced emf.

$$e_2 = N_2 \frac{d\Phi_1}{dt} = M_{21} \frac{di_1}{dt}$$

The proportionality constant  $M_{21}$  between induced emf in the second coil and rate of change of current in the first coil is called mutual inductance. Any two such coils are said to be magnetically coupled.

The unit of mutual inductance is HENRY. The mutual inductance between two coils is said to be 1 Henry when a change of current of 1 Amp/Sec in one coil produces a mutual induced emf of 1 volt in the other coil.



Self induced emf in coil1,

$$e_1 = L_{11} \frac{di_1}{dt}$$

Mutual induced emf in the coil2,

$$e_2 = M_{21} \frac{di_1}{dt}$$

Let us assume that second coil also carries a current of  $i_2$  as shown in fig, which in turn produces a self induced emf in coil2 and a mutual induced emf in coil1.

Self induced emf in coil2

$$L_{22} \frac{di_2}{dt}$$

Mutual induced emf in the coil1=

$$M_{21} \frac{di_2}{dt}$$

In practice all the flux produced by current in one coil may not completely link with the other coil. Depending on the position and orientation of the two coils, only a fraction of the flux may be linking with the other coil. Then the two circuits are said to be loosely coupled and if all the flux is linking with the other coil, then they are said to be tightly coupled.

If  $\Phi_1$  is the total flux produced by  $i_1$  and only  $\Phi_{21}$  is common and linking with second coil, then the fraction of the flux linking with coil2 is  $\frac{\Phi_{21}}{\Phi_1}$ . Similarly  $\Phi_2$  is the total flux produced by  $i_2$  and only  $\Phi_{12}$  is common and linking with first coil, then the fraction of the flux linking with coil1 is  $\frac{\Phi_{12}}{\Phi_2}$ . These fractions indicate the degree of coupling between the two coils. If the two coils are very close to one another and properly oriented then these fractions approach to unity.

### 3. Discuss series and parallel magnetic circuits?

#### **Solution: Composite Magnetic Circuits**

Consider a circular ring made from different materials of lengths  $l_1$ ,  $l_2$  and  $l_3$ , cross-sectional areas  $a_1$ ,  $a_2$  and  $a_3$  and relative permeability  $\mu_{r1}$ ,  $\mu_{r2}$  and  $\mu_{r3}$  respectively with a cut of length  $l_g$  known as air-gap. The total reluctance is the arithmetic sum of individual reluctances as they are joined in series.

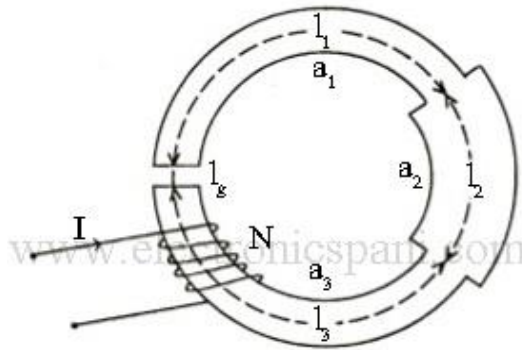


Figure 3: Composite Magnetic Circuit

$$S = \frac{l_1}{\mu_0 \mu_r a_1} + \frac{l_2}{\mu_0 \mu_r a_2} + \frac{l_3}{\mu_0 \mu_r a_3} + \frac{l_g}{\mu_0 a_g}$$

$$\text{Total } mmf = \phi \times S$$

$$= \phi \left[ \frac{l_1}{\mu_0 \mu_r a_1} + \frac{l_2}{\mu_0 \mu_r a_2} + \frac{l_3}{\mu_0 \mu_r a_3} + \frac{l_g}{\mu_0 a_g} \right]$$

Or Total ampere-turns required

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g + l_g$$

= Sum of ampere-turns required for individual parts of the magnetic circuit.

### Parallel Magnetic Circuits

In series circuits, all parts of the magnetic circuit carry same flux and total ampere-turns required to create a given flux is the arithmetic sum of the ampere-turns required for individual parts of the circuit.



But if the various paths of the magnetic circuit are in parallel, as shown in Fig. 4 the ampere-turns required for the combination is equal to the ampere-turns required to create the given flux in one path.

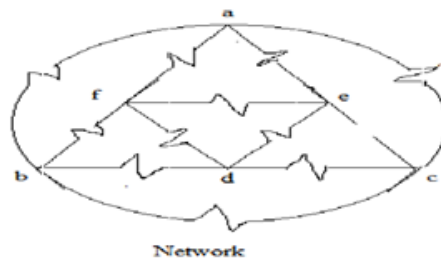
For example, for the circuit shown in Fig. 4 paths ABCD and AFED are in parallel, so ampere-turns required creating flux  $\phi_1$  in path ABCD is equal to ampere-turns required to create flux  $\phi_1$  in path AFED and also equal to the ampere-turns required for both of the paths.

Hence total ampere-turns required for magnetic circuit shown in Fig. 4.

$$= \text{AT for path DA} + \text{AT for path ABCD}$$

$$= \text{AT for path DA} + \text{AT for path AFED}$$

- 4. For the given network draw a graph and a tree. Select suitable tree branch voltage and write the cut-set schedule. Write equation for the branch voltages in terms of tree branch?**

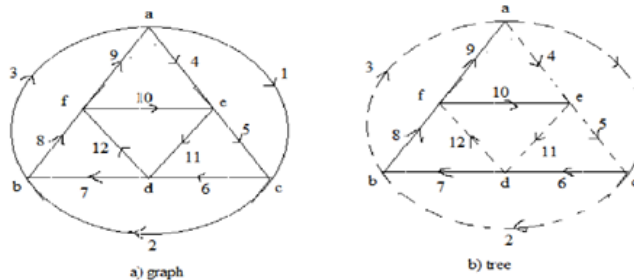


**Solution:**

No of nodes of a tree,  $n_t = 6$

No of tree branches,  $n = 6 - 1 = 5$

Cut-set schedule:



Treebranch voltages	Branch voltages (v)
---------------------	---------------------

	1	2	3	4	5	6	7	8	9	10	11	12
1	-1	0	1	0	-1	0	0	1	0	0	-1	1
2	-1	0	1	-1	0	0	0	0	1	0	0	0
3	0	0	0	1	-1	0	0	0	0	1	-1	0
4	-1	1	0	0	-1	0	1	0	0	0	-1	1
5	-1	1	0	0	-1	1	0	0	0	0	0	0

Tree Branch	Basic Cut-Set
e1	1, 3, 5, 8, 11, 12
e2	1, 4, 3, 9
e3	4, 5, 10, 11
e4	1, 2, 5, 7, 11, 12
e5	1, 2, 5, 6

The equations are,  $r_1 = -e_1 - e_2 - e_4 - e_5$

$$r_2 = e_4 + e_5$$

$$r_3 = e_1 + e_2$$

$$r_4 = -e_2 + e_3$$

$$r_5 = -e_1 - e_3 - e_4 - e_5$$

$$r_6 = e_5$$

$$r_7 = e_4$$

$$r_8 = e_1$$

$$r_9 = e_2$$

$$r_{10} = e_3$$

$$r_{11} = -e_1 - e_3 - e_4$$

$$r_{12} = e_1 + e_4$$

### 16.4. Objective questions with answers

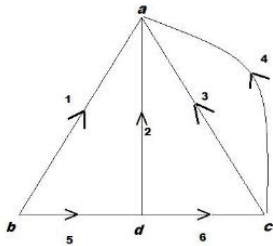
1. The current in a closed path in a loop is called?

- a) Loop current
- b) branch current
- c) link current
- d) twig current

2. Tie-set is also called?

- a) f loop
- b) g loop
- c) d loop
- d) e loop

3. Consider the graph shown below. If a tree of the graph has branches 4, 5, 6, then one of the twigs will be?



- a) 1
- b) 2
- c) 3
- d) 4

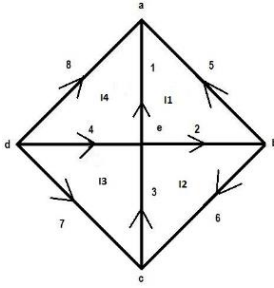
4. Consider the graph shown in the question 3 above. If a tree of the graph has branches 4, 5, 6, then one of the links will be?

- a) 3
- b) 4
- c) 5
- d) 6

5. The loop current direction of the basic loop formed from the tree of the graph is?

- a) same as the direction of the branch current
- b) opposite to the direction of the link current
- c) same as the direction of the link current
- d) opposite to the direction of the branch current

6. Consider the graph shown below. The direction of the loop currents will be? (ACW – Anticlockwise, CW – Clockwise).



- a)  $I_1$  ACW
- b)  $I_2$  ACW
- c)  $I_3$  CW
- d)  $I_4$  ACW

7. For Tie-set matrix, if the direction of current is same as loop current, then we place \_\_\_\_ in the matrix.

- a) +1
- b) -1
- c) 0
- d) +1 or -1

8. If a row of the tie set matrix is as given below, then its corresponding equation will be?

1 2 3 4 5 6 7 8

$I_1$  -1 +1 0 0 +1 0 0 0

- a)  $-V_1+V_2+V_3=0$
- b)  $-I_1+I_2+I_3=0$
- c)  $-V_1+V_2-V_3=0$
- d)  $-I_1+I_2-I_3=0$

9. The matrix formed by link branches of a tie set matrix is?

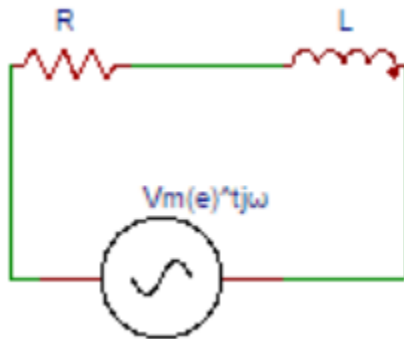
- a) Row matrix
- b) Column matrix
- c) Diagonal matrix
- d) Identity matrix

Answer Key: 1.c 2.a 3.d 4.a 5.c 6.a 7.a 8.a 9.d

### 16.5 Fill in the blanks of questions with answers

1. Impedance is a complex quantity having the real part as \_\_\_\_\_ and the imaginary part as \_\_\_\_\_

2. The voltage function  $v(t)$  in the circuit shown below is?



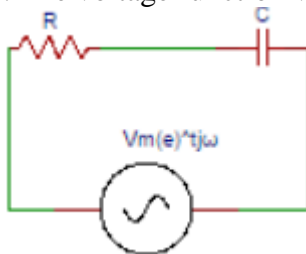
3. The current  $i(t)$  in the circuit shown above is?

4. The impedance of the circuit shown below is?

5. The magnitude of the impedance of the circuit shown above.

6. The phase angle between current and voltage in the circuit shown above is?

7. The voltage function  $v(t)$  in the circuit shown below is?



8. The impedance of the circuit shown above is?

9. The magnitude of the impedance of the circuit shown above is?

10. The angle between resistance and impedance in the circuit shown above.

**Answer Key:**

1. resistance, reactance 2.  $v(t) = V_m e^{jt\omega}$

2.  $v(t) = V_m e^{jt\omega}$

3.  $i(t) = (V_m / (R + j\omega L)) e^{jt\omega}$

4.  $R + j\omega L$

5.  $\sqrt{R^2 + (\omega L)^2}$

6.  $\tan^{-1}(\omega L / R)$

7.  $v(t) = V_m e^{jt\omega}$

8.  $R + 1/j\omega C$

9.  $\sqrt{R^2 + (1/\omega C)^2}$

10.  $\tan^{-1} 1/\omega RC$

## UNIT –II

### Two marks of questions with answers

1. Determine the resonant curve frequencies.

**Solution: Resonance curve definition:**

A curve whose abscissas are frequencies lying near to and on both sides of the natural frequency of a vibrating system and whose ordinates are the corresponding amplitudes of the near-resonant vibrations

- Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a simple pendulum).
- However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.
- Some systems have multiple, distinct, resonant frequencies.
- Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions.
- Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters).

2. Obtain the step response of R-L and R-C series circuits for dc excitation?

**Solution:**

Using the Laplace transform as part of your circuit analysis provides you with a prediction of circuit response. Analyze the poles of the Laplace transform to get a general idea of output behavior. Real poles, for instance, indicate exponential output behavior.

Follow these basic steps to analyze a circuit using Laplace techniques:

- Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.
- Apply the Laplace transformation of the differential equation to put the equation in the s-domain.
- Algebraically solve for the solution, or response transform.
- Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

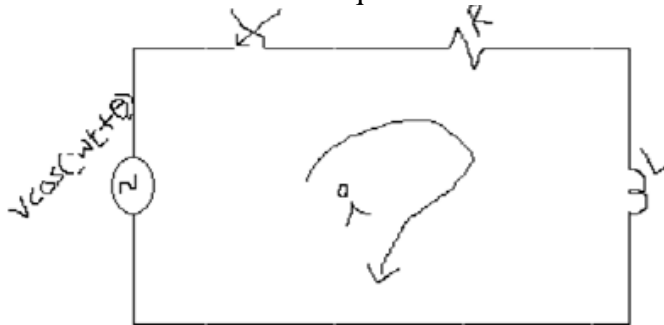
To get comfortable with this process, you simply need to practice applying it to different types of circuits such as an RC (resistor-capacitor) circuit, an RL (resistor-inductor) circuit, and an RLC (resistor-inductor-capacitor) circuit.

### 3. Derive the expression for transient response of R-L series circuit for ac excitation?

**Solution:**

#### Sinusoidal Response of R-L Circuit:

Consider a circuit consisting of resistance and inductance are connected as shown in fig. the switch S is closed at  $t=0$ . At  $t=0$ , sinusoidal voltage  $V\cos(\omega t + \theta)$  applied to RL circuit, where  $V$  is the amplitude of the wave and  $\theta$  is the phase angle. Application of Kirchhoff's laws we can determine the differential equations.



$$V\cos(\omega t + \theta) = iR + L \frac{di}{dt} \text{ ----- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \cos(\omega t + \theta) \text{ ----- (2)}$$

The corresponding characteristic equation is

$$(D + \frac{R}{L}) i = \frac{V}{L} \cos(\omega t + \theta) \text{ ----- (3)}$$

For the above equation, the solution consists of two parts.

One is complementary function and other is particular integral.

The complementary function of the solution is

$$i_c = c e^{-t(\frac{R}{L})} \text{ ----- (4)}$$

The particular solution can be obtained by using undetermined co-efficient.



By assuming,

$$i_p = A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \text{ ----- (5)}$$

$$i_p^1 = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) \text{ ----- (6)}$$

Substituting equations 5 & 6 in 3 we get,

$$\{-A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) + \frac{R}{L}[A \cos(\omega t + \theta) + B \sin(\omega t + \theta)]\} = \frac{V}{L} \cos(\omega t + \theta)$$

$$(-A\omega + \frac{BR}{L})\sin(\omega t + \theta) + (B\omega + \frac{AR}{L})\cos(\omega t + \theta) = \frac{V}{L} \cos(\omega t + \theta)$$

4. Analyze the open loop poles, zeros and a number of branches using root locus method.

General steps for drawing the Root Locus of the given system:

- 1. Procedure for determining the open loop poles, zeros and a number of branches from given  $G(s)H(s)$ .
  2. Draw the pole-zero plot and determine the region of real axis for which the root locus exists. Also, determine the number of breakaway points (This will be explained while solving the problems).
  3. Calculate the angle of asymptotes.
  4. Determine the centroid.
  5. Calculate the breakaway points (if any).
  6. Calculate the intersection point of root locus with the imaginary axis.
  7. Calculate the angle of departure or angle of arrivals if any.
  8. From above steps draw the overall sketch of the root locus.
  9. Predict the stability and performance of the given system by the root locus.

**Three marks of questions with answers**

4. Draw the resonant curve frequencies.

**Solution: Resonance curve definition:**

A curve whose abscissas are frequencies lying near to and on both sides of the natural frequency of a vibrating system and whose ordinates are the corresponding amplitudes of the near-resonant vibrations

- Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a simple pendulum).
  - However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.
  - Some systems have multiple, distinct, resonant frequencies.
  - Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions.
  - Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters).
5. For a unity feedback system,  $G(s) = K/[s(s+4)(s+2)]$ . Sketch the nature of root locus showing all details on it. Comment on the stability of the system

Solution:

Given system is unity feedback system. Therefore  $H(s) = 1$ .

Therefore  $G(s)H(s) = K/[s(s+4)(s+2)]$ .

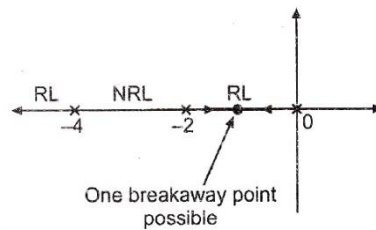
Step 1:

Poles = 0, -4, -2. Therefore  $P=3$ .

Zeros = there are no zeros.  $Z=0$ .

So all  $(P-Z=3)$  branches terminate at infinity.

Step 2: Pole-zero plot and sections of the real axis.



The pole-zero plot of the system is shown in the figure below. Here RL denotes Root Locus existence region and NRL denotes the non-existence region of root locus. These sections of real axis identified as a part of the root locus as to the right sum of poles and zeros is odd for those sections.

Step 3: Angle of asymptotes 'A line to which root locus touches at infinity is called asymptotes.'

Number of asymptotes = P-Z = 3. Therefore 3 asymptotes are approaching to infinity.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2$$

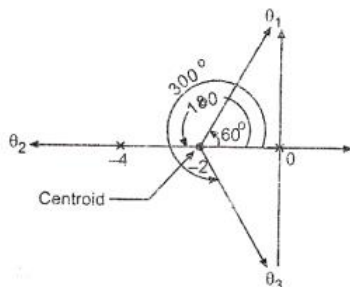
$$\therefore \theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{(2+1)180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{(2 \times 2 + 1)180^\circ}{3} = 300^\circ$$

Step 4: Centroid or Centre of asymptotes.

Asymptote touches real axis at a point called centroid.

Branches will approach infinity along these lines which are asymptotes.

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z} = \frac{0 - 2 - 4}{3} = -2$$



Step 5: To find breakaway point, we have characteristic equation as,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\therefore s^3 + 6s^2 + 8s + K = 0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$$

$$\text{Le } 3s^2 + 12s + 8 = 0$$

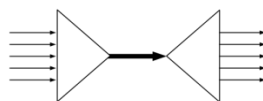
$$\text{Roots i.e. breakaway points} = \frac{-12 \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3} = -0.845, -3.15$$

- As there is no root locus between -2 to -4, -3.15 cannot be a breakaway point. It also can be confirmed by calculating 'K' for  $s = -3.15$ . It will be negative that confirms  $s = -3.15$  is not a breakaway point.

### 3. Analyze the circuit switching

#### **Circuit switching:**

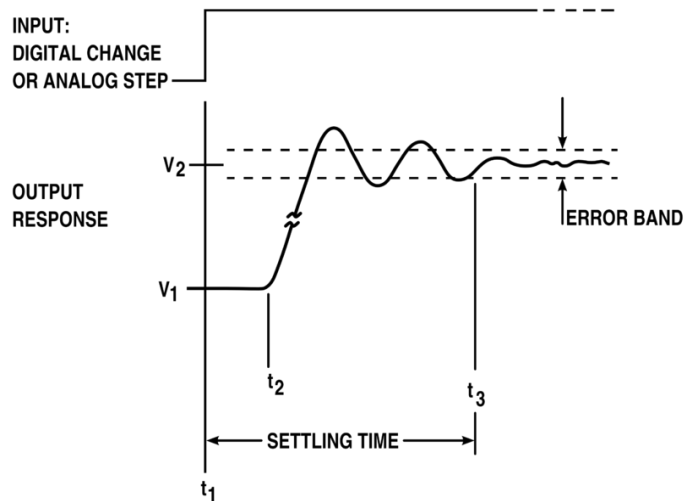
- Circuit switching is a method of implementing a telecommunications network in which two network nodes establish a dedicated communications channel (circuit) through the network before the nodes may communicate.
- The circuit guarantees the full bandwidth of the channel and remains connected for the duration of the communication session. The circuit functions as if the nodes were physically connected as with an electrical circuit.
- The defining example of a circuit-switched network is the early analog telephone network. When a call is made from one telephone to another, switches within the telephone exchanges create a continuous wire circuit between the two telephones, for as long as the call lasts.



Multiplexing

- Circuit switching contrasts with packet switching which divides the data to be transmitted into packets transmitted through the network independently.

- In packet switching, instead of being dedicated to one communication session at a time, network links are shared by packets from multiple competing communication sessions, resulting in the loss of the quality of service guarantees that are provided by circuit switching.
- In circuit switching, the bit delay is constant during a connection, as opposed to packet switching, where packet queues may cause varying and potentially indefinitely long packet transfer delays.
- No circuit can be degraded by competing users because it is protected from use by other callers until the circuit is released and a new connection is set up. Even if no actual communication is taking place, the channel remains reserved and protected from competing users.
- Virtual circuit switching is a packet switching technology that emulates circuit switching, in the sense that the connection is established before any packets are transferred, and packets are delivered in order.
- While circuit switching is commonly used for connecting voice circuits, the concept of a dedicated path persisting between two communicating parties or nodes can be extended to signal content other than voice.
- Its advantage is that it provides for continuous transfer without the overhead associated with packets making maximal use of available bandwidth for that communication. Its disadvantage is that it can be relatively inefficient because unused capacity guaranteed to a connection cannot be used by other connections on the same network.
- The step response of a system in a given initial state consists of the time evolution of its outputs when its control inputs are Heaviside step functions.
- In electronic engineering and control theory, step response is the time behavior of the outputs of a general system when its inputs change from zero to one in a very short time.
- The concept can be extended to the abstract mathematical notion of a dynamical system using an evolution parameter.
- From a practical standpoint, knowing how the system responds to a sudden input is important because large and possibly fast deviations from the long term steady state may have extreme effects on the component itself and on other portions of the overall system dependent on this component.
- In addition, the overall system cannot act until the component's output settles down to some vicinity of its final state, delaying the overall system response.
- Formally, knowing the step response of a dynamical system gives information on the stability of such a system, and on its ability to reach one stationary state when starting from another.



5. General steps for drawing the Root Locus of the given system:

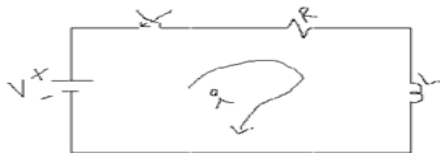
- 2. Draw the pole-zero plot and determine the region of real axis for which the root locus exists. Also, determine the number of breakaway points (This will be explained while solving the problems).
- 3. Calculate the angle of asymptotes.
- 4. Determine the centroid.
- 5. Calculate the breakaway points (if any).
- 6. Calculate the intersection point of root locus with the imaginary axis.
- 7. Calculate the angle of departure or angle of arrivals if any.
- 8. From above steps draw the overall sketch of the root locus.
- 9. Predict the stability and performance of the given system by the root locus.

**Five marks of questions with answers**

**6. Derive the expression for transient response of R-L series circuit for dc excitation?**

**Solution:**

**DC Response of an R-L Circuit:**



Consider a circuit consisting of a resistance and inductance as shown in fig. the inductor in the circuit is initially uncharged and is in series with the resistor. When switch S is closed, we can find the complete solution for current. Application of Kirchoff's law to the circuit results in following differential equations.

$$V = iR + L \frac{di}{dt} \text{ ----- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} I = \frac{V}{L} \text{ ----- (2)}$$

In the above equation, the current  $i$  is the solution to be found and  $V$  is the applied constant voltage. The voltage  $V$  is applied to the circuit only when the switch  $S$  is closed. The above equation is linear differential equation of the first order comparing with the non homogenous differential equation

$$\frac{dx}{dt} + P X = K \text{ whose solution is } x = e^{-pt} \int K e^{+pt} dt + c e^{-pt}$$

Where  $c$  is an arbitrary constant,

in similar way we can write the current equation as

$$i = c e^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \int \frac{V}{L} e^{\left(\frac{R}{L}\right)t} dt$$

$$i = c e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R} .$$

To determine the value of ' $c$ ', in equation (5) we use initial conditions. In the circuit shown in fig the switch  $S$  is closed at  $t=0$ . At  $t=0^-$ , i.e. just before closing the switch  $S$ , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at  $t=0^+$  just after the switch is closed, the current remains zero.

Substituting above conditions we get,

$$0 = c + (V/R)$$

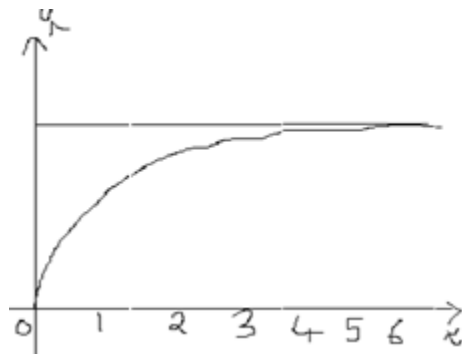
Therefore,  $c = -V/R$

Hence from equation

$$i = -\frac{V}{R} e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$$

$$i = \frac{V}{R} (1 - e^{-\left(\frac{R}{L}\right)t})$$

Above equation consists of two parts, the steady state part ( $V/R$ ) and other is transient part.



$$\tau = \frac{L}{R} \text{ seconds}$$

The transient part of solution is,  $i(\tau) = -\frac{V}{R} e^{-\frac{\tau}{\tau}}$

At time constant is one,  $i(\tau) = -\frac{V}{R} e^{-\frac{\tau}{\tau}} = -\frac{V}{R} e^{-1} = -0.368 \frac{V}{R}$

The transient response reaches 36.8 percent of its initial value.

Similarly,  $i(2\tau) = -\frac{V}{R} e^{-\frac{2\tau}{\tau}} = -\frac{V}{R} e^{-2} = -0.135 \frac{V}{R}$

$$i(3\tau) = -\frac{V}{R} e^{-\frac{3\tau}{\tau}} = -\frac{V}{R} e^{-3} = -0.0498 \frac{V}{R}$$

$$i(5\tau) = -\frac{V}{R} e^{-\frac{5\tau}{\tau}} = -\frac{V}{R} e^{-5} = -0.0067 \frac{V}{R}$$



After 5, the transient part reaches more than 99 percent of its final value. In fig we can find out the voltages and powers across each element by using the current.

Voltage across the resistor,  $V_R = R i = R * \frac{V}{R} (1 - e^{-\frac{R}{L}t})$

$$V_R = V (1 - e^{-\frac{R}{L}t})$$

Similarly, the voltage across the inductor,  $V_L = L \frac{di}{dt}$

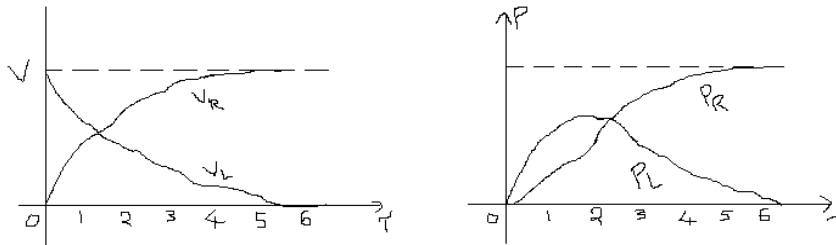
$$V_L = L * \frac{V}{R} e^{-\frac{R}{L}t} \frac{R}{L} = V e^{-\frac{R}{L}t}$$

Power in the resistor,  $P_R = V_R i = V (1 - e^{-\frac{R}{L}t}) * \frac{V}{R} (1 - e^{-\frac{R}{L}t})$

$$= \frac{V^2}{R} (1 - 2 e^{-\frac{R}{L}t} + e^{-\frac{2R}{L}t})$$

Power in the inductor,  $P_L = V_L i = V e^{-\frac{R}{L}t} * \frac{V}{R} (1 - e^{-\frac{R}{L}t})$

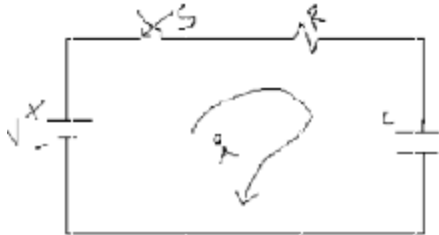
$$= \frac{V^2}{R} (e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t})$$



7. Derive the expression for transient response of R-C series circuit for dc excitation?

**Solution:**

### DC Response of an R-C Circuit:



Consider a circuit consisting of resistance and capacitance as shown in fig. the capacitor in the circuit is initially uncharged, and is in series with resistor. When the switch S is closed at  $t=0$ , we can determine the complete solution for current. Application of Kirchoff's laws we can determine the differential equations.

$$V = R i + \frac{1}{C} \int i dt$$

By differentiating the above equation we get,

$$0 = R \frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} + \frac{i}{RC} = 0 -$$

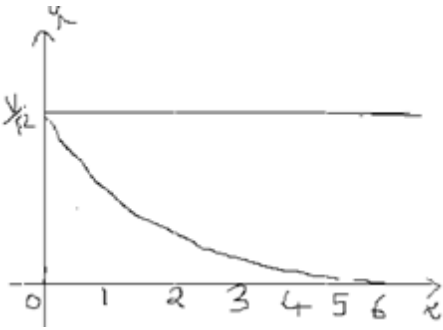
Equation is linear differential equation with only the complementary function. The particular solution for the above equation is zero. The solution for this type of differential equation is

$$i = c e^{-\frac{t}{RC}}$$

Here, to find the value of c, we use the initial conditions. In the circuit shown in fig switch S is closed at  $t=0$ . Since the capacitor never allows sudden changes in voltage, it will act as short at  $t=0+$ . So, the current in the circuit at  $t=0+$  is  $V/R$ .

Substituting the i value in equation we get,

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$



When switch S is closed, the response decays with time as shown in fig. In the solution, the quantity RC is the time constant, and is denoted by  $\tau$ , where  $\tau = RC$  seconds.

After 5, the transient part reaches more than 99 percent of its final value. In fig we can find out the voltage across each element by using the current equation.

Voltage across the resistor

$$V_R = R i = R * \frac{V}{R} e^{-\frac{t}{RC}} = V e^{-\frac{t}{RC}}$$

Similarly, voltage across the capacitor

$$\begin{aligned} V_C &= \frac{1}{C} \int i dt \\ &= \frac{1}{C} \int \frac{V}{R} e^{-\frac{t}{RC}} dt \\ &= -\left(\frac{V}{RC} * RC e^{-\frac{t}{RC}}\right) + c \\ &= -V e^{-\frac{t}{RC}} + c \end{aligned}$$

At  $t=0$ , voltage across the capacitor is zero.

$$c = V$$

$$V_C = V (1 - e^{-\frac{t}{RC}})$$

Power in the resistor,

$$P_R = V_R i = V e^{-\frac{t}{RC}} * \frac{V}{R} e^{-\frac{t}{RC}}$$

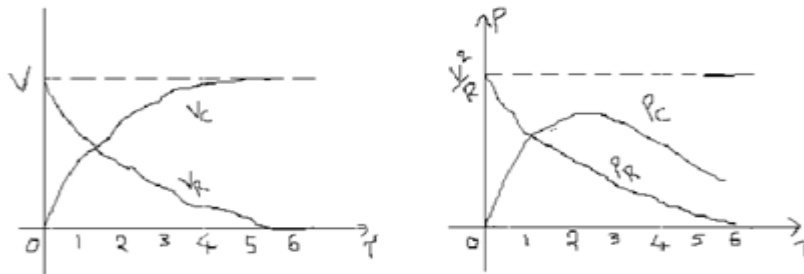
$$= \frac{V^2}{R} e^{-\frac{2t}{RC}}$$

Power in the capacitor,

$$P_C = V_C i = V (1 - e^{-\frac{t}{RC}}) * \frac{V}{R} e^{-\frac{t}{RC}}$$

$$= \frac{V^2}{R} (e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}})$$

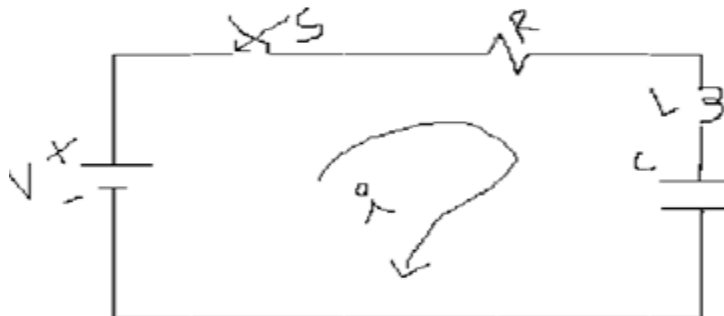
The responses are shown in fig.



8. Derive the expression for transient response of R-L-C series circuit for dc excitation?

**Solution:**

**DC Response of an R-L-C Circuit:**



Consider a circuit consisting of resistance, inductance and capacitance as shown in fig. the capacitor and inductor are initially uncharged, and are in series with a resistor. When the switch

S is closed at  $t=0$ , we can determine the complete solution for current. Application of Kirchoff's laws we can determine the differential equations.

$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt \text{ ----- (1)}$$

By differentiating above equation we get,

$$0 = R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{1}{C} i \text{ ----- (2)}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0 \text{ ----- (3)}$$

The above equation is a second order linear differential equation, with only complementary function. The particular solution for the above equation is zero. Characteristic equation for the above differential equation is

$$D^2 + \frac{R}{L} D + \frac{1}{LC} = 0 \text{ ----- (4)}$$

The roots above equation are,

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

By assuming,

$$K_1 = -\frac{R}{2L} \text{ and } K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$D_1 = K_1 + K_2$$

$$D_2 = K_1 - K_2$$

Here  $K_2$  may be positive or negative or zero.

$K_2$  is positive,

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

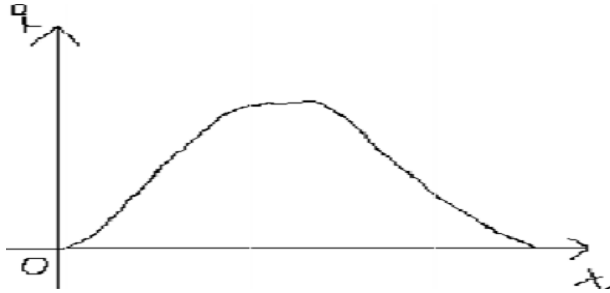
The roots are real and unequal, and give the over damped response as shown in fig. then equation (4) becomes

$$[D - (K_1 + K_2)][[D - (K_1 - K_2)]]i = 0$$

The solution for the above equation is,

$$i = c_1 e^{(K_1+K_2)t} + c_2 e^{(K_1-K_2)t}$$

The current curve for the over damped case is shown in fig.



K Is negative,

$$\text{when } \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}.$$

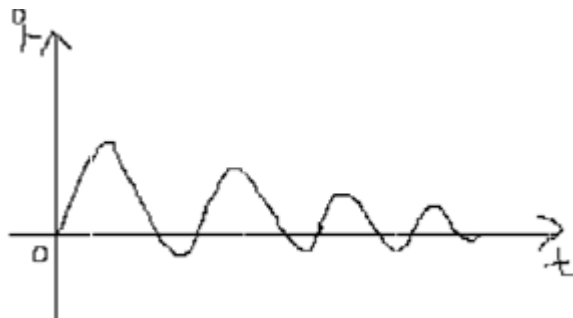
The roots are complex conjugate, and give the under damped response as shown in fig. the equation as shown in becomes

$$[D - (K_1+jK_2)][[D - (K_1-jK_2)]]i=0$$

The solution for above equation is,

$$i = e^{K_1 t} [c_1 \cos K_2 t + c_2 \sin K_2 t ]$$

The current curve for the under damped case is shown in fig.



K Is zero,

When

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

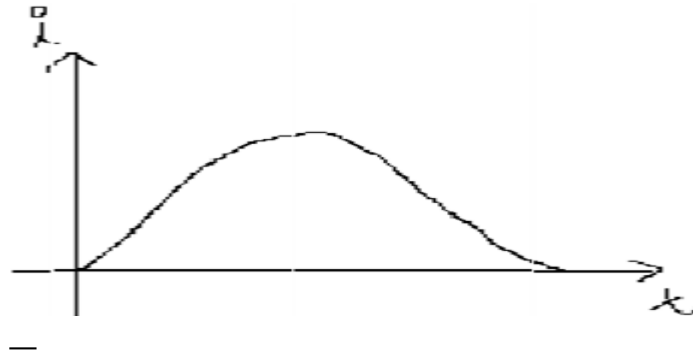
The roots are equal, and give the critically damped response as shown the equation becomes

$$[D - K_1][[D - K_2]]i=0$$

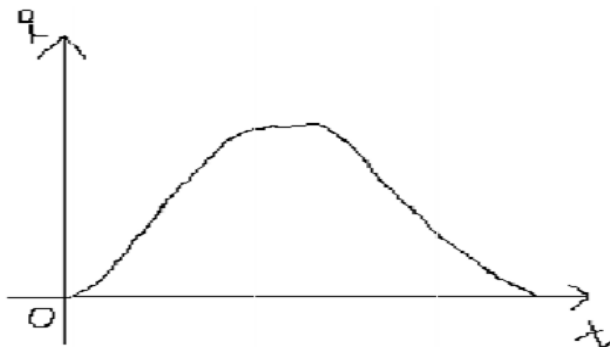
The solution for above equation is

$$i = e^{K_1 t} [c_1 + c_2 t]$$

The current curve for critically damped case is as shown in fig.



The current curve for the over damped case is shown in fig.



K is negative,

$$\text{when } \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

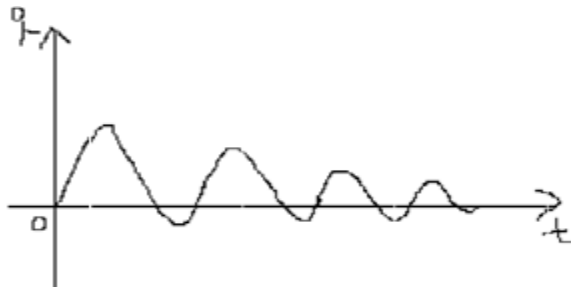
The roots are complex conjugate, and give the under damped response as shown in fig. the equation as shown in becomes

$$[D - (K_1 + jK_2)][[D - (K_1 - jK_2)]]i = 0$$

The solution for above equation is,

$$i = e^{K_1 t} [c_1 \cos K_2 t + c_2 \sin K_2 t]$$

The current curve for the under damped case is shown in fig.



K is zero,

When

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

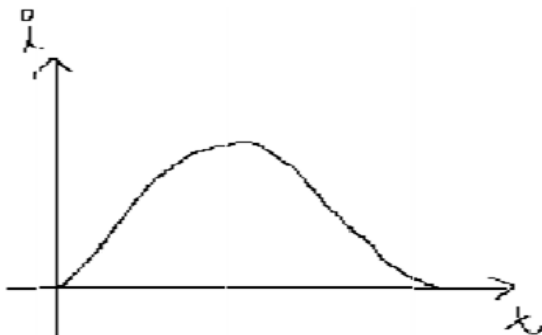
The roots are equal, and give the critically damped response as shown the equation becomes

$$[D - K_1][[D - K_2]]i = 0$$

The solution for above equation is

$$i = e^{K_1 t} [c_1 + c_2 t]$$

The current curve for critically damped case is as shown in fig.



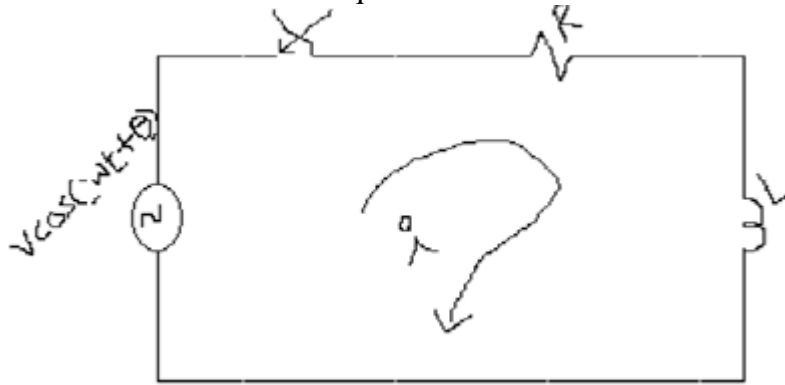


**9. Derive the expression for transient response of R-L series circuit for ac excitation?**

**Solution:**

**Sinusoidal Response of R-L Circuit:**

Consider a circuit consisting of resistance and inductance are connected as shown in fig. the switch S is closed at  $t=0$ . At  $t=0$ , sinusoidal voltage  $V\cos(\omega t + \theta)$  applied to RL circuit, where  $V$  is the amplitude of the wave and  $\theta$  is the phase angle. Application of Kirchhoff's laws we can determine the differential equations.



$$V\cos(\omega t + \theta) = iR + L \frac{di}{dt} \text{ ----- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \cos(\omega t + \theta) \text{ ----- (2)}$$

The corresponding characteristic equation is

$$(D + \frac{R}{L}) i = \frac{V}{L} \cos(\omega t + \theta) \text{ ----- (3)}$$

For the above equation, the solution consists of two parts.  
One is complementary function and other is particular integral.

The complementary function of the solution is

$$i_c = c e^{-t(\frac{R}{L})} \text{ ----- (4)}$$

The particular solution can be obtained by using undetermined co-efficient.

By assuming,

$$i_p = A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \text{ ----- (5)}$$

$$i_p^1 = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) \text{ ----- (6)}$$

Substituting equations 5 & 6 in 3 we get,

$$\{-A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) + \frac{R}{L}[A \cos(\omega t + \theta) + B \sin(\omega t + \theta)]\} = \frac{V}{L} \cos(\omega t + \theta)$$

$$(-A\omega + \frac{BR}{L})\sin(\omega t + \theta) + (B\omega + \frac{AR}{L})\cos(\omega t + \theta) = \frac{V}{L} \cos(\omega t + \theta)$$

Comparing cosine terms and sine terms, we get

$$-A\omega + \frac{BR}{L} = 0$$

$$B\omega + \frac{AR}{L} = \frac{V}{L}$$

From the above equations we have

$$A = V \frac{R}{R^2 + (\omega L)^2}$$

$$B = V \frac{\omega L}{R^2 + (\omega L)^2}$$

Substituting the A and B values in equation (5) we get

$$i_p = V \frac{R}{R^2 + (\omega L)^2} \cos(\omega t + \theta) + V \frac{\omega L}{R^2 + (\omega L)^2} \sin(\omega t + \theta) \text{ ----- (7)}$$

$$\text{Putting } M \cos \phi = V \frac{R}{R^2 + (\omega L)^2}$$

$$M \sin \phi = V \frac{\omega L}{R^2 + (\omega L)^2}$$

To find M and  $\phi$ , we divide one equation by the other

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{wL}{R}$$

Squaring both equations and adding, we get

$$M^2 \cos^2 \phi + M^2 \sin^2 \phi = V \frac{V}{R^2 + (wL)^2}$$

$$M = \frac{V}{\sqrt{R^2 + (wL)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2 + (wL)^2}} \cos\left(\omega t + \theta - \tan^{-1} \frac{wL}{R}\right) \text{----- (8)}$$

The complete solution for the current,  $i = i_c + i_p$

$$i = c e^{-t\left(\frac{R}{L}\right)} + \frac{V}{\sqrt{R^2 + (wL)^2}} \cos\left(\omega t + \theta - \tan^{-1} \frac{wL}{R}\right)$$

Since the inductor does not allow sudden changes in currents, at  $t=0$ ,  $i=0$

$$c = \frac{-V}{\sqrt{R^2 + (wL)^2}} \cos\left(\theta - \tan^{-1} \frac{wL}{R}\right)$$

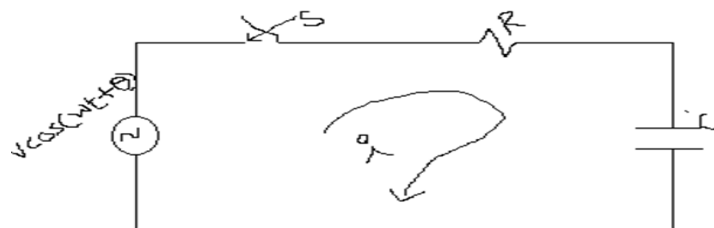
The complete solution for the current is,

$$i = e^{-t\left(\frac{R}{L}\right)} \left[ \frac{-V}{\sqrt{R^2 + (wL)^2}} \cos\left(\theta - \tan^{-1} \frac{wL}{R}\right) \right] + \frac{V}{\sqrt{R^2 + (wL)^2}} \cos\left(\omega t + \theta - \tan^{-1} \frac{wL}{R}\right)$$

**10. Derive the expression for transient response of R-C series circuit for ac excitation?**

**Solution:**

**Sinusoidal Response of R-C Circuit:**



Consider a circuit consisting of resistance and capacitance in series as shown in fig. the switch S is closed at t=0. At t=0, sinusoidal voltage  $V\cos(\omega t + \theta)$  applied to RL circuit, where V is the amplitude of the wave and  $\theta$  is the phase angle. Application of Kirchhoff's laws we can determine the differential equations.

$$V\cos(\omega t + \theta) = Ri + \frac{1}{C} \int i dt \text{ ----- (1)}$$

$$- V\omega\sin(\omega t + \theta) = R\frac{di}{dt} + \frac{i}{C} \text{ ----- (2)}$$

$$(D + \frac{1}{RC}) i = \frac{-V\omega}{R} \sin(\omega t + \theta) \text{ ----- (3)}$$

The complementary function,

$$i_c = c e^{\frac{-t}{RC}}$$

The particular solution can be obtained by using undetermined coefficients.

$$i_p = A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \text{ ----- (4)}$$

$$i_p^1 = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) \text{ ----- (5)}$$

Substituting equation 4 & 5 in 3 we get

$$\{-A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta)\} +$$

$$\frac{1}{RC} \{ A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \} = \frac{-V\omega}{R} \sin(\omega t + \theta)$$

Comparing both sides,

$$-AW + \frac{B}{RC} = \frac{-Vw}{R}$$

$$Bw + \frac{A}{RC} = 0$$

So we get,

$$A = \frac{VR}{R^2 + \left(\frac{1}{wC}\right)^2}$$

$$B = \frac{-V}{wC[R^2 + \left(\frac{1}{wC}\right)^2]}$$

Substituting A and B values in 4 we get

$$i_p = \frac{VR}{R^2 + \left(\frac{1}{wC}\right)^2} \cos(wt + \theta) - \frac{V}{wC[R^2 + \left(\frac{1}{wC}\right)^2]} \sin(wt + \theta)$$

$$\text{Putting } M \cos \phi = \frac{VR}{R^2 + \left(\frac{1}{wC}\right)^2}$$

$$M \sin \phi = \frac{V}{wC[R^2 + \left(\frac{1}{wC}\right)^2]}$$

To find M and  $\phi$ , we divide one equation by the other

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{1}{wCR}$$

Squaring both equations and adding, we get

$$M^2 \cos^2 \phi + M^2 \sin^2 \phi = V \frac{V}{R^2 + (\frac{1}{\omega C})^2}$$

$$M = \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos(\omega t + \theta + \tan^{-1} \frac{1}{\omega CR}) \text{ ----- (8)}$$

The complete solution for the current,  $i = i_c + i_p$

$$i = c e^{-t(\frac{1}{RC})} + \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos(\omega t + \theta + \tan^{-1} \frac{1}{\omega CR})$$

Since the capacitor does not allow sudden changes in voltages,

$$t=0, i = \frac{V}{R} \cos \theta$$

$$\frac{V}{R} \cos \theta = c + \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos(\theta + \tan^{-1} \frac{1}{\omega CR})$$

Therefore,

$$c = \frac{V}{R} \cos \theta - \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos(\theta + \tan^{-1} \frac{1}{\omega CR})$$

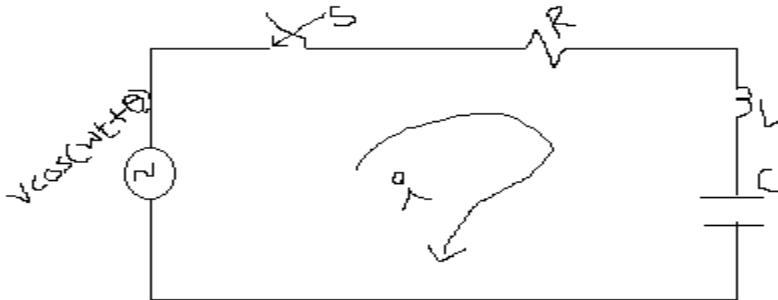
The complete solution for the current is,

$$i = e^{-t(\frac{1}{RC})} \left[ \frac{V}{R} \cos \theta - \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos(\theta + \tan^{-1} \frac{1}{\omega CR}) \right] + \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos(\omega t + \theta + \tan^{-1} \frac{1}{\omega CR})$$

**11. Derive the expression for transient response of R-L-C series circuit for ac excitation?**

**Solution:**

**Sinusoidal Response of R-L-C Circuit:**



Consider a circuit consisting of resistance, inductance and capacitance in series as shown in fig. Switch  $s$  is closed at  $t=0$ . At  $t=0$ , a sinusoidal voltage  $V\cos(\omega t + \theta)$  applied to RLC circuit, where  $V$  is the amplitude of the wave and  $\theta$  is the phase angle. Application of Kirchoff's laws we can determine the differential equations.

$$V\cos(\omega t + \theta) = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt \text{ ----- (1)}$$

By differentiating above equation we get,

$$- V\omega \sin(\omega t + \theta) = R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{1}{C} i \text{ ----- (2)}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = - V\omega \sin(\omega t + \theta) \text{ ----- (3)}$$

$$(D^2 + \frac{R}{L} D + \frac{1}{LC}) i = - \frac{V\omega}{L} \sin(\omega t + \theta) \text{ ----- (4)}$$

The particular solution can be obtained by using undetermined coefficients.

$$i_p = A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \text{ ----- (5)}$$

$$i_p^1 = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) \text{ ----- (6)}$$

$$i_p^{11} = -Aw^2 \cos(\omega t + \theta) - Bw^2 \sin(\omega t + \theta) \text{ ----- (7)}$$

Substituting 5,6& 7 in equation 4 we get,

$$\begin{aligned} & \{-Aw^2 \cos(\omega t + \theta) - Bw^2 \sin(\omega t + \theta)\} + \frac{R}{L}\{-Aw \sin(\omega t + \theta) + Bw \cos(\omega t + \theta)\} \\ & + \frac{1}{LC}\{A \cos(\omega t + \theta) + B \sin(\omega t + \theta)\} = -\frac{Vw}{L} \sin(\omega t + \theta) \end{aligned}$$

Comparing both the sides, sine and cosine coefficients we get,

$$-Bw^2 - A \frac{wR}{L} + \frac{B}{LC} = -\frac{Vw}{L}$$

$$A \left(\frac{wR}{L}\right) + B \left(w^2 - \frac{1}{LC}\right) = \frac{Vw}{L} \text{ ----- (8)}$$

$$-Aw^2 + B \frac{wR}{L} + \frac{A}{LC} = 0$$

$$A \left(w^2 - \frac{1}{LC}\right) - B \left(\frac{wR}{L}\right) + = 0 \text{ ----- (9)}$$

$$A = \frac{V * \frac{w^2 R}{L^2}}{\left[\left(\frac{wR}{L}\right)^2 - \left(w^2 - \frac{1}{LC}\right)^2\right]}$$

$$B = \frac{\left(w^2 - \frac{1}{LC}\right) * Vw}{L \left[\left(\frac{wR}{L}\right)^2 - \left(w^2 - \frac{1}{LC}\right)^2\right]}$$

Substituting A and B values in 5 we get



$$i_p = \frac{V * \frac{w^2 R}{L^2}}{[(\frac{wR}{L})^2 - (w^2 - \frac{1}{LC})^2]} \cos(wt + \theta) + \frac{(w^2 - \frac{1}{LC}) * VW}{L[(\frac{wR}{L})^2 - (w^2 - \frac{1}{LC})^2]} \sin(wt + \theta) \text{ ---- (10)}$$

$$\text{Putting } M \cos \phi = \frac{V * \frac{w^2 R}{L^2}}{[(\frac{wR}{L})^2 - (w^2 - \frac{1}{LC})^2]}$$

$$M \sin \phi = \frac{(w^2 - \frac{1}{LC}) * VW}{L[(\frac{wR}{L})^2 - (w^2 - \frac{1}{LC})^2]}$$

To find M and  $\phi$ , we divide one equation by the other

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{(wL - \frac{1}{wC})}{R}$$

Squaring both equations and adding, we get

$$M^2 \cos^2 \phi + M^2 \sin^2 \phi = V \frac{V}{R^2 + (\frac{1}{wC} - wL)^2}$$

$$M = \frac{V}{\sqrt{R^2 + (\frac{1}{wC} - wL)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2 + (\frac{1}{wC} - wL)^2}} \cos\left(wt + \theta + \tan^{-1} \frac{\frac{1}{wC} - wL}{R}\right)$$

The

complementary function is similar to that of DC series RLC circuit.

$$D^2 + \frac{R}{L} D + \frac{1}{LC} = 0$$

The roots above equation are,

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

By assuming,

$$K_1 = -\frac{R}{2L} \text{ and } K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$D_1 = K_1 + K_2$$

$$D_2 = K_1 - K_2$$

Here  $K_2$  may be positive or negative or zero.

$K_2$  Is positive, when

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

The roots are real and unequal, and give the over damped response as shown in fig. then equation (4) becomes

$$[D - (K_1 + K_2)][[D - (K_1 - K_2)]]i = 0$$

The solution for the above equation is,

$$i_c = c_1 e^{(K_1 + K_2)t} + c_2 e^{(K_1 - K_2)t}$$

Therefore the complete solution is,  $i = i_c + i_p$

$$i = c_1 e^{(K_1 + K_2)t} + c_2 e^{(K_1 - K_2)t} + \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \cos\left(\omega t + \theta + \tan^{-1} \frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

$K_2$  Is negative, when

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

The roots are complex conjugate, and give the under damped the equation as shown in becomes

$$[D - (K_1 + jK_2)][[D - (K_1 - jK_2)]]i = 0$$

The solution for above equation is,

$$i_c = e^{K_1 t} [c_1 \cos K_2 t + c_2 \sin K_2 t]$$

Therefore the complete solution is,  $i = i_c + i_p$

$$i = e^{K_1 t} [c_1 \cos K_2 t + c_2 \sin K_2 t] + \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \cos\left(\omega t + \theta + \tan^{-1} \frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

$K_2$  Is zero, when

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

The roots are equal, and give the critically damped response as shown in fig. the equation becomes

$$[D - K_1][[D - K_2]]i = 0$$

The solution for above equation is

$$i_c = e^{K_1 t} [c_1 + c_2 t]$$

Therefore the complete solution is,  $i = i_c + i_p$

$$i = e^{K_1 t} [c_1 + c_2 t] + \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \cos\left(\omega t + \theta + \tan^{-1} \frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

**12. Obtain the step response of R-L and R-C series circuits for dc excitation?**

### Solution:

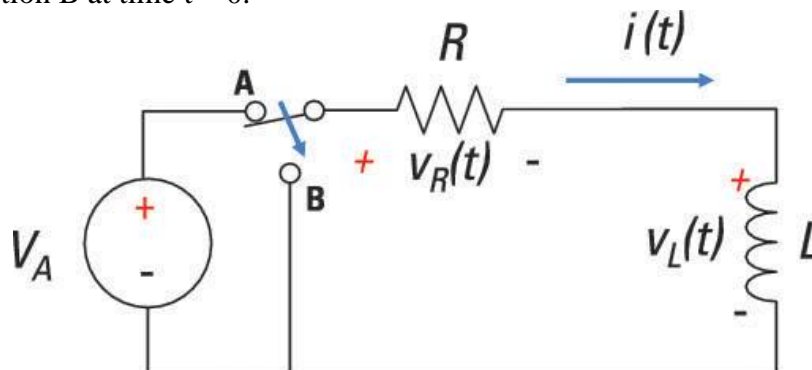
Using the Laplace transform as part of your circuit analysis provides you with a prediction of circuit response. Analyze the poles of the Laplace transform to get a general idea of output behavior. Real poles, for instance, indicate exponential output behavior.

Follow these basic steps to analyze a circuit using Laplace techniques:

- Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.
- Apply the Laplace transformation of the differential equation to put the equation in the s-domain.
- Algebraically solve for the solution, or response transform.
- Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

To get comfortable with this process, you simply need to practice applying it to different types of circuits such as an RC (resistor-capacitor) circuit, an RL (resistor-inductor) circuit, and an RLC (resistor-inductor-capacitor) circuit.

Here is an RL circuit that has a switch that's been in Position A for a long time. The switch moves to Position B at time  $t = 0$ .



For this circuit, you have the following KVL equation:

$$v_R(t) + v_L(t) = 0$$

Next, formulate the element equation (or i-v characteristic) for each device. Using Ohm's law to describe the voltage across the resistor, you have the following relationship:

$$v_R(t) = i_L(t)R$$

The inductor's element equation is

$$v_L(t) = L \frac{di_L(t)}{dt}$$

Substituting the element equations,  $v_R(t)$  and  $v_L(t)$ , into the KVL equation gives you the desired first-order differential equation:

$$L \frac{di_L(t)}{dt} + i_L(t)R = 0$$

On to Step 2: Apply the Laplace transform to the differential equation:

$$\mathcal{L}\left[L\frac{di_L(t)}{dt} + i_L(t)R\right] = 0$$

$$\mathcal{L}\left[L\frac{di_L(t)}{dt}\right] + \mathcal{L}[i_L(t)R] = 0$$

The preceding equation uses the linearity property which says you can take the Laplace transform of each term. For the first term on the left side of the equation, you use the differentiation property:

$$\mathcal{L}\left[L\frac{di_L(t)}{dt}\right] = L[sI_L(s) - I_0]$$

This equation uses  $I_L(s) = \mathcal{L}[i_L(t)]$ , and  $I_0$  is the initial current flowing through the inductor.

The Laplace transform of the differential equation becomes

$$I_L(s)R + L[sI_L(s) - I_0] = 0$$

Solve for  $I_L(s)$ :

$$I_L(s) = \frac{I_0}{s + \frac{R}{L}}$$

For a given initial condition, this equation provides the solution  $i_L(t)$  to the original first-order differential equation. You simply perform an inverse Laplace transform of  $I_L(s)$  — or look for the appropriate transform pair in this table — to get back to the time-domain.

Signal Description	Time-Domain Waveform, $f(t)$	$s$ -Domain Waveform, $F(s)$
Step	$u(t)$	$\frac{1}{s}$
Exponential	$[e^{-\alpha t}]u(t)$	$\frac{1}{s + \alpha}$
Impulse	$\delta(t)$	1
Ramp, $r(t)$	$tu(t)$	$\frac{1}{s^2}$
Sine	$[\sin \beta t]u(t)$	$\frac{\beta}{s^2 + \beta^2}$
Cosine	$[\cos \beta t]u(t)$	$\frac{s}{s^2 + \beta^2}$
<b>Damped Pairs</b>		
Damped ramp	$te^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^2}$
Damped sine	$[e^{-\alpha t} \sin \beta t]u(t)$	$\frac{\beta}{(s + \alpha)^2 + \beta^2}$
Damped cosine	$[e^{-\alpha t} \cos \beta t]u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$

The preceding equation has an exponential form for the Laplace transform pair. You wind up with the following solution:

$$I_L(s) = \frac{I_0}{s + \frac{R}{L}} \leftrightarrow i_L(t) = I_0 e^{-\left(\frac{R}{L}\right)t}$$

The result shows as time  $t$  approaches infinity, the initial inductor current eventually dies out to zero after a long period of time — about 5 time constants ( $L/R$ ).

Using the Laplace transform as part of your circuit analysis provides you with a prediction of circuit response. Analyze the poles of the Laplace transform to get a general idea of output behavior. Real poles, for instance, indicate exponential output behavior.

Follow these basic steps to analyze a circuit using Laplace techniques:

Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.

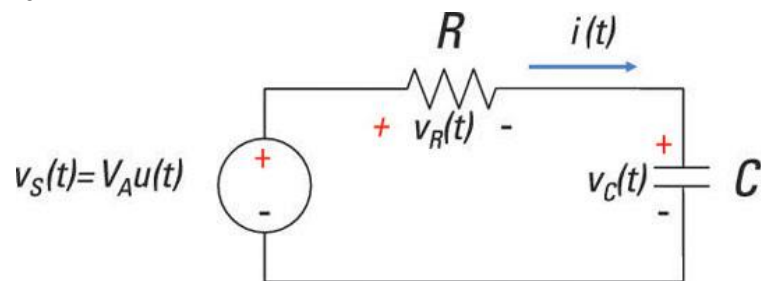
Apply the Laplace transformation of the differential equation to put the equation in the s-domain. Algebraically solve for the solution, or response transform.

Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

To get comfortable with this process, you simply need to practice applying it to different types of circuits such as an RC (resistor-capacitor) circuit, an RL (resistor-inductor) circuit, and an RLC (resistor-inductor-capacitor) circuit.

Consider the simple first-order RC series circuit shown here. To set up the differential equation for this series circuit, you can use Kirchhoff's voltage law (KVL), which says the sum of the voltage rises and drops around a loop is zero. This circuit has the following KVL equation around the loop:

$$-v_S(t) + v_R(t) + v_C(t) = 0$$



Next, formulate the element equation (or i-v characteristic) for each device. The element equation for the source is

$$v_S(t) = V_A u(t)$$

Use Ohm's law to describe the voltage across the resistor:

$$v_R(t) = i(t)R$$

The capacitor's element equation is given as

$$i(t) = C \frac{dv_C(t)}{dt}$$

Substituting this expression for  $i(t)$  into  $v_R(t)$  gives you the following expression:

$$v_R(t) = i(t)R = RC \frac{dv_C(t)}{dt}$$

Substituting  $v_R(t)$ ,  $v_C(t)$ , and  $v_S(t)$  into the KVL equation leads to

$$-v_S(t) + v_R(t) + v_C(t) = 0$$

$$-V_A u(t) + RC \frac{dv_C(t)}{dt} + v_C(t) = 0$$

Now rearrange the equation to get the desired first-order differential equation:

$$RC \frac{dv_c(t)}{dt} + v_c(t) = V_A u(t)$$

Now you're ready to apply the Laplace transformation of the differential equation in the s-domain. The result is

$$\mathcal{L} \left[ RC \frac{dv_c(t)}{dt} + v_c(t) \right] = \mathcal{L} [V_A u(t)]$$

$$\mathcal{L} \left[ RC \frac{dv_c(t)}{dt} \right] + \mathcal{L} [v_c(t)] = \mathcal{L} [V_A u(t)]$$

On the left, the linearity property was used to take the Laplace transform of each term. For the first term on the left side of the equation, you use the differentiation property, which gives you

$$\mathcal{L} \left[ RC \frac{dv_c(t)}{dt} \right] = RC [sV_c(s) - V_0]$$

This equation uses  $V_c(s) = \mathcal{L}[v_c(t)]$ , and  $V_0$  is the initial voltage across the capacitor.

Using the following table, the Laplace transform of a step function provides you with this:

$$\mathcal{L} [V_A u(t)] = \frac{V_A}{s}$$

Signal Description	Time-Domain Waveform, $f(t)$	s-Domain Waveform, $F(s)$
Step	$u(t)$	$\frac{1}{s}$
Exponential	$[e^{-\alpha t}]u(t)$	$\frac{1}{s+\alpha}$
Impulse	$\delta(t)$	1
Ramp, $r(t)$	$tu(t)$	$\frac{1}{s^2}$
Sine	$[\sin \beta t]u(t)$	$\frac{\beta}{s^2 + \beta^2}$
Cosine	$[\cos \beta t]u(t)$	$\frac{s}{s^2 + \beta^2}$
<b>Damped Pairs</b>		
Damped ramp	$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
Damped sine	$[e^{-\alpha t} \sin \beta t]u(t)$	$\frac{\beta}{(s+\alpha)^2 + \beta^2}$
Damped cosine	$[e^{-\alpha t} \cos \beta t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2 + \beta^2}$

Based on the preceding expressions for the Laplace transforms, the differential equation becomes the following:

$$RC [sV_c(s) - V_0] + V_c(s) = \frac{V_A}{s}$$

Next, rearrange the equation:

$$\left[ s + \frac{1}{RC} \right] V_c(s) = \frac{V_A}{RC} \left( \frac{1}{s} \right) + V_0$$

Solve for the output  $V_c(s)$  to get the following transform solution:

$$V_c(s) = \frac{V_A}{RC} \left[ \frac{1}{s(s + \frac{1}{RC})} \right] + \frac{V_0}{s + \frac{1}{RC}}$$

By performing an inverse Laplace transform of VC(s) for a given initial condition, this equation leads to the solution vC(t) of the original first-order differential equation.

On to Step 3 of the process. To get the time-domain solution vC(t), you need to do a partial fraction expansion for the first term on the right side of the preceding equation:

$$\frac{V_A}{RC} \left[ \frac{1}{s(s + \frac{1}{RC})} \right] = \frac{A}{s} + \left( \frac{B}{s + \frac{1}{RC}} \right)$$

You need to determine constants A and B. To simplify the preceding equation, multiply both sides by  $s(s + 1/RC)$  to get rid of the denominators:

$$\frac{V_A}{RC} = A \left( s + \frac{1}{RC} \right) + Bs$$

Algebraically rearrange the equation by collecting like terms:

$$(A+B)s + \frac{1}{RC}(A - V_A) = 0$$

In order for the left side of the preceding equation to be zero, the coefficients must be zero ( $A + B = 0$  and  $A - V_A = 0$ ). For constants A and B, you wind up with  $A = V_A$  and  $B = -V_A$ . Substitute these values into the following equation:

$$\frac{V_A}{RC} \left[ \frac{1}{s(s + \frac{1}{RC})} \right] = \frac{A}{s} + \left( \frac{B}{s + \frac{1}{RC}} \right)$$

The substitution leads you to:

$$\frac{V_A}{RC} \left( \frac{1}{s(s + \frac{1}{RC})} \right) = \frac{V_A}{s} + \frac{-V_A}{s + \frac{1}{RC}}$$

Now substitute the preceding expression into the VC(s) equation to get the transform solution:

$$\begin{aligned} V_c(s) &= \frac{V_A}{RC} \left( \frac{1}{s(s + \frac{1}{RC})} \right) + \frac{V_0}{s + \frac{1}{RC}} \\ &= \frac{V_A}{s} + \frac{-V_A}{s + \frac{1}{RC}} + \frac{V_0}{s + \frac{1}{RC}} \\ &= \frac{V_A}{s} + \frac{-V_A + V_0}{s + \frac{1}{RC}} \end{aligned}$$

That completes the partial fraction expansion. You can then use the table given earlier to find the inverse Laplace transform for each term on the right side of the preceding equation.

The first term has the form of a step function, and the last two terms have the form of an exponential, so the inverse Laplace transform of the preceding equation leads you to the following solution vC(t) in the time-domain:



$$v_c(t) = V_A u(t) - V_A e^{-\frac{t}{RC}} u(t) + V_0 e^{-\frac{t}{RC}} u(t)$$

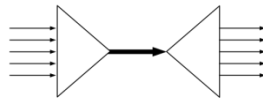
$$v_c(t) = V_A \left(1 - e^{-\frac{t}{RC}}\right) u(t) + V_0 e^{-\frac{t}{RC}} u(t)$$

The result shows as time  $t$  approaches infinity, the capacitor charges to the value of the input  $V_A$ . Also, the initial voltage of the capacitor eventually dies out to zero after a long period of time (about 5 time constants,  $RC$ ).

### 13. Analyze the circuit switching

#### **Circuit switching:**

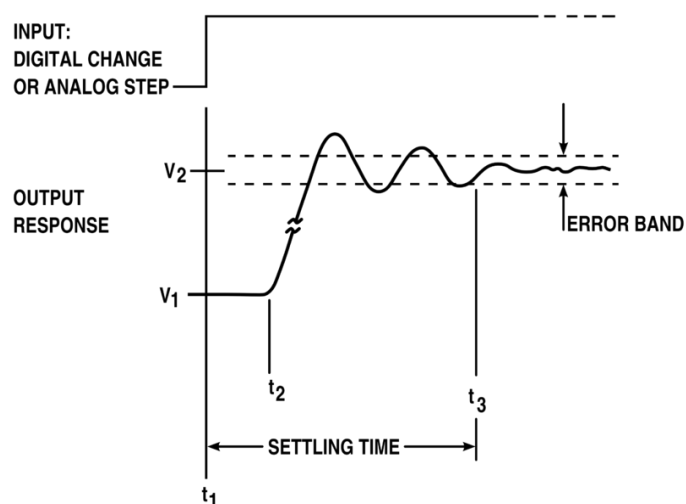
- Circuit switching is a method of implementing a telecommunications network in which two network nodes establish a dedicated communications channel (circuit) through the network before the nodes may communicate.
- The circuit guarantees the full bandwidth of the channel and remains connected for the duration of the communication session. The circuit functions as if the nodes were physically connected as with an electrical circuit.
- The defining example of a circuit-switched network is the early analog telephone network. When a call is made from one telephone to another, switches within the telephone exchanges create a continuous wire circuit between the two telephones, for as long as the call lasts.



#### Multiplexing

- Circuit switching contrasts with packet switching which divides the data to be transmitted into packets transmitted through the network independently.
- In packet switching, instead of being dedicated to one communication session at a time, network links are shared by packets from multiple competing communication sessions, resulting in the loss of the quality of service guarantees that are provided by circuit switching.
- In circuit switching, the bit delay is constant during a connection, as opposed to packet switching, where packet queues may cause varying and potentially indefinitely long packet transfer delays.
- No circuit can be degraded by competing users because it is protected from use by other callers until the circuit is released and a new connection is set up. Even if no actual communication is taking place, the channel remains reserved and protected from competing users.

- Virtual circuit switching is a packet switching technology that emulates circuit switching, in the sense that the connection is established before any packets are transferred, and packets are delivered in order.
- While circuit switching is commonly used for connecting voice circuits, the concept of a dedicated path persisting between two communicating parties or nodes can be extended to signal content other than voice.
- Its advantage is that it provides for continuous transfer without the overhead associated with packets making maximal use of available bandwidth for that communication. Its disadvantage is that it can be relatively inefficient because unused capacity guaranteed to a connection cannot be used by other connections on the same network.
- The step response of a system in a given initial state consists of the time evolution of its outputs when its control inputs are Heaviside step functions.
- In electronic engineering and control theory, step response is the time behavior of the outputs of a general system when its inputs change from zero to one in a very short time.
- The concept can be extended to the abstract mathematical notion of a dynamical system using an evolution parameter.
- From a practical standpoint, knowing how the system responds to a sudden input is important because large and possibly fast deviations from the long term steady state may have extreme effects on the component itself and on other portions of the overall system dependent on this component.
- In addition, the overall system cannot act until the component's output settles down to some vicinity of its final state, delaying the overall system response.
- Formally, knowing the step response of a dynamical system gives information on the stability of such a system, and on its ability to reach one stationary state when starting from another.



14. Analyze the open loop poles, zeros and a number of branches using root locus method.

General steps for drawing the Root Locus of the given system:

- 1. Determine the open loop poles, zeros and a number of branches from given  $G(s)H(s)$ .
- 2. Draw the pole-zero plot and determine the region of real axis for which the root locus exists. Also, determine the number of breakaway points (This will be explained while solving the problems).
- 3. Calculate the angle of asymptotes.
- 4. Determine the centroid.
- 5. Calculate the breakaway points (if any).
- 6. Calculate the intersection point of root locus with the imaginary axis.
- 7. Calculate the angle of departure or angle of arrivals if any.
- 8. From above steps draw the overall sketch of the root locus.
- 9. Predict the stability and performance of the given system by the root locus.

EX:

Question: For a unity feedback system,  $G(s) = K/[s(s+4)(s+2)]$ . Sketch the nature of root locus showing all details on it. Comment on the stability of the system

Solution:

Given system is unity feedback system. Therefore  $H(s) = 1$ .

Therefore  $G(s)H(s) = K/[s(s+4)(s+2)]$ .

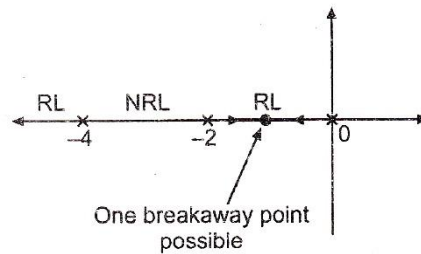
Step 1:

Poles = 0, -4, -2. Therefore  $P=3$ .

Zeros = there are no zeros.  $Z=0$ .

So all  $(P-Z=3)$  branches terminate at infinity.

Step 2: Pole-zero plot and sections of the real axis.



The pole-zero plot of the system is shown in the figure below. Here RL denotes Root Locus existence region and NRL denotes the non-existence region of root locus. These sections of real axis identified as a part of the root locus as to the right sum of poles and zeros is odd for those sections.

Step 3: Angle of asymptotes ‘A line to which root locus touches at infinity is called asymptotes.’

Number of asymptotes = P-Z = 3. Therefore 3 asymptotes are approaching to infinity.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2$$

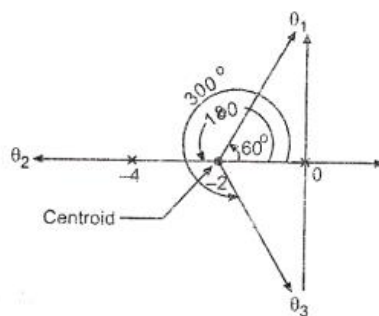
$$\therefore \theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{(2+1)180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{(2 \times 2 + 1)180^\circ}{3} = 300^\circ$$

Step 4: Centroid or Centre of asymptotes.

Asymptote touches real axis at a point called centroid.

Branches will approach infinity along these lines which are asymptotes.

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z} = \frac{0 - 2 - 4}{3} = -2$$



Step 5: To find breakaway point, we have characteristic equation as,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\therefore s^3 + 6s^2 + 8s + K = 0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$$

$$\text{Le } 3s^2 + 12s + 8 = 0$$

$$\text{Roots i.e. breakaway points} = \frac{-12 \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3} = -0.845, -3.15$$

As there is no root locus between -2 to -4, -3.15 cannot be a breakaway point. It also can be confirmed by calculating 'K' for  $s = -3.15$ . It will be negative that confirms  $s = -3.15$  is not a breakaway point.

For  $s = -3.15$ ,  $K = -3.079$  (Substituting in equation for K). But as there has to be breakaway point between '0' and '-2',  $s = -0.845$  is a valid breakaway point.

For  $s = -0.845$ ,  $K = +3.079$ . As K is positive  $s = -0.845$  is valid breakaway point.

Step 6: Intersection point with the imaginary axis.

Characteristic equation

$$s^3 + 6s^2 + 8s + K = 0$$

Roth's array:

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 6 & K \\ s^1 & \frac{48-K}{6} & 0 \\ s^0 & K & \end{array}$$

$K_{\text{marginal}} = 48$  which makes row of  $s^1$  as row of zeros.

$$A(s) = 6s^2 + K = 0$$

$$K_{\text{max}} = 48$$

$$\therefore 6s^2 + 48 = 0$$

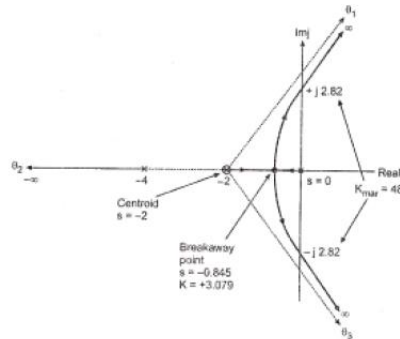
$$s^2 = -8$$

$$\therefore s = \pm j\sqrt{8} = \pm j2.828$$

Intersection of root locus with imaginary axis is at  $\pm j2.828$  and corresponding value of  $K(\text{marginal}) = 48$ .

Step 7 : As there are no complex conjugate poles or zeros, no angles of departures or arrivals are required to be calculated.

Step 8: The complete root locus is as shown in the figure below.



Step 9: Prediction about stability:

For  $0 < K < 48$ , all the roots are in left half of s-plane hence system is absolutely stable. For  $K(\text{marginal}) = +48$ , a pair of dominant roots on imaginary axis with remaining root in left half. So the system is marginally stable oscillating at 2.82 rad/sec. For  $48 < K < \infty$ , dominant roots are located in right half of s-plane hence system is unstable. Stability is predicted by locations of dominant roots. Dominant roots are those which are located closest to the imaginary axis.

15. Determine the resonant curve frequencies.

**Solution: Resonance curve definition:**

A curve whose abscissas are frequencies lying near to and on both sides of the natural frequency of a vibrating system and whose ordinates are the corresponding amplitudes of the near-resonant vibrations

- Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a simple pendulum).
- However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.
- Some systems have multiple, distinct, resonant frequencies.

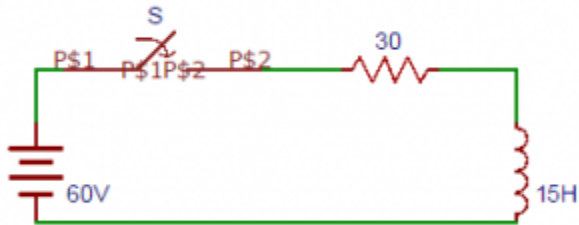
- Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions.
- Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters).

### Objective Type Questions

1. The expression of current in R- L circuit is?  
a)  $i = (V/R)(1 + \exp^{-t/(R/L)})$   
b)  $i = -(V/R)(1 - \exp^{-t/(R/L)})$   
c)  $i = -(V/R)(1 + \exp^{-t/(R/L)})$   
d)  $i = (V/R)(1 - \exp^{-t/(R/L)})$
2. The steady state part in the expression of current in the R-L circuit is?  
a)  $(V/R)(\exp^{-t/(R/L)})$   
b)  $(V/R)(-\exp^{-t/(R/L)})$   
c)  $V/R$   
d)  $R/V$
3. In the expression of current in the R-L circuit the transient part is?  
a)  $R/V$   
b)  $(V/R)(-\exp^{-t/(R/L)})$   
c)  $(V/R)(\exp^{-t/(R/L)})$   
d)  $V/R$
4. The value of the time constant in the R-L circuit is?  
a)  $L/R$   
b)  $R/L$   
c)  $R$   
d)  $L$
5. After how many time constants, the transient part reaches more than 99 percent of its final value?  
a) 2  
b) 3  
c) 4

d)5

6. A series R-L circuit with  $R = 30\Omega$  and  $L = 15H$  has a constant voltage  $V = 60V$  applied at  $t = 0$  as shown in the figure. Determine the current (A) in the circuit at  $t = 0+$ .



- a)1
- b)2
- c)3
- d)0

7. The expression of current obtained from the circuit in terms of differentiation from the circuit shown in the question 6?

- a) $di/dt+i=4$
- b) $di/dt+2i=0$
- c) $di/dt+2i=4$
- d) $di/dt-2i=4$

8. The expression of current from the circuit shown in the question 6 is?

- a) $i=2(1-e^{-2t})A$
- b) $i=2(1+e^{-2t})A$
- c) $i=2(1+e^{2t})A$
- d) $i=2(1+e^{2t})A$

9. The expression of voltage across resistor in the circuit shown in the question 6 is?

- a) $V_R=60(1+e^{2t})V$
- b) $V_R=60(1-e^{-2t})V$
- c) $V_R=60(1-e^{2t})V$
- d) $V_R=60(1+e^{-2t})V$

10. Determine the voltage across the inductor in the circuit shown in the question 6 is?

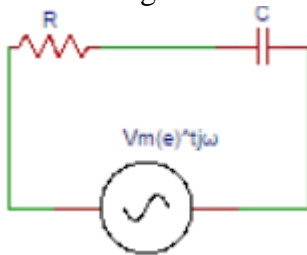
- a) $V_L=60(-e^{-2t})V$
- b) $V_L=60(e^{2t})V$
- c) $V_L=60(e^{-2t})V$
- d) $V_L=60(-e^{2t})V$



Answer Key: 1.d 2.c 3.b 4.a 5.d 6.d 7.c 8.a 9.b 10.c

**Fill in the blanks of questions with answers**

1. The current  $i(t)$  in the circuit shown above is-----
2. The impedance of the circuit at resonance condition is-----
3. The magnitude of the impedance of the circuit -----
4. The phase angle between current and voltage in the circuit shown above is?
5. The voltage function  $v(t)$  in the circuit shown below is-----



6. The impedance of the circuit shown above is-----
7. In inductor, the energy delivered by source is \_\_\_\_\_ by inductor.
8. In capacitor, the energy delivered by source is \_\_\_\_\_ by capacitor.
9. If there is complex impedance in a circuit, part of energy is \_\_\_\_\_ by reactive part and part of its energy is \_\_\_\_\_ by the resistance.

10. The equation of instantaneous power is-----

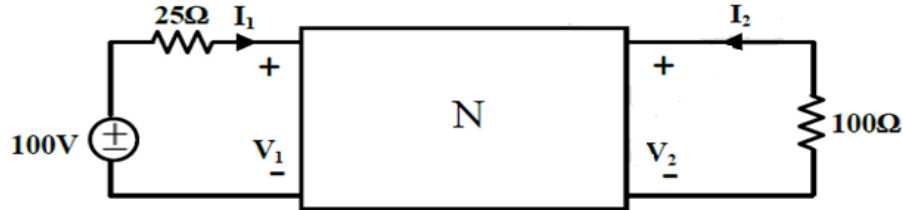
Answer Key: 1.  $i(t) = \frac{V_m}{R + j\omega L} e^{jt\omega}$  2. Purely Resistive 3.  $\sqrt{R^2 + (\omega L)^2}$  4.  $\tan^{-1} \frac{\omega L}{R}$  5.  $v(t) = V_m e^{jt\omega}$  6.  $R + 1/j\omega C$  7. : stored as magnetic field 8. stored as electric field

9. Alternately stored and returned, dissipated 10.  $P(t) = (V_m I_m / 2) (\cos(2\omega t + \theta) + \cos(\theta))$

### UNIT-III

#### Two marks of questions with answers

2. In the circuit shown below, the network N is described by the following Y matrix:  
 $Y = [0.1S \ -0.01S; \ 0.01S \ 0.1]$ . Determine the voltage gain  $V_2/V_1$ ?



Sol:

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$= 0.01V_1 + 0.1V_2 \text{ ----- (i)}$$

$$V_2 = -I_2 R_L = -100I_2$$

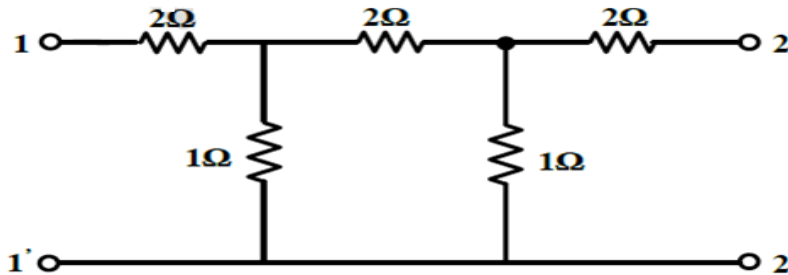
$$I_2 = -V_2/100$$

Substituting the value of  $I_2$  in equation (i)

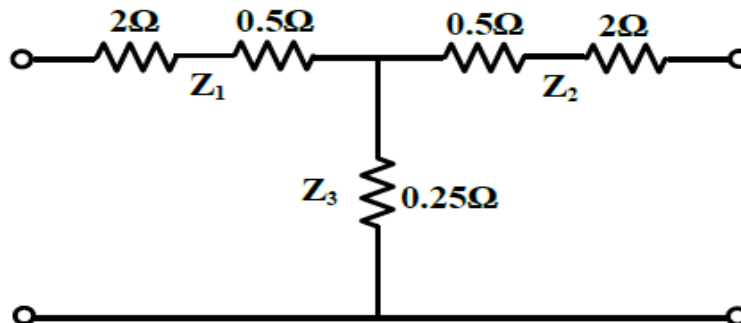
$$(-V_2/100) = 0.01V_1 + 0.1V_2$$

$$V_2/V_1 = -1/11$$

3. Determine the impedance parameters  $Z_{11}$  and  $Z_{12}$  of the two-port network shown in the figure?



Soln. Using  $\Delta - Y$  conversion, the circuit reduces to



Sol:

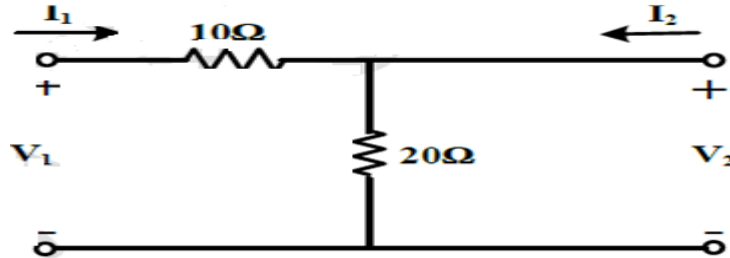
$$Z_{11} = Z_1 + Z_3$$

$$= 2.5 + 0.25$$

$$= 2.75\Omega$$

$$Z_{12}=Z_{33}=0.25\Omega$$

3 . Find out h parameters of the circuit shown in the figure?



Soln:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Writing KVL in LHS and RHS Loop

$$V_1 = 10I_1 + 20(I_1 + I_2) \text{-----(i)}$$

$$V_2 = 20(I_2 + I_1) \text{-----(ii)}$$

$$\text{Or } V_1 = 10I_1 + V_2$$

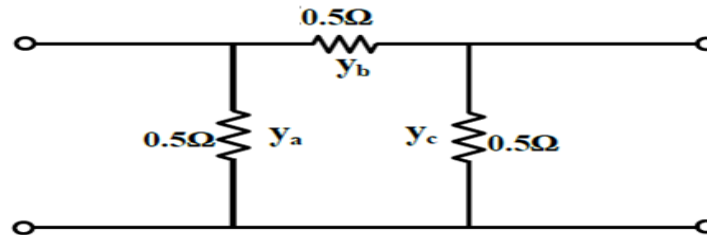
$$\text{At } V_2=0; \quad h_{11} = V_1/I_1 | V_2=0 = 10$$

$$\text{At } V_2=0; \quad h_{21} = I_2/I_1 | V_2=0 = -1$$

$$\text{At } I_1=0; \quad h_{12} = V_1/V_2 | I_1=0 = 1$$

$$\text{At } I_1=0; \quad h_{22} = I_2/V_2 | I_1=0 = 1/20 = 0.05\Omega$$

6. For the two-port network shown, find the short-circuit admittance parameter matrix?



Soln. The short circuit admittance parameters of a two port  $\pi$  network:

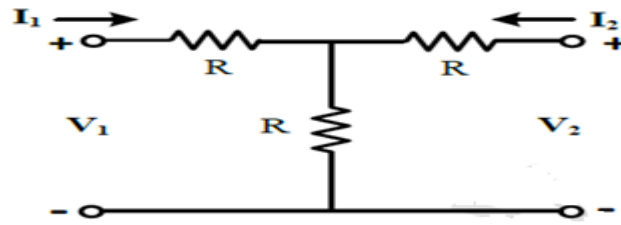
$$y_{11} = y_a + y_b = 1/0.5 + 1/0.5 = 4\Omega$$

$$y_{12} = y_{21} = -y_b = -1/0.5 = -2\Omega$$

$$y_{22} = y_b + y_c = 1/0.5 + 1/0.5 = 4\Omega$$

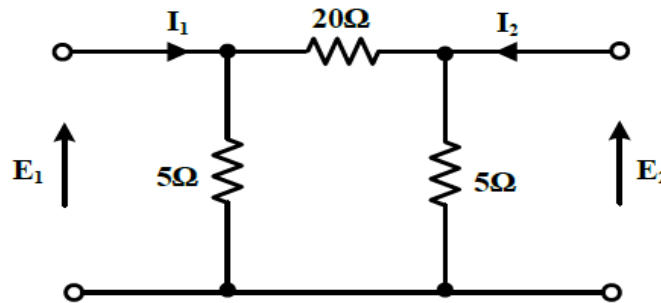
### Three marks of questions with answers

1) A 2-port network is shown in the figure. Determine the parameter  $h_{21}$  for this network?



Soln.  $V_1 = h_{11}I_1 + h_{12}V_2$   
 $I_2 = h_{21}I_1 + h_{22}V_2$   
 $h_{21} = I_2/I_1 | V_2 = 0$   
 $V_2 = RI_2 + R(I_1 + I_2)$   
 Or  $2RI_2 + RI_1 = V_2$   
 When  $V_2 = 0$ ,  $2RI_2 + RI_1 = 0$   
 So,  $I_2/I_1 = -R/2R = -1/2$

2) The Z parameters  $Z_{11}$  and  $Z_{21}$  for the 2-port network in the figure are



Sol. For z – parameters  
 $E_1 = Z_{11}I_1 + Z_{12}I_2$   
 $E_2 = Z_{21}I_1 + Z_{22}I_2$   
 Writing KVL in LHS loop

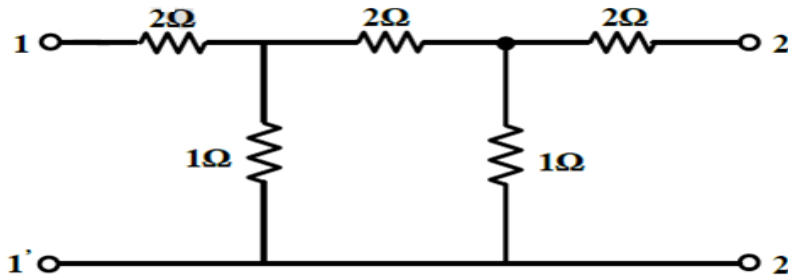
$E_1 = 2I_1 + 4I_1 + 4I_2 - 10E_1$   
 Or  $11E_1 = 6I_1 + 4I_2$  ----- (I)  
 At  $I_2 = 0$ ;  $Z_{11} = E_1/I_1 = 6/11 \Omega$

At  $I_1 = 0$   $Z_{12} = E_1/I_2 = 4/11 \Omega$   
 Writing KVL in RHS Loop

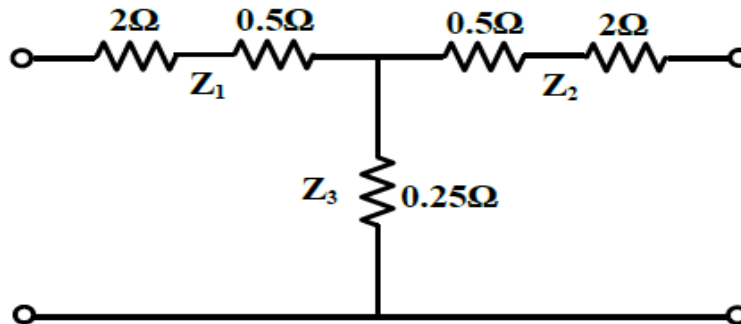
$E_2 = 4(I_1 + I_2) - 10E_1$  ----- (II)  
 Substituting  $E_1 = 6I_1 + 4I_2/11$  in equation ----- (III)

$E_2 = 4(I_1 + I_2) - 10(6I_1 + 4I_2)/11$   
 At  $I_2 = 0$ ;  $Z_{21} = E_2/I_1 = -16/11 \Omega$

3. Determine the impedance parameters  $Z_{11}$  and  $Z_{12}$  of the two-port network shown in the figure?



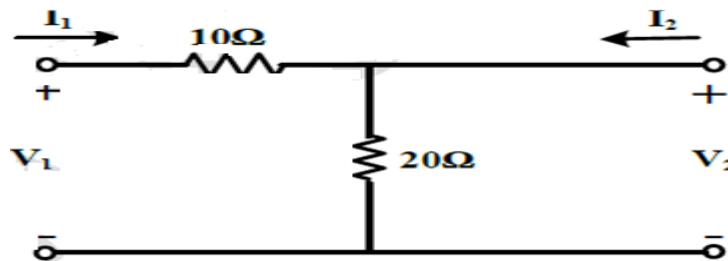
Soln. Using  $\Delta - Y$  conversion, the circuit reduces to



Sol:

$$\begin{aligned}
 Z_{11} &= Z_1 + Z_3 \\
 &= 2.5 + 0.25 \\
 &= 2.75\Omega \\
 Z_{12} &= Z_3 = 0.25\Omega
 \end{aligned}$$

4. Find out h parameters of the circuit shown in the figure?



Soln:

$$\begin{aligned}
 V_1 &= h_{11}I_1 + h_{12}V_2 \\
 I_2 &= h_{21}I_1 + h_{22}V_2
 \end{aligned}$$

Writing KVL in LHS and RHS Loop

$$\begin{aligned}
 V_1 &= 10I_1 + 20(I_1 + I_2) \text{-----(i)} \\
 V_2 &= 20(I_2 + I_1) \text{-----(ii)}
 \end{aligned}$$

Or  $V_1 = 10I_1 + V_2$

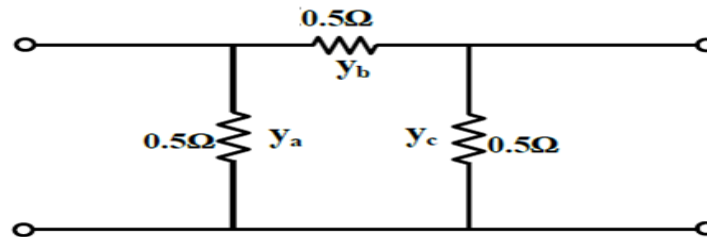
At  $V_2 = 0$ ;  $h_{11} = V_1/I_1|_{V_2=0} = 10$

At  $V_2 = 0$ ;  $h_{21} = I_2/I_1|_{V_2=0} = -1$

At  $I_1 = 0$ ;  $h_{12} = V_1/V_2|_{I_1=0} = 1$

At  $I_1 = 0$ ;  $h_{22} = I_2/V_2|_{I_1=0} = 1/20 = 0.05\Omega$

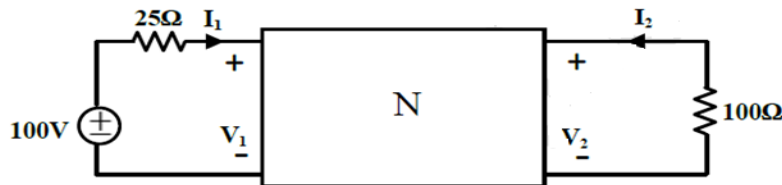
5. For the two-port network shown, find the short-circuit admittance parameter matrix?



Soln. The short circuit admittance parameters of a two port  $\pi$  network:

$$\begin{aligned}
 y_{11} &= y_a + y_b = 1/0.5 + 1/0.5 = 4\Omega \\
 y_{12} &= y_{21} = -y_b = -1/0.5 = -2\Omega \\
 y_{22} &= y_b + y_c = 1/0.5 + 1/0.5 = 4\Omega
 \end{aligned}$$

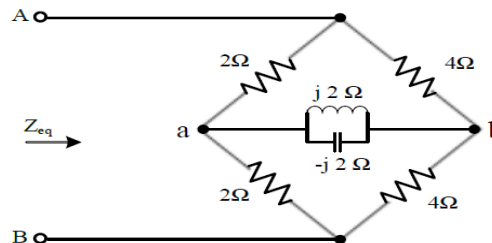
6. In the circuit shown below, the network N is described by the following Y matrix:  $Y = [0.1S \ -0.01S; \ 0.01S \ 0.1]$ . Determine the voltage gain  $V_2/V_1$ ?



Sol:

$$\begin{aligned}
 I_2 &= y_{21}V_1 + y_{22}V_2 \\
 &= 0.01V_1 + 0.1V_2 \text{ ----- (i)} \\
 V_2 &= -I_2 R_L = -100I_2 \\
 I_2 &= -V_2/100 \\
 \text{Substituting the value of } I_2 \text{ in equation (i)} \\
 (-V_2/100) &= 0.01V_1 + 0.1V_2 \\
 V_2/V_1 &= -1/11
 \end{aligned}$$

7. In the circuit of figure, the equivalent impedance seen across terminals A, B is



Soln.

The product of the opposite arms is equal, so the bridge is balanced.  
 The point a and b are at the same potential.

$$\begin{aligned}
 Z_{eq} &= (2\parallel 4) + (2\parallel 4) \\
 &= 4/3 + 4/3 = 8/3
 \end{aligned}$$

**Five marks of questions with answers**

**1. What is two-port network? Obtain the Z, Y, Hybrid and ABCD parameters?**

**Solution:**

A network containing two pairs of terminals is called as *two port network*.

Normally one pair of terminals coming together to supply power or to withdraw power or to measure the parameters, are called as *port*. To achieve simplicity, the whole network is shown with a single block.

A typical two port network is as shown below in fig (a)



Fig(a) Two port network.

**Z-parameters** can be defined by the following equations

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \dots\dots\dots (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \dots\dots\dots (2)$$

In Matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \dots\dots\dots (3)$$

If port 2-2<sup>1</sup> is open circuited, i.e., I<sub>2</sub> = 0, then Z<sub>11</sub> = V<sub>1</sub>/I<sub>1</sub> & Z<sub>21</sub> = V<sub>2</sub>/I<sub>1</sub>

If port 1-1<sup>1</sup> is open circuited, i.e., I<sub>1</sub> = 0, then Z<sub>12</sub> = V<sub>1</sub>/I<sub>2</sub> & Z<sub>22</sub> = V<sub>2</sub>/I<sub>2</sub>

Here, **Z<sub>11</sub>** is the **driving point impedance** at port 1-1<sup>1</sup> with 2-2<sup>1</sup> open circuited. It can also be called as **open circuit input impedance**.

**Z<sub>21</sub>** is the **transfer impedance** at port 1-1<sup>1</sup> with 2-2<sup>1</sup> open circuited. It can also be called as **open circuit forward transfer impedance**.

**Z<sub>12</sub>** is the **transfer impedance** at port 2-2<sup>1</sup> with 1-1<sup>1</sup> open circuited. It can also be called as **open circuit reverse transfer impedance** and

**Z<sub>22</sub>** is the **driving point impedance** at port 2-2<sup>1</sup> with 1-1<sup>1</sup> open circuited. It can also be called as **open circuit output impedance**.



Network is

- a) Reciprocal then  $V_1/I_2$  (where  $I_1 = 0$ ) =  $V_2/I_1$  (where  $I_2 = 0$ ) i.e.,  $Z_{12} = Z_{21}$
- b) Symmetrical then  $V_1/I_1$  (where  $I_2 = 0$ ) =  $V_2/I_2$  (where  $I_1 = 0$ ) i.e.,  $Z_{11} = Z_{22}$

**SHORT CIRCUIT ADMITTANCE PARAMETERS (*Y*-parameters):**

*Y*-parameters can be defined by the following equations

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \dots\dots\dots (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \dots\dots\dots (2)$$

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} * \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \dots\dots\dots (3)$$

If port 2-2<sup>1</sup> is short circuited, i.e.  $V_2 = 0$  then  $Y_{11} = I_1/V_1$  &  $Y_{21} = I_2/V_1$

If port 1-1<sup>1</sup> is short circuited, i.e.  $V_1 = 0$  then  $Y_{12} = I_1/V_2$  &  $Y_{22} = I_2/V_2$

If the network is

- a) Reciprocal then  $I_2/V_1$  (where  $V_2 = 0$ ) =  $I_1/V_2$  (where  $V_1 = 0$ ) i.e.  $Y_{21} = Y_{12}$
- b) Symmetrical then  $I_1/V_1$  (where  $V_2 = 0$ ) =  $I_2/V_2$  (where  $V_1 = 0$ ) i.e.  $Y_{11} = Y_{22}$

**Hybrid Parameters (*h*-Parameters):**

*h*-parameters can be defined by the following equations

$$V_1 = h_{11}I_1 + h_{12}V_2 \dots\dots\dots (1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \dots\dots\dots (2)$$

In matrix form:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \dots\dots\dots (3)$$

If port 2-2<sup>1</sup> is short circuited, i.e.  $V_2 = 0$  then

$$h_{11} = \frac{v_1}{I_1} \text{ \& } h_{21} = \frac{I_2}{I_1}$$

$h_{11}$  is called input impedance and  $h_{21}$  is called forward current gain.

If port 1-1<sup>1</sup> is open circuited, i.e.,  $I_1=0$  then

$$h_{12} = \frac{v_1}{v_2} \text{ \& } h_{22} = \frac{I_2}{v_2}$$

$h_{22}$  is called output admittance and  $h_{12}$  is called reverse voltage gain.

**ABCD Parameters:**

ABCD parameters can be defined by the following equations

$$V_1 = AV_2 + B(-I_2) \dots \dots \dots (1)$$

$$I_1 = CV_2 + D(-I_2) \dots \dots \dots (2)$$

**In matrix form**

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \dots \dots \dots (3)$$

If port 2-2<sup>1</sup> is open circuited i.e.,  $I_2=0$  then

$$A = \frac{V_1}{V_2} \text{ \& } C = \frac{I_1}{V_2}$$

A is called reverse voltage ratio and C is known as transfer admittance.

If port 2-2<sup>1</sup> is short circuited i.e.,  $V_2=0$  then

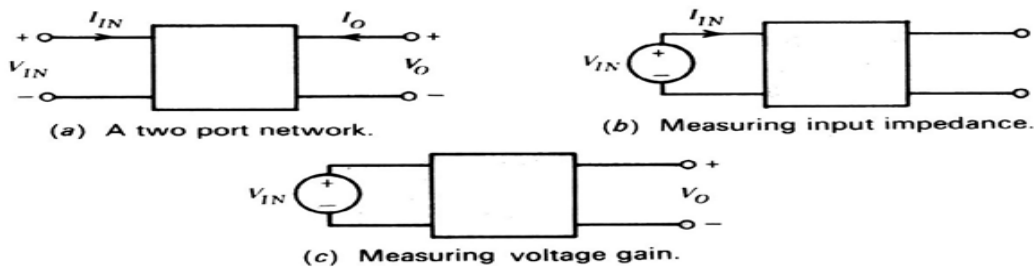
$$B = \frac{V_1}{-I_2} \text{ \& } D = \frac{I_1}{-I_2}$$

B is called transfer impedance and D is called reverse current ratio.

## 2. Briefly discuss about network functions and network parameters?

### Solution:

A network function is the Laplace transform of an impulse response. Its format is a ratio of two polynomials of the complex frequencies. Consider the general two-port network shown in Figure a. The terminal voltages and currents of the two-port can be related by two classes of network functions, namely, the driving point functions and the transfer functions.



The driving point functions relate the voltage at a port to the current at the same port. Thus, these functions are a property of a single port. For the input port the driving point impedance function  $Z_{IN}(s)$  is defined as

$$Z_{IN}(s) = V_{IN}(s) / I_{IN}(s)$$

This function can be measured by observing the current  $I_{IN}$  when the input port is driven by a voltage source  $V_{IN}$ . The driving point admittance function  $Y_{IN}(s)$  is the reciprocal of the impedance function, and is given by

$$Y_{IN}(s) = \frac{I_{IN}(s)}{V_{IN}(s)}$$

The output port driving point functions are defined in a similar way. The transfer functions of the two-port relate the voltage (or current) at one port to the voltage (or current) at the other port. The possible forms of transfer functions are:

1. The voltage transfer function, which is a ratio of one voltage to another voltage.
2. The current transfer function, which is a ratio of one current to another current.
3. The transfer impedance function, which is the ratio of a voltage to a current.
4. The transfer admittance function, which is the ratio of a current to a voltage.

The voltage transfer functions are defined with the output port an open circuit, as:

$$\text{voltage gain} = \frac{V_O(s)}{V_{IN}(s)}$$

$$\text{voltage loss (attenuation)} = \frac{V_{IN}(s)}{V_O(s)}$$

To evaluate the voltage gain, for example, the output voltage  $V_O$  is measured with the input port driven by a voltage source  $V_{IN}$ . The other three types of transfer functions can be defined in a similar manner. Of the four types of transfer functions, the voltage transfer function is the one most often specified in the design of filters

The functions defined above, when realized using resistors, inductors, capacitors, and active devices, can be shown to be the ratios of polynomials in  $s$  with real coefficients. This is so because the network functions are obtained by solving simple algebraic node equations, which involve at most the terms  $R$ ,  $sL$ ,  $sC$  and their reciprocals. The active device, if one exists, the solution still involves only the addition and multiplication of simple terms, which can only lead to a ratio of polynomials in  $s$ . In addition, all the coefficients of the numerator and denominator polynomials will be real.

Thus, the general form of a network function is

$$H(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_0}{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_0}$$

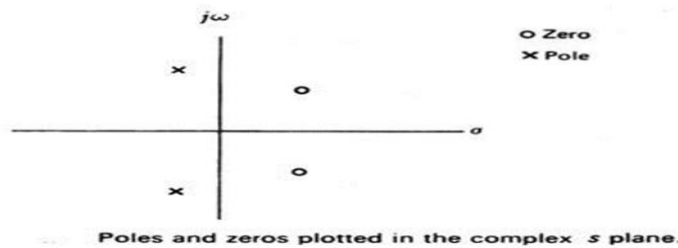
where  $a_n \neq 0$        $b_m \neq 0$

and all the coefficients  $a_i$  and  $b_i$  are real. If the numerator and denominator polynomials are factored, an alternate form of  $H(s)$  is obtained:

$$H(s) = \frac{a_n (s - z_1)(s - z_2) \dots (s - z_n)}{b_m (s - p_1)(s - p_2) \dots (s - p_m)}$$

Where  $z_1, z_2, \dots, z_n$  are called the zeros of  $H(s)$ , because  $H(s) = 0$  when  $s = z_i$ . The roots of the denominator  $p_1, p_2, \dots, p_m$  are called the poles of  $H(s)$ . It can be seen that  $H(s) = \infty$  at the poles,  $s = p_i$ .

The poles and zeros can be plotted on the complex  $s$  plane ( $s = \sigma + j\omega$ ), which has the real part  $\sigma$  for the abscissa, and the imaginary part  $j\omega$  for the ordinate.



Properties of all Network Functions:

A consequence of this property is that complex poles (and zeros) must occur in conjugate pairs. To demonstrate this fact consider a complex root at  $(s=-a-jb)$  which leads to the factor  $(s+a+jb)$  in the network function. The  $jb$  term will make some of the coefficients complex in the polynomial, unless the conjugate of the complex root at  $(s=-a+jb)$  is also present in the polynomial. The product of a complex factor and its conjugate is which can be seen to have real coefficients.

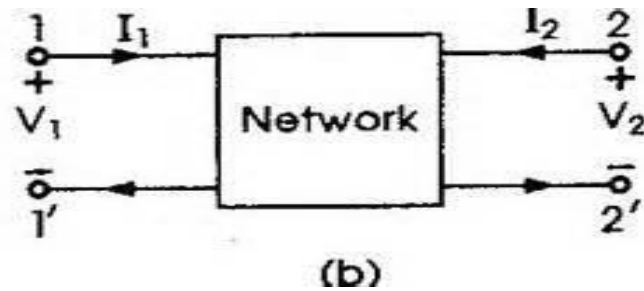
$$(s + a + jb)(s + a - jb) = s^2 + 2as + a^2 + b^2$$

Further important properties of network functions are obtained by restricting *the networks to be stable*, by which we mean that a bounded input excitation to the network must yield a bounded response. Put differently, the output of a stable network cannot be made to increase indefinitely by the application of a bounded input excitation. Passive networks are stable by their very nature, since they do not contain energy sources that might inject additional energy into the network. Active networks, however, do contain energy sources that could join forces with the input excitation to make the output increase indefinitely. Such unstable networks, however, have no use in the world of practical filters and are therefore precluded from all our future discussions.

### 3. Explain the concept of two port network functions using transformed variables?

**Solution:**

- Consider a two port network with voltages and currents at ports 1-1' and 2-2' as  $V_1(t)$ ,  $I_1(t)$  and  $V_2(t)$ ,  $I_2(t)$  respectively as shown in figure .



**Network functions for Two-Port Network are as follows:**

1. Driving point functions:

- Driving point impedance functions
- Driving point admittance functions

2. Transfer Functions:

- Voltage transfer functions
- Current transfer functions
- Transfer impedance functions
- Transfer admittance functions

**Driving point impedance functions:**

- The ratio of Laplace transform of voltage and current at port 1-1' or 2-2' is defined as driving point impedance function.
- Thus there are two driving point impedance functions.
  - At port 1-1' denoted as  $Z_{11}(s)$ :  $Z_{11}(s) = V_1(s)/I_1(s)$
  - At port 2-2' denoted as  $Z_{22}(s)$ :  $Z_{22}(s) = V_2(s)/I_2(s)$

**Driving point admittance functions:**

- The ratio of Laplace transform of current and voltage at port 1-1' or 2-2' is defined as driving point admittance function.

Thus there are two driving point admittance functions.

- At port 1-1' denoted as  $Y_{11}(s)$

$$Y_{11}(s) = I_1(s)/V_1(s)$$

- At port 2-2' denoted as  $Y_{11}(s)$

$$Y_{22}(s) = I_2(s)/V_2(s)$$

### Transfer Functions:

Voltage Transfer Function:

- It is defined as the ratio of Laplace transform of voltage at one port and voltage at another port. It is denoted as  $G(s)$ .

$$G_{12}(s) = V_1(s)/V_2(s) \quad \text{and} \quad G_{21}(s) = V_2(s)/V_1(s)$$

Current Transfer Function:

- It is defined as the ratio of Laplace transform of current at one port and current at another port. It is denoted as  $\alpha(s)$ .

$$\alpha_{12}(s) = I_1(s)/I_2(s) \quad \text{and} \quad \alpha_{21}(s) = I_2(s)/I_1(s)$$

Transfer Impedance Function:

- It is defined as the ratio of Laplace transform of voltage at one port and current at another port.

$$Z_{12}(s) = V_1(s)/I_2(s) \quad \text{and} \quad Z_{21}(s) = V_2(s)/I_1(s)$$

Transfer Admittance Function:

- It is defined as the ratio of Laplace transform of current at one port and voltage at another port.

$$Y_{12}(s) = I_1(s)/V_2(s) \quad \text{and} \quad Y_{21}(s) = I_2(s)/V_1(s)$$

- 4. Explain the concept of poles, zeros, their significance and necessary conditions for driving point functions and transfer functions?**

**Solution:**

### System Poles and Zeros:

The transfer function provides a basis for determining important system response characteristics without solving the complete differential equation. As defined, the transfer function is a rational function in the complex variable  $s = \sigma + j\omega$ , that is

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors

$$H(s) = \frac{N(s)}{D(s)} = K \frac{(s - z_1)(s - z_2) \dots (s - z_{m-1})(s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_{n-1})(s - p_n)}$$

Where the numerator and denominator polynomials, N(s) and D(s), have real coefficients defined by the system's differential equation and  $K = b_m/a_n$ . As written in Eq. (2) the  $z_i$ 's are the roots of the equation

$$N(s) = 0$$

and are defined to be the system zeros, and the  $p_i$ 's are the roots of the equation

$$D(s) = 0,$$

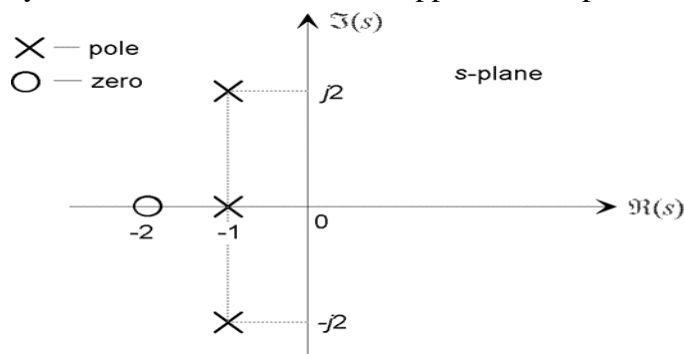
and are defined to be the system poles. In Eq. (2) the factors in the numerator and denominator are written so that when  $s = z_i$  the numerator  $N(s) = 0$  and the transfer function vanishes, that is

$$\lim_{s \rightarrow z_i} H(s) = 0$$

and similarly when  $s = p_i$  the denominator polynomial  $D(s) = 0$  and the value of the transfer function becomes unbounded,

$$\lim_{s \rightarrow p_i} H(s) = \infty$$

All of the coefficients of polynomials N(s) and D(s) are real, therefore the poles and zeros must be either purely real, or appear in complex conjugate pairs. In general for the poles, either  $p_i = \sigma_i$ , or else  $p_i, p_{i+1} = \sigma_i + j\omega_i$ . The existence of a single complex pole without a corresponding conjugate pole would generate complex coefficients in the polynomial D(s). Similarly, the system zeros are either real or appear in complex conjugate pairs.





A system is characterized by its poles and zeros in the sense that they allow reconstruction of the input/output differential equation. In general, the poles and zeros of a transfer function may be complex, and the system dynamics may be represented graphically by plotting their locations on the complex s-plane, whose axes represent the real and imaginary parts of the complex variable s. Such plots are known as pole-zero plots. It is usual to mark a zero location by a circle (°) and a pole location a cross (×). The location of the poles and zeros provide qualitative insights into the response characteristics of a system

**System Stability:** The stability of a linear system may be determined directly from its transfer function. An nth order linear system is asymptotically stable only if all of the components in the homogeneous response from a finite set of initial conditions decay to zero as time increases, or

$$\lim_{t \rightarrow \infty} \sum_{i=1}^n c_i e^{p_i t} = 0$$

Where the  $p_i$  are the system poles. In a stable system all components of the homogeneous response must decay to zero as time increases. If any pole has a positive real part there is a component in the output that increases without bound, causing the system to be unstable.

In order for a linear system to be stable, all of its poles must have negative real parts that are they must all lie within the left-half of the s-plane. An unstable pole, lying in the right half of the s-plane, generates a component in the system homogeneous response that increases without bound from any finite initial conditions.

A system having one or more poles lying on the imaginary axis of the s-plane has non-decaying oscillatory components in its homogeneous response, and is defined to be marginally stable.

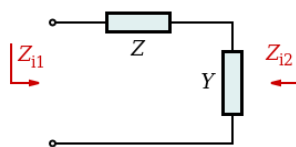
### **Properties of Driving Point (Positive Real) Functions:**

**These conditions are required to satisfy to be positive realness:**

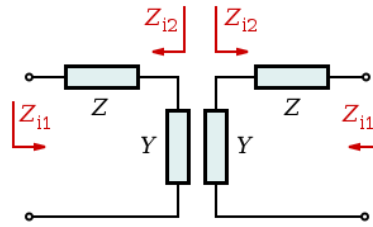
- Y(s) must be a rational function in s with real coefficients, i.e., the coefficients of the numerator and denominator polynomials is real and positive.
- The poles and zeros of Y(s) have either negative or zero real parts, i.e., Y(s) not have poles or zeros in the right half s plane.
- Poles of Y(s) on the imaginary axis must be simple and their residues must be real and positive, i.e., Y(s) not has multiple poles or zeros on the  $j\omega$  axis. The same statement applies to the poles of  $1/Y(s)$ .
- The degrees of the numerator and denominator polynomials in Y(s) differ at most by 1. Thus the number of finite poles and finite zeros of Y(s) differ at most by 1.

- The terms of lowest degree in the numerator and denominator polynomials of  $Y(s)$  differ in degree at most by 1. So  $Y(s)$  has neither multiple poles nor zeros at the origin.
  - There are no missing terms in numerator and denominator polynomials unless all even or all odd terms are missing.
5. Derive the image impedance and iterative impedance of L, PI and T networks .
- Image impedance is a concept used in electronic network design and analysis and most especially in filter design.
  - The term image impedance applies to the impedance seen looking into a port of a network. Usually a two-port network is implied but the concept can extend to networks with more than two ports.
  - The definition of image impedance for a two-port network is the impedance,  $Z_{i1}$ , seen looking into port 1 when port 2 is terminated with the image impedance,  $Z_{i2}$ , for port 2.
  - In general, the image impedances of ports 1 and 2 will not be equal unless the network is symmetrical (or anti-symmetrical) with respect to the ports.
  - As an example, the derivation of the image impedances of a simple 'L' network is given below. The L network consists of a series impedance,  $Z$ , and a shunt admittance,  $Y$ .
  - The difficulty here is that in order to find  $Z_{i1}$  it is first necessary to terminate port 2 with  $Z_{i2}$ .
  - However,  $Z_{i2}$  is also an unknown at this stage. The problem is solved by terminating port 2 with an identical network: port 2 of the second network is connected to port 2 of the first network and port 1 of the second network is terminated with  $Z_{i1}$ .
  - The second network is terminating the first network in  $Z_{i2}$  as required. Mathematically, this is equivalent to eliminating one variable from a set of simultaneous equations.
  - The network can now be solved for  $Z_{i1}$ . Writing out the expression for input impedance gives;

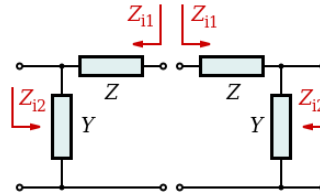
Simple 'L' network with series impedance  $Z$  and shunt admittance  $Y$ . Image impedances  $Z_{i1}$  and  $Z_{i2}$  are shown



Showing how a T section is made from two cascaded L half sections.  $Z_{i2}$  is facing  $Z_{i2}$  to provide matching impedances



Showing how a  $\Pi$  section is made from two cascaded L half sections.  $Z_{i1}$  is facing  $Z_{i1}$  to provide matching impedances



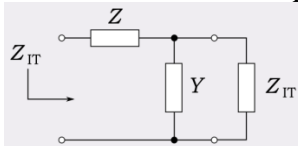
$$Z_{i1} = Z + \frac{1}{2Y + \frac{1}{Z + Z_{i1}}}$$

$$Z_{i1}^2 = Z^2 + \frac{Z}{Y}$$

$$Y_{i2}^2 = Y^2 + \frac{Y}{Z}$$

### Iterative impedance

- Iterative impedance is the input impedance of an infinite chain of identical networks. It is related to the image impedance used in filter design, but has a simpler, more straightforward definition.
- A simple generic L-circuit is shown in the diagram consisting of a series impedance  $Z$  and a shunt admittance  $Y$ . The iterative impedance of this network,  $Z_{IT}$ , in terms of its output load (also  $Z_{IT}$ ) is given by,



Iterative impedance of a simple generic L-circuit

$$Z_{IT} = Z + Y \parallel Z_{IT}$$

$$Z_{IT} = \frac{Z}{2} \pm \sqrt{\frac{Z^2}{4} + \frac{Z}{Y}}$$

$$Y_{IT} = \frac{Y}{2} \pm \sqrt{\frac{Y^2}{4} + \frac{Y}{Z}}$$

$$Y_{IT} = \frac{1}{Z_{IT}}$$

**Relationship to image impedance:**

$$Z_{IT} = \frac{Z}{2} + Z_{IM}$$

## 6. Driving Points and Transfer functions –using Transformed (S)Variables

Properties of Driving Point (Positive Real) Functions:

These conditions are required to satisfy to be positive realness  $Y(s)$  must be a rational function in  $s$  with real coefficients, i.e., the

- Coefficients of the numerator and denominator polynomials are real and positive. The poles and zeros of  $Y(s)$  have either negative or zero real parts, i.e.,  $Y(s)$  not have poles or zeros in the right half  $s$  plane. Poles of  $Y(s)$  on the imaginary axis must be simple and their residues must be real and positive, i.e.,  $Y(s)$  not has multiple poles or zeros on the  $j\omega$  axis. The same statement applies to the poles of  $1/Y(s)$ .

The degrees of the numerator and denominator polynomials in  $Y(s)$  differ at most by 1. Thus the number of finite poles and finite zeros of  $Y(s)$  differ at most by 1.

The terms of lowest degree in the numerator and denominator polynomials of  $Y(s)$  differ in degree at most by 1. So  $Y(s)$  has neither multiple poles nor zeros at the origin. There are no missing terms in numerator and denominator polynomials unless all even or all odd terms are missing. Test for necessary and sufficient conditions:

$Y(s)$  must be real when  $s$  is real. ♣ If  $Y(s) = p(s)/q(s)$ , then  $p(s) + q(s)$  must be Hurwitz. This requires that: ♣i. the continued fraction expansion of the Hurwitz test give only real and positive coefficients, and ii. the continued fraction expansion not end prematurely. In order that  $\text{Re} [Y(j\omega)] \geq 0$  for all  $\omega$ , it is necessary and sufficient that

$$A(\omega^2) = m_1 m_2 - n_1 n_2 \Big|_{s=j\omega}$$

Have no real positive roots of odd multiplicity. This may be determined by factoring  $A(\omega^2)$  or by the use of Sturm's theorem.

EX: the function

### Objective Questions with Answers

1. The ratio of voltage transform at first port to the voltage transform at the second port is called?

- a) Voltage transfer ratio
- b) Current transfer ratio
- c) Transfer impedance
- d) Transfer admittance

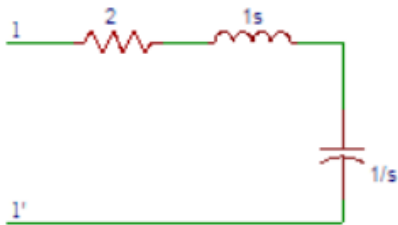
2. The ratio of the current transform at one port to current transform at other port is called?

- a) Transfer admittance
- b) Transfer impedance
- c) Current transfer ratio
- d) Voltage transfer ratio

3. The ratio of voltage transform at first port to the current transform at the second port is called?

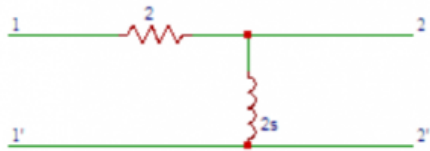
- a) Voltage transfer ratio
- b) Transfer admittance
- c) Current transfer ratio
- d) Transfer impedance

4. For the network shown in the figure, find the driving point impedance.



- a)  $(s^2 - 2s + 1)/s$
- b)  $(s^2 + 2s + 1)/s$
- c)  $(s^2 - 2s - 1)/s$
- d)  $(s^2 + 2s - 1)/s$

5. Obtain the transfer function  $G_{21}(S)$  in the circuit shown below.



- a)  $(s+1)/s$

- b)  $s+1$
- c)  $s$
- d)  $s/(s+1)$

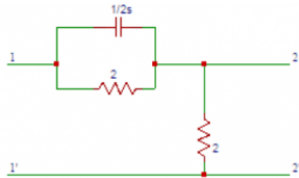
6. Determine the transfer function  $Z_{21}(S)$  in the circuit shown in question 5.

- a)  $s$
- b)  $2s$
- c)  $3s$
- d)  $4s$

7. Find the driving point impedance  $Z_{11}(S)$  in the circuit shown in question 5.

- a)  $2(s+2)$
- b)  $(s+2)$
- c)  $2(s+1)$
- d)  $(s+1)$

8. Obtain the transfer function  $G_{21}(s)$  in the circuit shown below.



- a)  $(8S+2)/(8S+1)$
- b)  $(8S+2)/(8S+2)$
- c)  $(8S+2)/(8S+3)$
- d)  $(8S+2)/(8S+4)$

9. Obtain the transfer function  $Z_{21}(s)$  in the circuit shown in question 8.

- a) 1
- b) 2
- c) 3
- d) 4

10. Determine the driving point impedance  $Z_{11}(S)$  in the circuit shown in question 8.

- a)  $(8S+4)/(4S+4)$
- b)  $(8S+4)/(4S+3)$
- c)  $(8S+4)/(4S+2)$
- d)  $(8S+4)/(4S+1)$

AnswerKey:1.a 2.c 3.d 4.b 5.d 6.b 7.c 8.d 9.b 10.d

**Fill in the blanks of questions with answers**

1. The driving point function is the ratio of polynomials in  $s$ . Polynomials is obtained from the \_\_\_\_\_ of the elements and their combinations.
2. The pole is that finite value of  $S$  for which  $N(S)$  becomes \_\_\_\_\_
3. A function  $N(S)$  is said to have a pole (or zero) at infinity, if the function  $N(1/S)$  has a pole (or zero) at  $S = ?$
4. The number of zeros including zeros at infinity is \_\_\_\_\_ the number of poles including poles at infinity.
5. The poles of driving point impedance are those frequencies corresponding to \_\_\_\_\_ conditions?
6. The zeros of driving point impedance are those frequencies corresponding to \_\_\_\_\_ conditions?
7. In the driving point admittance function, a zero of  $Y(s)$  means a \_\_\_\_\_ of  $I(S)$ .
8. In the driving point admittance function, a pole of  $Y(s)$  means a \_\_\_\_\_ of  $V(S)$ .
9. The real part of all zeros and poles must be?
10. Poles or zeros lying on the  $j\omega$  axis must be?

**Answer key**

1. transform impedance
2.  $\infty$
3. 0
4. equal to
5. open circuit
6. short circuit
7. Zero
8. Zero
- 9: negative or zero
10. simple

### Five marks of questions with answers

1. Analyze the T, Pi and L with suitable values of YA(s), YB(s) and Z(s)

Solution:

Any passive reciprocal two-port network requires only three parameters to describe it.

The fourth parameter will be provided by the reciprocity condition.

The network is being a reciprocal one, has three parameters. It has three impedances also.

Therefore, it must be possible to determine YA(s), YB(s) and Z(s) such that the resulting network has a pre specified set of parameters (z, y, h, g or ABCD).

Therefore, it follows that any reciprocal network will have a T-equivalent – i.e., given any reciprocal network, it will be possible to find a T-network with suitable values of YA(s), YB(s) and Z(s) such that the T-network and the original two-port network will have the same two-port parameter set we have already obtained the ABCD matrix of the symmetric  $\pi$ -network in Example.

$$\begin{bmatrix} 1 + \frac{1}{2}Z(s)Y(s) & Z(s) \\ Y(s)(1 + \frac{1}{4}Z(s)Y(s)) & 1 + \frac{1}{2}Z(s)Y(s) \end{bmatrix}$$

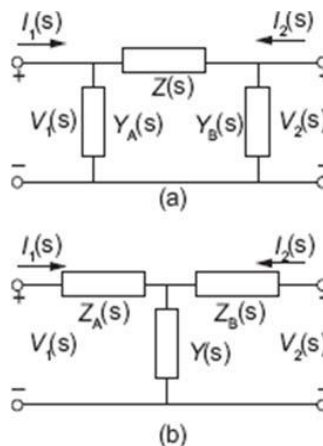


Fig. General Passive T and  $\pi$  Networks in s-Domain



Fig. Standard Symmetric Passive T and Equivalent Circuits for an Arbitrary Passive Symmetric Two-Port Network

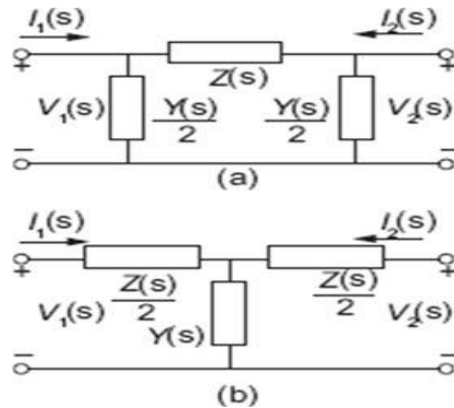


Fig. Standard Symmetric Passive T and Equivalent Circuits for an Arbitrary Passive Symmetric Two-Port Network

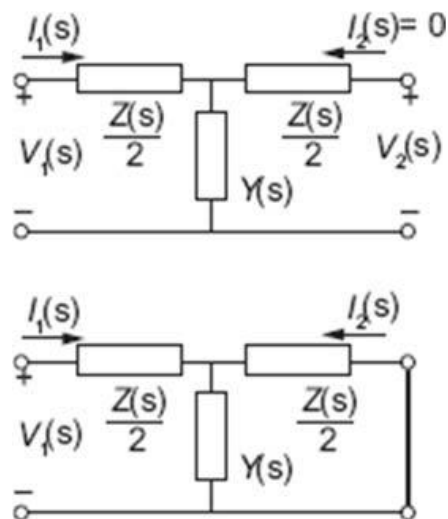


Fig. Circuits to be Solved for Determining ABCD Parameters of a Symmetric T-Network.

7. Analyze the image transfer constant, designing of attenuator.

The third parameter needed to complete the description is obtained by determining the ratios and when the second port is terminated in  $Z_{im2}$  and  $v_1$  is applied at the first port. The geometric mean of these two ratios is expressed as the exponential of a number  $\gamma$  and that  $\gamma$  is called the Image Transfer Constant. Image transfer constant defined.

$$e^{\gamma} = \sqrt{\frac{v_1}{v_2} \times \frac{i_1}{(-i_2)}} \text{ with } v_2 \text{ developed across } Z_{im2}.$$

Image transfer constant can be expressed in terms of the ABCD parameters.

$$v_1 = Av_2 - Bi_2$$

$$i_1 = Cv_2 - Di_2$$

But, with  $Z_{im2}$  termination,  $-i_2 = \frac{v_2}{Z_{im2}}$ . Substituting this in the first ABCD equation,

we get,  $\frac{v_1}{v_2} = (A + \frac{B}{Z_{im2}})$ . Substituting  $v_2 = -Z_{im2}i_2$  in the second ABCD equation, we get,

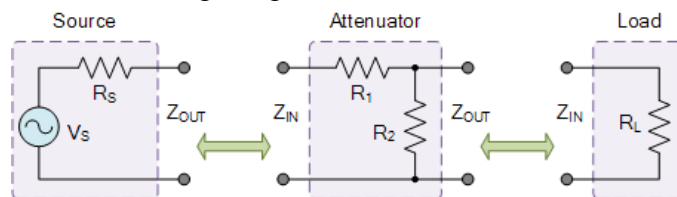
$$\frac{i_1}{(-i_2)} = (D + CZ_{im2}). \text{ But } Z_{im2} = \sqrt{\frac{DB}{CA}}.$$

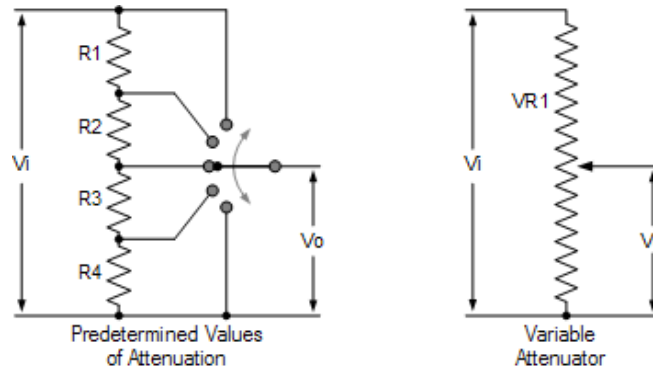
$$\therefore \frac{v_1}{v_2} = A + \frac{B}{Z_{im2}} = A + \frac{\sqrt{ABCD}}{D} \text{ and } \frac{i_1}{(-i_2)} = D + CZ_{im2} = D + \frac{\sqrt{ABCD}}{A}$$

$$\begin{aligned} \therefore e^{\gamma} &= \sqrt{\frac{v_1}{v_2} \times \frac{i_1}{(-i_2)}} = \sqrt{AD + BC + 2\sqrt{ABCD}} = \sqrt{(\sqrt{AD} + \sqrt{BC})^2} \\ &= \sqrt{AD} + \sqrt{BC} = \sqrt{AD} + \sqrt{AD - 1} \end{aligned}$$

$$AD - BC = 1$$

A *passive attenuator* reduces the amount of power being delivered to the connected load by either a single fixed amount, a variable amount or in a series of known switchable steps. Attenuators are generally used in radio, communication and transmission line applications to weaken a stronger signal.





$$dB_v = 20 \log_{10} \frac{V_{out}}{V_{in}} \text{ (dB)}$$

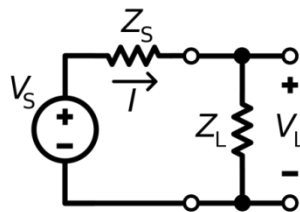
### 3. Impedance Matching Networks

In electronics, impedance matching is the practice of designing the input impedance of an electrical load or the output impedance of its corresponding signal source to maximize the power transfer or minimize signal reflection from the load.

In the case of a complex source impedance  $Z_S$  and load impedance  $Z_L$ , maximum power transfer is obtained when

$$Z_S = Z_L^*$$

where the asterisk indicates the complex conjugate of the variable. Where  $Z_S$  represents the characteristic impedance of a transmission line, minimum reflection is obtained when



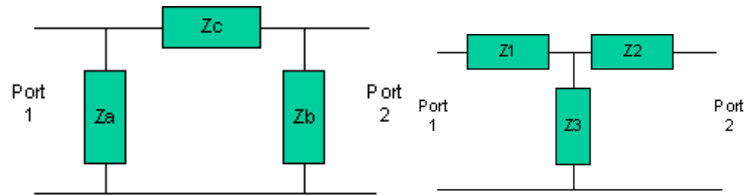
$$Z_S = Z_L$$

- The concept of impedance matching found first applications in electrical engineering, but is relevant in other applications in which a form of energy, not necessarily electrical, is transferred between a source and a load.
- An alternative to impedance matching is impedance bridging, in which the load impedance is chosen to be much larger than the source impedance and maximizing voltage transfer, rather than power, is the goal.

4. Formulate Pi and T networks, T and Pi conversion

**Transforming from Pi to T and vice versa**

Any pi network can be transformed to an equivalent T network. This is also known as the Wye-Delta transformation, which is the terminology used in power distribution and electrical engineering. The pi is equivalent to the Delta and the T is equivalent to the Wye (or Star) form.



**B) T NETWORK**

### A) PI NETWORK

The impedances of the pi network ( $Z_a$ ,  $Z_b$ , and  $Z_c$ ) can be found from the impedances of the T network with the following equations:

$$Z_a = \frac{(Z_1 * Z_2) + (Z_1 * Z_3) + (Z_2 * Z_3)}{Z_2}$$

$$Z_b = \frac{(Z_1 * Z_2) + (Z_1 * Z_3) + (Z_2 * Z_3)}{Z_1}$$

$$Z_c = \frac{(Z_1 * Z_2) + (Z_1 * Z_3) + (Z_2 * Z_3)}{Z_3}$$

Note the common numerator in all these expressions which can prove useful in reducing the amount of computation necessary.

The impedances of the T network ( $Z_1$ ,  $Z_2$ , and  $Z_3$ ) can be found from the impedances of the equivalent pi network with the following equations:

$$Z_1 = \frac{Z_a * Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_b * Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a * Z_b}{Z_a + Z_b + Z_c}$$

Note the common denominator in these expressions.

In the case where all the impedances are reactive (i.e. they are all in the form  $jX$ ), it is handy to note that the -1 factors from squaring  $j*j$  on the top cancels the -1 from bringing the  $j$  in the denominator up top.

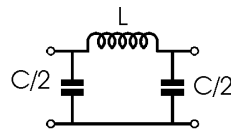
#### 5. Draw the T and Pi section filters.

Low pass filters are used in a wide number of applications. Particularly in radio frequency applications, low pass filters are made in their LC form using inductors and capacitors.

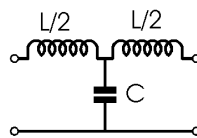
Typically they may be used to filter out unwanted signals that may be present in a band above the wanted pass band. In this way, this form of filter only accepts signals below the cut-off frequency.

Low pass filters using LC components, i.e. inductors and capacitors are arranged in either a pi or T network. For the pi section filter, each section has one series component and either side a component to ground.

The T network low pass filter has one component to ground and either side there is a series in line component. In the case of a low pass filter the series component or components are inductors whereas the components to ground are capacitors.



Pi section filter



T section filter

### LC Pi and T section low pass filters:

There is a variety of different filter variants that can be used dependent upon the requirements in terms of in band ripple, rate at which final roll off is achieved, etc. The type used here is the constant-k and this produces some manageable equations:

$$L = Z_o / (\pi \times F_c) \text{ Henries}$$

$$C = 1 / (Z_o \times \pi \times F_c) \text{ Farads}$$

$$F_c = 1 / (\pi \times \text{square root} (L \times C)) \text{ Hz}$$

Where

$Z_o$  = characteristic impedance in ohms

$C$  = Capacitance in Farads

$L$  = Inductance in Henries

$F_c$  = Cut-off frequency in Hertz

### 6. Wright short notes on filters?

**Solution:**

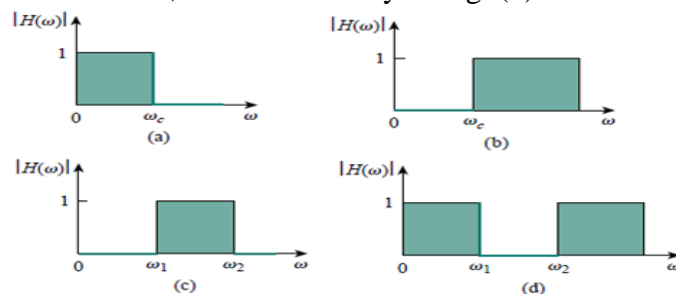
The concept of filters has been an integral part of the evolution of electrical engineering from the beginning.

A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the

Circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment. A filter is a passive filter if it consists of only passive elements R, L, and C. It is said to be an active filter if it consists of active elements (such as transistors and op amps) in addition to passive elements R, L, and C. There are other kinds of filters—such as digital filters, electromechanical filters, and microwave filters—which are beyond the level of the text. As shown in Fig. , there are four types of filters whether passive or active:

1. A low pass filter passes low frequencies and stops high frequencies, as shown ideally in Fig. (a).
2. A high pass filter passes high frequencies and rejects low frequencies, as shown ideally in Fig. (b).
3. A band pass filter passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. (c).
4. A band stop filter passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. (d).



## 7. Discuss the following terms?

- a) Low pass filter
- b) High pass filter

### Solution:

Low pass Filter:

A typical low pass filter is formed when the output of an RC circuit is taken off the capacitor as shown in Fig... The transfer function is

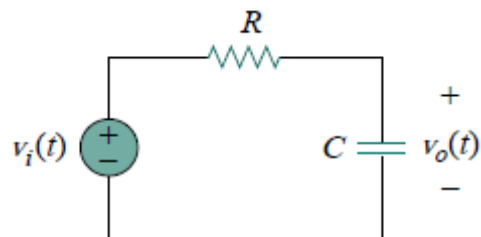
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

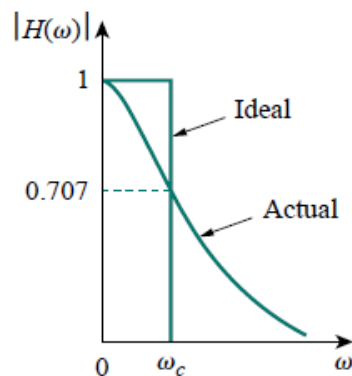
Note that  $H(0) = 1$ ,  $H(\infty) = 0$ . Figure shows the plot of  $|H(\omega)|$ , along with the ideal characteristic. The half-power frequency, which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as the cutoff frequency  $\omega_c$ , is obtained by setting the magnitude of  $H(\omega)$  equal to  $1/\sqrt{2}$ , thus

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{RC}$$



**Figure** A lowpass filter.



**Figure** Ideal and actual frequency response of a lowpass filter.

The cutoff frequency is the frequency at which the transfer function  $H$  drops in magnitude to 70.71% of its maximum value. It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.

The cut-off frequency is also called the roll off frequency.

A low pass filter is designed to pass only frequencies from dc up to the cutoff frequency  $\omega_c$ .

A low pass filter can also be formed when the output of an RL circuit is taken off the resistor. Of course, there are many other circuits for low pass filters.

High pass Filter:

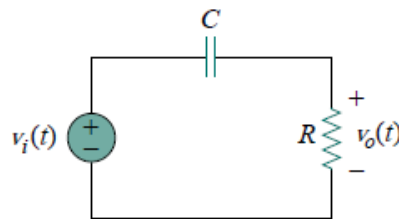
A high pass filter is formed when the output of an RC circuit is taken off the resistor as shown in Fig. The transfer function is

$$\mathbf{H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + 1/j\omega C}}$$

$$\mathbf{H(\omega) = \frac{j\omega RC}{1 + j\omega RC}}$$

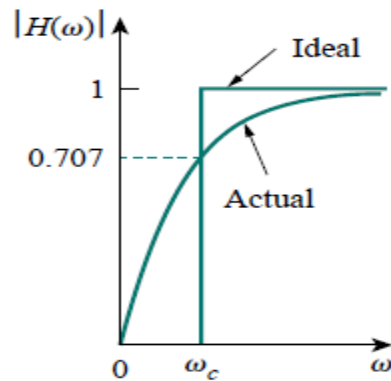
Note that  $H(0) = 0$ ,  $H(\infty) = 1$ . Figure shows the plot of  $|H(\omega)|$ . Again, the corner or cutoff frequency is

$$\omega_c = \frac{1}{RC}$$



**Figure** A highpass filter.





**Figure** Ideal and actual frequency response of a highpass filter.

A high pass filter is designed to pass all frequencies above its cutoff frequency  $\omega_c$ .

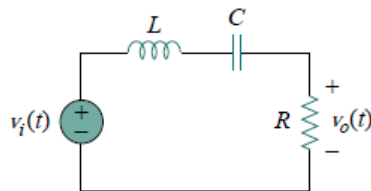
A high pass filter can also be formed when the output of an RL circuit is taken off the inductor.

### 8. Explain about band pass and band reject filters?

#### Solution:

Band pass Filter:

The RLC series resonant circuit provides a band pass filter when the output is taken off the resistor as shown in Fig. The transfer function is

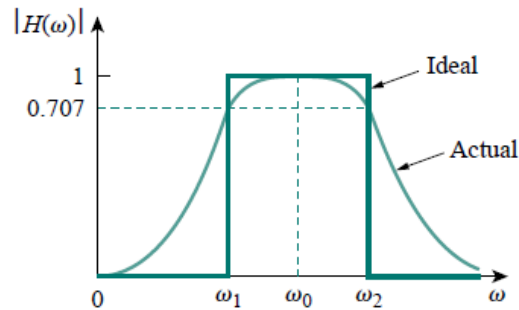


**Figure** A bandpass filter.

$$\mathbf{H}(\omega) = \frac{V_o}{V_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

We observe that  $H(0) = 0$ ,  $H(\infty) = 0$ . Figure 14.36 shows the plot of  $|H(\omega)|$ . The band pass filter passes a band of frequencies ( $\omega_1 < \omega < \omega_2$ ) centered on  $\omega_0$ , the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



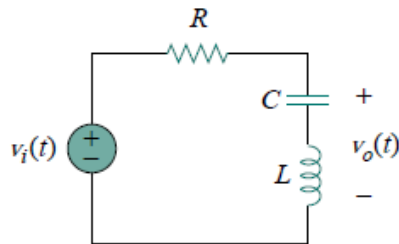
**Figure** Ideal and actual frequency response of a bandpass filter.

A band pass filter is designed to pass all frequencies within a band of frequencies,  $\omega_1 < \omega < \omega_2$ .

Since the band pass filter in Fig is a series resonant circuit, the half power frequencies, the bandwidth, and the quality factor are determined as in Section. A band pass filter can also be formed by cascading the low pass filter (where  $\omega_2 = \omega_c$ ) with the high pass filter (where  $\omega_1 = \omega_c$ ).

**Band stop Filter:**

Filter that prevents a band of frequencies between two designated values ( $\omega_1$  and  $\omega_2$ ) from passing is variably known as a band stop, band reject, or notch filter. A band stop filter is formed when the output RLC series resonant circuit is taken off the LC series combination as shown in Fig. The transfer function is



**Figure** A bandstop filter.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

Notice that  $H(0) = 1$ ,  $H(\infty) = 1$ . Figure shows the plot of  $|H(\omega)|$ . Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

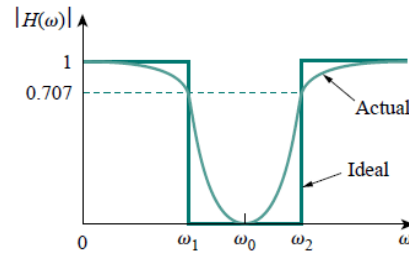


Figure Ideal and actual frequency response of a bandstop filter.

While the half-power frequencies, the bandwidth, and the quality factor is calculated using the formulas in Section for a series resonant circuit. Here,  $\omega_0$  is called the frequency of rejection, while the corresponding bandwidth ( $B = \omega_2 - \omega_1$ ) is known as the bandwidth of rejection. Thus,

A band stop filter is designed to stop or eliminate all frequencies within a band of frequencies,  $\omega_1 < \omega < \omega_2$ .

### 9. What is constant K filter?

#### Solution:

In constant k filters,  $z_1$  and  $z_2$  are opposite type of reactances.

$$z_1 z_2 = k^2$$

Where k is a constant independent of frequency.

There are two types of constant k type filters:

- (i) constant k low pass filter
- (ii) constant k high pass filter

Figure shows constant k low pass filter.

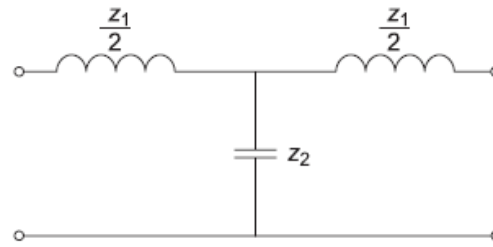


Fig.

Let

$$z_1 = j\omega L$$

$$z_2 = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$z_1 z_2 = \frac{L}{C} = k^2$$

$$k = \sqrt{\frac{L}{C}}$$

Determination of pass band and stop band:

(i) when

$$\frac{z_1}{4z_2} = 0$$

$$\frac{j\omega L}{-j/\omega C} = 0$$

$$\frac{-\omega^2 LC}{4} = 0$$

$$\omega = 0$$

$$f = 0$$

(ii) When

$$\frac{z_1}{4z_2} = -1$$

$$\frac{j\omega L}{-4j\omega C} = -1$$

$$\frac{\omega^2 LC}{4} = 1$$

$$\frac{4\pi^2 f^2 LC}{4} = 1$$

$$f = f_c = \frac{1}{\pi \sqrt{LC}}$$

The passband starts at  $f = 0$  and continues up to  $f_c$ , the cutoff frequency. All the frequencies above.

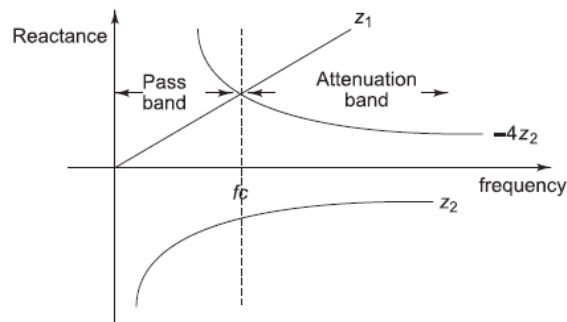


Fig.

Cut off frequency  $f_c$  are in the attenuation or stop band. Thus, the network is called a low-pass filter.

$$\begin{aligned} \sinh \frac{\gamma}{2} &= \sqrt{\frac{z_1}{4z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{j\omega \sqrt{LC}}{2} \\ &= \frac{j2\pi f}{2\pi f_c} = j \frac{f}{f_c} \end{aligned}$$

We also know that in the pass band

$$-1 < \frac{z_1}{4z_2} < 0$$

$$-1 < \frac{-\omega^2 LC}{4} < 0$$

$$-1 < -\left(\frac{f}{f_c}\right)^2 < 0$$

$$\frac{f}{f_c} < 1$$

$$\beta = 2 \sin^{-1} \left( \frac{f}{f_c} \right)$$

$$\alpha = 0$$

In the attenuation band,

$$\frac{z_1}{4z_2} < -1$$

$$\frac{f}{f_c} > 1$$

$$\alpha = 2 \cosh^{-1} \left( \frac{z_1}{4z_2} \right) = 2 \cosh^{-1} \left( \frac{f}{f_c} \right)$$

$$\beta = \pi$$

The variation of  $\alpha$  and  $\beta$  is plotted in the Fig.

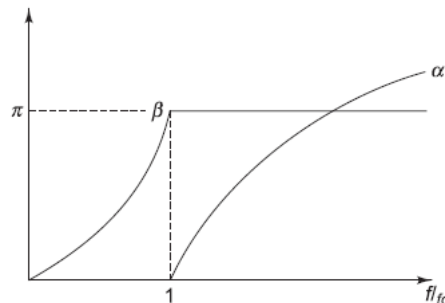


Fig.

The attenuation is zero throughout the pass band but increases gradually from the cutoff frequency. The phase shift  $\beta$  is zero at zero frequency and increases gradually through the pass band, reaches  $\pi$  at cutoff frequency  $f_c$ . It remains at  $\pi$  for all frequencies beyond  $f_c$ .

Determination of characteristic impedance:

The characteristic impedance of low pass filter will be given by

$$\begin{aligned}
 z_{0T} &= \sqrt{z_1 z_2 \left( 1 + \frac{z_1}{4z_2} \right)} \\
 &= \sqrt{\frac{L}{C} \left( 1 - \frac{\omega^2 LC}{4} \right)} \\
 &= k \sqrt{1 - \left( \frac{f}{f_c} \right)^2} \\
 z_{0\pi} &= \frac{z_1 z_2}{z_{0T}} = \frac{k}{\sqrt{1 - \left( \frac{f}{f_c} \right)^2}}
 \end{aligned}$$

The plots of characteristic impedance are shown in Fig.

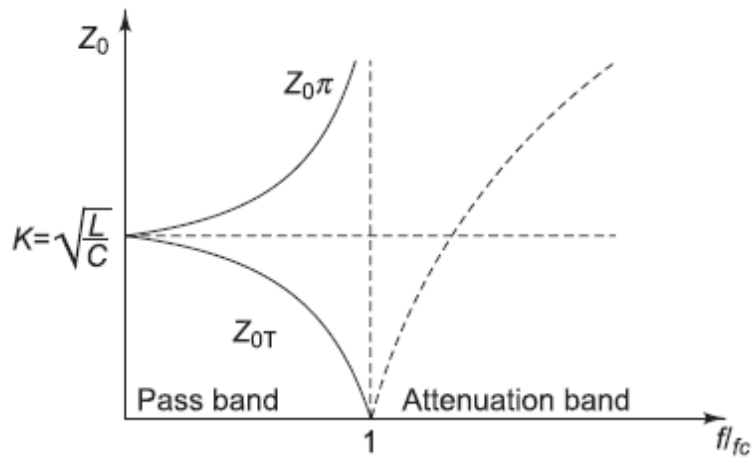


Fig.

$z_{0T}$  is real when  $f < f_c$  i.e. in the pass band. If  $f = f_c$ ,  $z_{0T} = 0$  and for  $f > f_c$ ,  $z_{0T}$  is imaginary in the attenuation band, rising to infinite reactance at infinite frequency.

$z_{0\pi}$  is real when  $f < f_c$ . If  $f = f_c$ ,  $z_{0\pi}$  is finite and for  $f > f_c$ ,  $z_{0\pi}$  is imaginary.

Constant  $k$  high pass filter is obtained by changing the positions of series and shunt arms of the constant  $k$  low pass filter. Figure shows a constant  $k$  high pass filter.

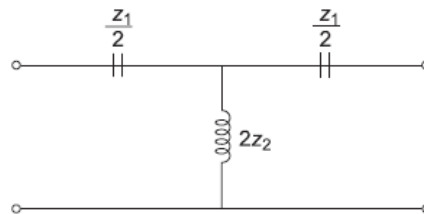


Fig.

Let

$$z_1 = \frac{-j}{\omega C}$$

$$z_2 = j\omega L$$

$$z_1 z_2 = \frac{-j}{\omega C} j\omega L = \frac{L}{C} = k^2$$

$$k = \sqrt{\frac{L}{C}}$$

Determination of pass band and stop band:

(i) when

$$z_1 = 0$$

$$\frac{-j}{\omega C} = 0$$

$$\omega = \infty$$

(ii) when

$$\frac{z_1}{4z_2} = -1$$

$$z_1 = -4z_2$$

$$\frac{-j}{\omega C} = -4j\omega L$$

$$\omega^2 LC = \frac{1}{4}$$

$$\omega^2 = \frac{1}{4LC}$$

$$f = \frac{1}{4\sqrt{LC}}$$

$$f = f_c = \frac{1}{4\sqrt{LC}}$$

The reactance  $z_1$  and  $z_2$  are shown in Fig.



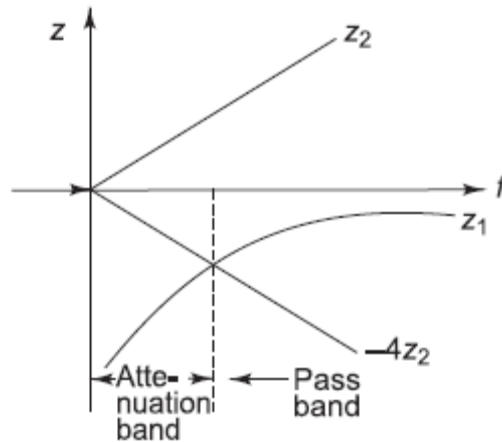


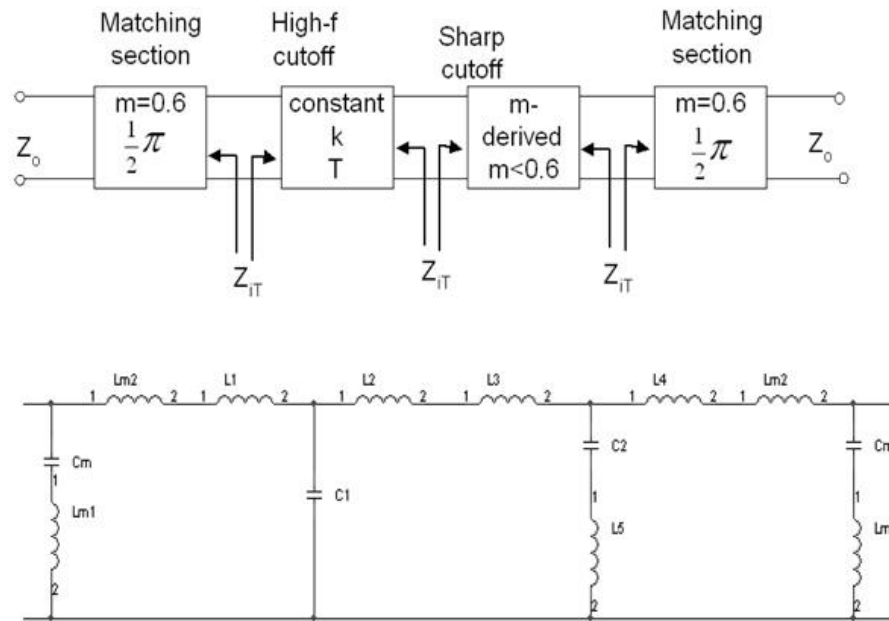
Fig.

The filter passes all the frequency beyond  $f_c$ . All frequencies below the cutoff frequency lie in attenuation or stop band. Hence the network is called a high pass filter.

$$\begin{aligned} \sinh \frac{l}{2} &= \sqrt{\frac{z_1}{4z_2}} = \sqrt{\frac{1}{4\omega^2 LC}} \\ &= \sqrt{\frac{(4\pi)^2 f_c^2}{4\omega^2}} \\ &= j \frac{f_c}{f} \end{aligned}$$

10. Gives the brief introduction of composite low pass filter.

- A planar composite low pass filter implemented in micro strip line, designed by image parameter method will be described.
- This composite filter combines four filter sections and presents an attenuation pole near the cut off frequency to ensure sharp cut off. This filter design also ensuring good matching properties in the pass band.
- The lumped-element schematic of the filter has been implemented, and the lumped elements are converted into micro strip line to become a planar composite filter.
- The simulations are done by Advanced Design System (ADS). The micro strip line simulation results show 1.5GHz as the cut off frequency of the filter, which fulfill the design specification.
- Measured results exhibit some losses compared to the simulation results, with +/- 0.1GHz tolerance from the cut off frequency and the insertion losses for the designs are around -1dB to -2 dB, which may caused by practical limitations.
- Measurement results exhibit rejection of the attenuated pole around 20 to 30dB.



- However, a high-order design is also required to simultaneously ensure a flat response in the pass band and a good out-of-band attenuation.
- In all cases, a compact planar design is practically hard to achieve due to the number and size of components to be implemented using the semi lumped component approach

### Objective type Questions

1. Consider a function  $Z(s)=5(s+1)(s+4)/(s+3)(s+5)$  . Find the value of  $R_1$  after performing the first form of Foster method.

- a)  $1/3$
- b)  $2/3$
- c)  $3/3$
- d)  $4/3$

2. The value of  $R_1$  in the question 1 is?

- a)  $4/3$
- b)  $5/3$
- c)  $3/5$
- d)  $3/4$

3. The value of  $L_1$  in the question 1 is?

- a)  $5/9$
- b)  $9/5$
- c)  $4/9$
- d)  $9/4$

4. The value of  $R_2$  in the question 1 is?

- a) 1
- b) 2
- c) 3
- d) 4

5. The value of  $L_2$  in the question 1 is?

- a)  $4/5$
- b)  $3/5$
- c)  $2/5$
- d)  $1/5$

6. Consider the admittance function,  $Y(s)=((2s^2+16s+30))/(s^2+6s+8)$ . Determine the value of  $L_1$  after performing the second form of Foster method.

- a)  $1/3$
- b)  $2/3$
- c)  $3/3$

d)  $4/3$

7. The value of  $R_1$  in the question 6 is?

a)  $4/3$

b)  $3/3$

c)  $2/3$

d)  $1/3$

8. The value of  $R_2$  in the question 6 is?

a) 1

b) 2

c) 3

d) 4

9. The value of  $L_2$  in the question 6 is?

a) 4

b) 1

c) 2

d) 3

10. The value of  $R_\infty$  in the question 6 is?

a) 3

b) 1

c) 2

d) 4

Answer Key: 1.d 2.b 3.a 4.b 5.c 6.a 7.c 8.d 9.b 10.c

**Fill In the Blanks with Answers**

1. The value of one decibel is equal to.....
2. A filter which passes without attenuation all frequencies up to the cut-off frequency  $f_c$  and attenuates all other frequencies greater than  $f_c$  is called.....
3. A filter which attenuates all frequencies below a designated cut-off frequency  $f_c$  and passes all other frequencies greater than  $f_c$  is called.....
4. A filter that passes frequencies between two designated cut-off frequencies and attenuates all other frequencies is called?
5. A filter that passes all frequencies lying outside a certain range, while it attenuates all frequencies between the two designated frequencies is called-----
6. The expression of the open circuit impedance  $Z_{oc}$  is-----
7. The expression of short circuit impedance  $Z_{sc}$  is-----
8. The relation between  $Z_{OT}$ ,  $Z_{oc}$ ,  $Z_{sc}$  is-----
9. The value of  $\sinh^{-1} \gamma/2$  in terms of  $Z_1$  and  $Z_2$  is-----

Answer Key: 1. 0.115 N 2. : low pass filter 3. high pass filter 4. band pass filter 5. band elimination filter 6.  $Z_{oc} = Z_1/2 + Z_2$  7.  $Z_{sc} = (Z_1^2 + 4Z_1Z_2)/(2Z_1 + 4Z_2)$  8.  $Z_{OT} = \sqrt{Z_{oc}Z_{sc}}$

9.  $\sinh^{-1} \gamma/2 = \sqrt{(Z_1/4Z_2)}$

## UNIT-IV

### Two marks question with answers

1. **Define the Transmission lines**

**Ans:** Another means of transmitting power or information is by guided structures. Guided structures serve to guide (or direct) the propagation of energy from the source to the load. Typical examples of such structures are transmission lines

2. **Define the Characteristic Impedance**

**Ans:** If RF voltage  $V$  is applied across the conductors of an infinite line, it causes a current  $I$  to flow. By this observation, the line is equivalent to impedance, which is known as the characteristic impedance,  $Z_0$ :

3. **Define the Lossless Line?**

**Ans:** A transmission line is said to be **lossless** if the conductors of the line are perfect ( $\sigma=\infty$ ) and the dielectric medium separating them is lossless ( $\sigma=0$ ).

4. **Define the distortion less Line**

**Ans:** A **distortion less line** is one in which the attenuation constant  $\alpha$  is frequency independent while the phase constant  $\beta$  is linearly dependent on frequency.

5. **Define the voltage reflection coefficient?**

**Ans:** the ratio of the voltage reflection wave to the incident wave at the load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

### Three marks question with answers

1. **Define the current reflection coefficient?**

**Ans:** The **current reflection coefficient** at any point on the line is negative of the voltage reflection coefficient at that point.

2. **Define the voltage standing wave ratio (VSWR)?**

**Ans:** the ratio of the maximum voltage to the minimum voltage

$$S = \frac{V_{\max}}{V_{\min}} = \frac{I_{\max}}{I_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

3. **Define the standing wave ratio (SWR)?**

**Ans:** the ratio of transmitted wave to the reflected wave

4. Write the formula of input impedance of lossy transmission line

Ans: 
$$Z_{in} = Z_o \left[ \frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} \right]$$

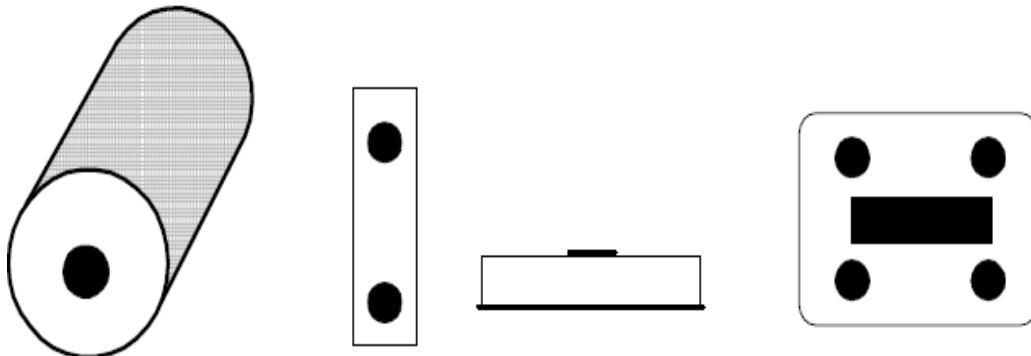
5. Write the formula of input impedance of lossless transmission line

Ans: 
$$Z_{in} = Z_o \left[ \frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \right]$$

**Five marks question with answers**

1. Explain the Transmission line parameters?

**Ans:** Transmission lines provide one media of transmitting electrical energy between the power sources to the load. Figure shows three different geometry types of lines used at microwave frequencies.



The open two-wire line is the most popular at lower frequencies, especially for TV application. Modern RF and microwave devices practice involves considerable usage of coaxial cables at frequencies from about 10 MHz up to 30 GHz and hollow waveguides from 1 to 300 GHz.

A uniform transmission line can be defined as a line with distributed elements, as shown in Figure.

$R'$  = Series resistance per unit length of line (/m). Resistance is related to the dimensions and conductivity of the metallic conductors, resistance is depended on frequency due to skin effect.

$G'$  = Shunt conductance per unit length of line ( $\text{S/m}$ ).

$G'$  is related to the loss tangent of the dielectric material between the two conductors. It is important to remember that  $G'$  is not a reciprocal of  $R'$ . They are independent quantities,  $R'$  being related to the various properties of the two conductors while  $G'$  is related to the properties of the insulating material between them.

$L'$  = Series inductance per unit length of line ( $\text{H/m}$ ).

$L'$  - Is associated with the magnetic flux between the conductors.

$C'$  = Shunt capacitance per unit length of line ( $\text{F/m}$ ).

$C'$  - Is associated with the charge on the conductors.

Naturally, a relatively long piece of line would contain identical sections as shown. Since, these sections can always be chosen to be small as compared to the operating wavelength. Hence the idea is valid at all frequencies. The series impedance and the shunt admittance per unit length of the transmission line are given by:

$$Z = R' + j\omega L'$$

$$Y = G' + j\omega C'$$

The expressions for voltage and current per unit length are given respectively by equations (1) and (2):

$$dV(z)/dz = -I(z)(R' + j\omega L') \quad (1)$$

$$dI(z)/dz = -V(z)(G' + j\omega C') \quad (2)$$

Where, the negative sign indicates on a decrease in voltage and current as  $z$  increases. The current and voltage are measured from the receiving end; at  $z = 0$  and line extends in negative  $z$ -direction. The differentiating equations,(3) and (4), associate the voltage and current:

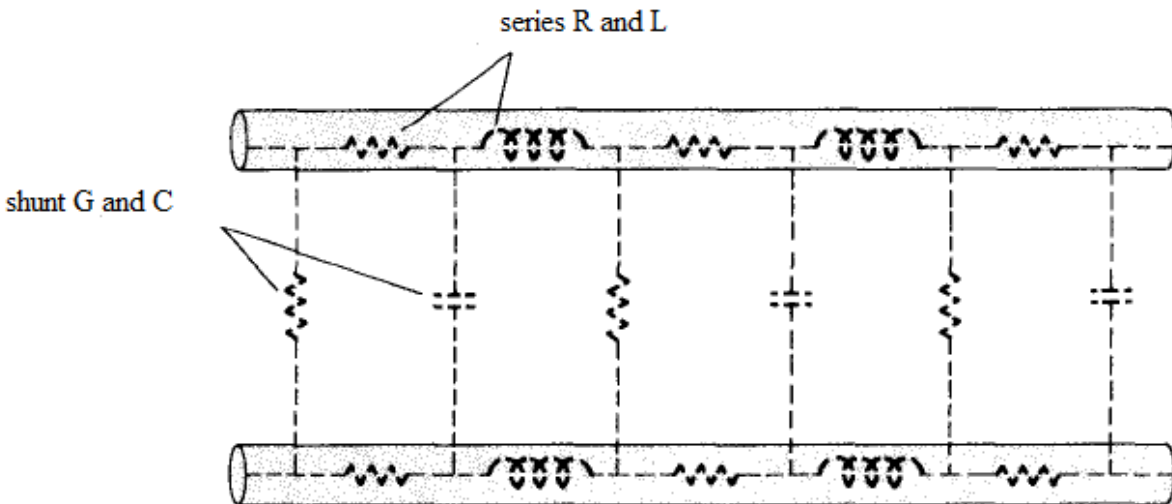
## 2. Derive the Transmission line equations

**Ans:** As mentioned in the previous section, a two-conductor transmission line supports a TEM wave; that is, the electric and magnetic fields on the line are transverse to the direction of wave propagation. An important property of TEM waves is that the fields  $E$  and  $H$  are uniquely related to voltage  $V$  and current  $I$  respectively:

$$V = - \int \mathbf{E} \cdot d\mathbf{l}, \quad I = \oint \mathbf{H} \cdot d\mathbf{l}$$

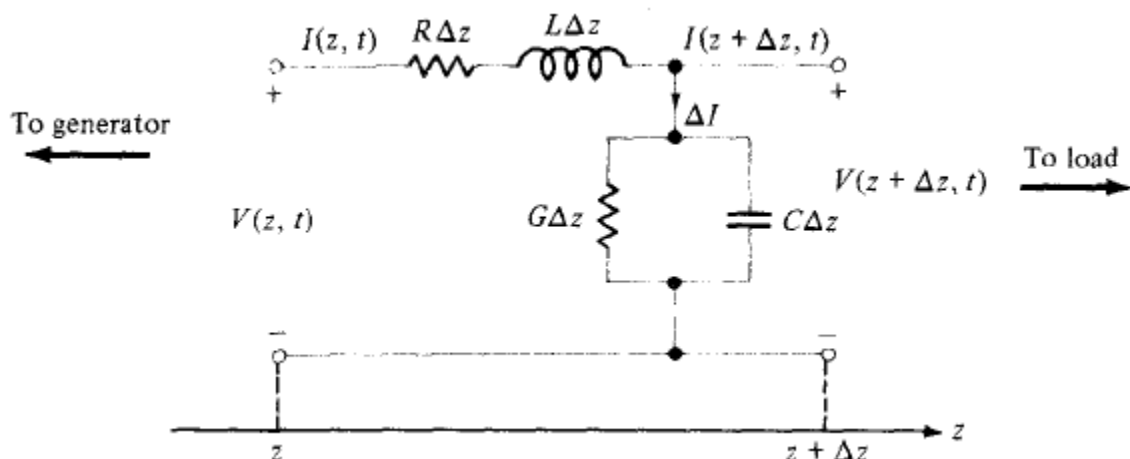


In view of this, we will use circuit quantities  $V$  and  $I$  in solving the transmission line problem instead of solving field quantities  $E$  and  $H$  (i.e., solving Maxwell's equations and boundary conditions). The circuit model is simpler and more convenient. Let us examine an incremental portion of length  $\Delta z$  of a two-conductor transmission line. We intend to find an equivalent circuit for this line and derive the line equations.



**Figure a:** Distributed parameters of a two-conductor transmission line.

From Figure a, we expect the equivalent circuit of a portion of the line to be as in Figure b. The model in Figure b is in terms of the line parameters  $R$ ,  $L$ ,  $G$ , and  $C$ , and may represent any of the two-conductor lines of Figure a. The model is called the L-type equivalent circuit; there are other possible types. In the model of Figure b, we assume that the wave propagates along the  $+z$ -direction, from the generator to the load.



**Figure b:** L-type equivalent circuit model of a differential length  $\Delta z$  of a two-conductor transmission line.

By applying Kirchhoff's voltage law to the outer loop of the circuit in Figure b, we obtain

Or

$$V(z, t) = R \Delta z I(z, t) + L \Delta z \frac{\partial I(z, t)}{\partial t} + V(z + \Delta z, t)$$

$$\frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = R I(z, t) + L \frac{\partial I(z, t)}{\partial t}$$

Taking the limit of above equations  $\Delta z \rightarrow 0$  leads to

$$-\frac{\partial V(z, t)}{\partial z} = R I(z, t) + L \frac{\partial I(z, t)}{\partial t}$$

Similarly, applying Kirchhoff's current law to the main node of the circuit in Figure b gives

$$I(z, t) = I(z + \Delta z, t) + \Delta I$$

$$= I(z + \Delta z, t) + G \Delta z V(z + \Delta z, t) + C \Delta z \frac{\partial V(z + \Delta z, t)}{\partial t}$$

Or

$$\frac{I(z + \Delta z, t) - I(z, t)}{\Delta z} = G V(z + \Delta z, t) + C \frac{\partial V(z + \Delta z, t)}{\partial t}$$

As  $\Delta z \rightarrow 0$ , above equation becomes

$$-\frac{\partial I(z, t)}{\partial z} = G V(z, t) + C \frac{\partial V(z, t)}{\partial t}$$

If we assume harmonic time dependence so that

$$V(z, t) = \text{Re} [V_s(z) e^{j\omega t}]$$

$$I(z, t) = \text{Re} [I_s(z) e^{j\omega t}]$$

Where,  $V_s(z)$  and  $I_s(z)$  are the phasor forms of  $V(z, t)$  and  $I(z, t)$ , respectively, from the above two equations and above equation becomes

$$-\frac{dV_s}{dz} = (R + j\omega L) I_s$$

$$-\frac{dI_s}{dz} = (G + j\omega C) V_s$$

In the differential form of above two equations,  $V_s$  and  $I_s$  are coupled. To separate them, we take the second derivative of  $V_s$  in eq. above two equations so that we obtain

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C) V_s$$

Or

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0$$

Where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

By taking the second derivative of  $I_s$  in above two equations, we get

$$\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0$$

### 3. Derive the velocity and characteristic impedance in lossless transmission line?

**Ans: A transmission line** is said to be **lossless** if the conductors of the line are perfect ( $\sigma=\alpha$ ) and the dielectric medium separating them is lossless ( $\sigma=0$ ).

For such a line, it is evident  $R=G=0$

Substitute this in following equations

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o$$

Hence  $\alpha = 0, \quad \gamma = j\beta = j\omega \sqrt{LC}$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

$$X_o = 0, \quad Z_o = R_o = \sqrt{\frac{L}{C}}$$

**4. Derive the velocity and characteristic impedance in distortion less transmission line?**

**Ans:** A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as  $\alpha$  is frequency dependent. This results in distortion.

**A distortion less line** is one in which the attenuation constant  $\alpha$  is frequency independent while the phase constant  $\beta$  is linearly dependent on frequency.

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

From the general expression for  $\alpha$  and  $\beta$

a distortion less line results if the line parameters are such that  $R/L=G/C$

Thus, for a distortion less line,

$$\begin{aligned}\gamma &= \sqrt{RG \left(1 + \frac{j\omega L}{R}\right) \left(1 + \frac{j\omega C}{G}\right)} \\ &= \sqrt{RG} \left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta\end{aligned}$$

Or

$$\alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC}$$

Showing that  $\alpha$  does not depend on frequency whereas  $\beta$  is a linear function of frequency.

Also

$$Z_0 = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_0 + jX_0$$

Or

$$R_0 \Rightarrow \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \quad X_0 = 0$$

And

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

**Note that**

1. The phase velocity is independent of frequency because the phase constant  $\beta$  depends on frequency. We have shape distortion of signals unless  $\alpha$  and  $u$  are independent of frequency.
2.  $u$  and  $Z_0$  remain the same as for lossless lines.
3. A lossless line is also a distortion less line, but a distortion less line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortion less.

**5. Derive the characteristic impedance  $Z_0$  of the transmission line?**

**Ans:** The solutions of the linear homogeneous differential equations and

$$\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0$$

Are similar to namely,

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$\longrightarrow +z \quad -z \longleftarrow$

And

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}$$

$\longrightarrow +z \quad -z \longleftarrow$

Where  $V^+$ ,  $V_o^-$ ,  $I_o^+$  and  $I_o^-$  are wave amplitudes; the + and — signs, respectively, denote wave travelling along +z and -z-directions, as is also indicated by the arrows. Thus, we obtain the instantaneous expression for voltage as

$$\begin{aligned} V(z, t) &= \text{Re} [V_s(z) e^{j\omega t}] \\ &= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z) \end{aligned}$$

The **characteristic impedance**  $Z_o$  of the line is the ratio of positively travelling voltage wave to current wave at any point on the line.

$Z_o$  is analogous to  $\eta$ , the intrinsic impedance of the medium of wave propagation. By substituting equations  $V_s(z)$  and  $I_s(z)$  into following equations and equating coefficients of terms  $e^{+z}$  and  $e^{-z}$  we obtain

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

Or

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o$$

Where,  $R_o$  and  $X_o$  are the real and imaginary parts of  $Z_o$ .  $R_o$  should not be mistaken for  $R$ —while  $R$  is in ohms per meter;  $R_o$  is in ohms. The propagation constant  $\Gamma$  and the characteristic impedance  $Z_o$  are important properties of the line because they both depend on the line parameters  $R$ ,  $L$ ,  $G$ , and  $C$  and the frequency of operation. The reciprocal of  $Z_o$  is the characteristic admittance  $Y_o$ , that is,  $Y_o = 1/Z_o$

### Multiple choice questions:

- For a distortion-less transmission line [     ]  
(A)  $R/L = G/C$  (B)  $RL=GC$  (C)  $RG=LC$  (D)  $RLGC=0$
- What would be the Standing Wave Ratio (SWR) for a line with reflection coefficient equal to 0.49? [     ]  
a. 0.01      b. 2.12      c. 2.921      d. 3.545
- Which primary constant of transmission line exhibits its dependency of value on the cross-sectional area of conductors [     ]  
a. Resistance (R)      b. Inductance (I)  
c. Conductance (G)      d. Capacitance (C)
- Which of the following parameters is not a primary parameter? [     ]  
a) Resistance    b) Attenuation constant    c) Capacitance    d) Conductance

5. The networks in which the R, L, C parameters are individually concentrated or lumped at discrete points in the circuit are called [     ]  
a)Lumped                      b)Distributed                      c) Parallel                      d) Paired
6. The lines having R, L, C distributed along the circuit are called [     ]  
a)Lumped      b)Distributed      c)Parallel      d) Paired
7. Which primary parameter is uniformly distributed along the length of the conductor? [     ]  
a) G    b) C    c) L    d) R
8. The primary parameter i.e. associated with the magnetic flux linkage [     ]  
a) R    b) L    c) C    d) G
9. The leakage current in the transmission lines is referred to as the [     ]  
a) Resistance    b) Radiation    c) Conductance                      d) Polarisation
10. Find the characteristic impedance expression in terms of the inductance and capacitance parameters [     ]  
a)  $Z_0 = \sqrt{LC}$     b)  $Z_0 = LC$     c)  $Z_0 = \sqrt{L/C}$                       d)  $Z_0 = L/C$

**Answer key**

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
a	C	a	b	a	b	d	B	c	c

**Fill in the blanks:**

1. When a transmission line has a load impedance same as that of the characteristic impedance, the line is said to be\_\_\_\_\_
2. After what wavelength does the nature of graph get reversed for the input impedance of open-circuited line\_\_\_\_\_
3. What is the phase variation range for reflection coefficient in the transmission lines\_\_

4. Identify the secondary parameter from the options given below\_\_\_\_\_
5. The condition for a quarter wave transformer is-\_\_\_\_\_
6. The reflection coefficient of a perfectly matched transmission line is\_\_\_\_\_
7. The purpose of the transmission line equation is to\_\_\_\_\_
8. The quarter wave transformer can be considered as a\_\_\_\_\_
9. Which transmission line is called as one to one transformer\_\_\_\_\_
10. The characteristic impedance of a transmission line is normally chosen to be\_\_\_\_\_

**Answer key**

1	2	3	4	5	6	7	8	9	10
Matched	$\lambda/4$	$0^\circ-180^\circ$	Phase constant	$Z_0^2 = Z_{in} Z_L$	0	Impedance matching	Impedance inverter	$L = \lambda/2$	50-75 $\Omega$



## UNIT-V

### Two marks question with answers

**1. Define the Brewster angle?**

**Ans:** When there is no reflection ( $E_r = 0$ ) and the incident angle at which this takes place is called the Brewster angle

**2. What is other name of Brewster angle?**

**Ans:** The Brewster angle is also known as the polarizing angle because an arbitrarily polarized incident wave will be reflected with only the component of E perpendicular to the plane of incidence.

**3. Transmission lines are commonly used as?**

**Ans:** Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies).

**4. Various types of Transmission lines?**

**Ans:** Various kinds of transmission lines such as the twisted-pair and coaxial cables (thin net and thick net) are used in computer networks such as the Ethernet and internet.

**5. Formulate the input impedance of lossy and lossless transmission line?**

$$Z_{in} = Z_o \left[ \frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} \right]$$

$$Z_{in} = Z_o \left[ \frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \right]$$

### Three marks question with answers

**1 Define the Shorted transmission line?**

**Ans:** when the transmission line is connected to load  $Z_L = 0$  (zero impedance) is called shorted circuited transmission line

**2 Define the open circuited transmission line?**

**Ans:** when the transmission line is connected to load  $Z_L = \alpha$  (infinity impedance) is called open circuited transmission line

**3. Define the matched transmission line?**

**Ans:** when the transmission line is connected to load  $Z_L = Z_0$  (characteristics impedance) is called matched transmission line

**4. Formulate the Shorted circuited transmission line?**

**Ans:**

$$Z_{sc} = Z_{in} \Big|_{Z_L=0} = jZ_0 \tan \beta \ell$$

**5. Formulate the open circuited transmission line?**

**Ans:**

$$Z_{oc} = \lim_{Z_L \rightarrow \infty} Z_{in} = \frac{Z_0}{j \tan \beta \ell} = -jZ_0 \cot \beta \ell$$

**5marks answer:**

**1. Derive the input impedance of transmission line**

**Ans:** Consider a transmission line of length  $l$ , characterized by  $\Gamma$  and  $Z_0$ ; connected to a load  $Z_L$  as shown in Figure a looking into the line, the generator sees the line with the load as input impedance  $Z_{in}$ . It is our intention in this section to determine the input impedance, the standing wave ratio (SWR), and the power flow on the line.

Let the transmission line extend from  $z = 0$  at the generator to  $z = l$  at the load. First of all, we need the voltage and current waves in equations.

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

Where, equation

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

Has been incorporated to find  $V_0^+$  and  $V_0^-$ , the terminal conditions must be given For example, if we are given the conditions at the input, say

$$V_0 = V(z = 0), I_0 = I(z = 0)$$

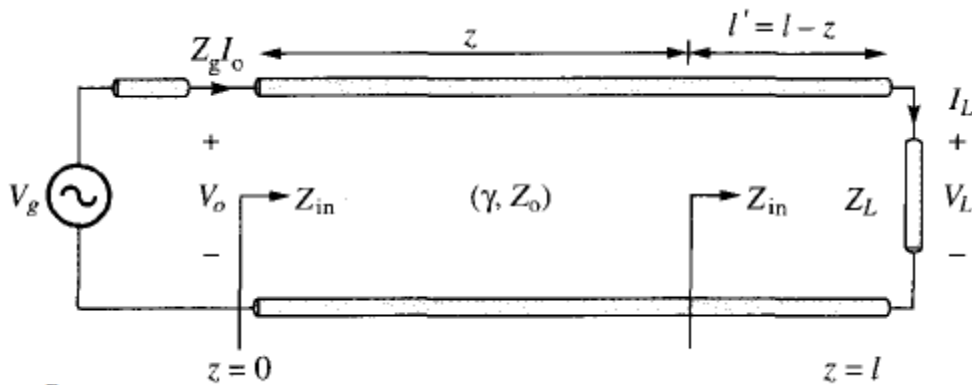


figure: a

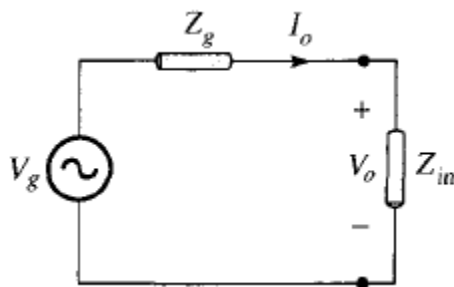


figure: b

**Figure a:** input impedance due to a line terminated by a load;

**(b)**Equivalent circuit for finding  $V_o$  and  $I_o$  in terms of  $Z_{in}$  at the input

Substituting these into equations  $V(z)$  and  $I_s(z)$  results in

$$V_o^+ = \frac{1}{2} (V_o + Z_o I_o)$$

$$V_o^- = \frac{1}{2} (V_o - Z_o I_o)$$

If the input impedance at the input terminals is  $Z_{in}$ , the input voltage  $V_o$  and the input current  $I_o$  are easily obtained from Figure (b) as On the other hand, if we are given the conditions at the load, say

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \quad I_o = \frac{V_g}{Z_{in} + Z_g}$$

On the other hand, if we are given the conditions at the load, say

$$V_L = V(z = l), \quad I_L = I(z = l)$$

Substituting these into equations  $V_s(z)$  and  $I_s(z)$  gives

$$V_o^+ = \frac{1}{2} (V_L + Z_o I_L) e^{\gamma \ell}$$

$$V_o^- = \frac{1}{2} (V_L - Z_o I_L) e^{-\gamma \ell}$$

Next, we determine the input impedance  $Z_{in} = V_s(z)/I_s(z)$  at any point on the line. At the generator, for example, equations  $V_s(z)$  and  $I_s(z)$  yield

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-}$$

Substituting in above two equations and utilizing the fact that

$$\frac{e^{\gamma \ell} + e^{-\gamma \ell}}{2} = \cosh \gamma \ell, \quad \frac{e^{\gamma \ell} - e^{-\gamma \ell}}{2} = \sinh \gamma \ell$$

Or

$$\tanh \gamma \ell = \frac{\sinh \gamma \ell}{\cosh \gamma \ell} = \frac{e^{\gamma \ell} - e^{-\gamma \ell}}{e^{\gamma \ell} + e^{-\gamma \ell}}$$

We, get

$$Z_{in} = Z_o \left[ \frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} \right]$$

Although above equation has been derived for the input impedance  $Z_{in}$  at the generation end, it is a general expression for finding  $Z_{in}$  at any point on the line. To find  $Z_{in}$  at a distance  $V$  from the load as in Figure a, we replace  $l$  by  $l'$ . A formula for calculating the hyperbolic tangent of a complex number, required in above equation. is found in.

For a lossless line,  $\Gamma = j\beta$ ,  $\tanh j\beta l = j \tan \beta l$ , and  $Z_o = R_o$ , so above equation becomes

$$Z_{in} = Z_o \left[ \frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \right]$$

Showing that the input impedance varies periodically with distance  $z$  from the load. The Quantity  $z/\lambda$  in eq. (11.34) is usually referred to as the electrical length of the line and can be expressed in degrees or radians.

We now define  $\Gamma$  as the voltage reflection coefficient (at the load).  $\Gamma$  is the ratio of the voltage reflection wave to the incident wave at the load, that is,

Substituting  $V^-$  and  $V^+$  in eq. (11.30) into eq. (11.35) and incorporating  $V_L = Z_L I_L$  gives

**2. Derive the reflection coefficient and voltage reflection coefficient of transmission line**

**Ans:** Consider a transmission line of length  $l$ , characterized by  $\Gamma$  and  $Z_0$ ; connected to a load  $Z_L$  as shown in Figure a looking into the line, the generator sees the line with the load as input impedance  $Z_{in}$ . It is our intention in this section to determine the input impedance, the standing wave ratio (SWR), and the power flow on the line.

Let the transmission line extend from  $z = 0$  at the generator to  $z = l$  at the load. First of all, we need the voltage and current waves in equations.

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I_s(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{\gamma z}$$

Where, equation 
$$Z_0 = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

Has been incorporated to find  $V_o^+$  and  $V_o^-$ , the terminal conditions must be given. For example, if we are given the conditions at the input, say

$$V_o = V(z = 0), I_o = I(z = 0)$$

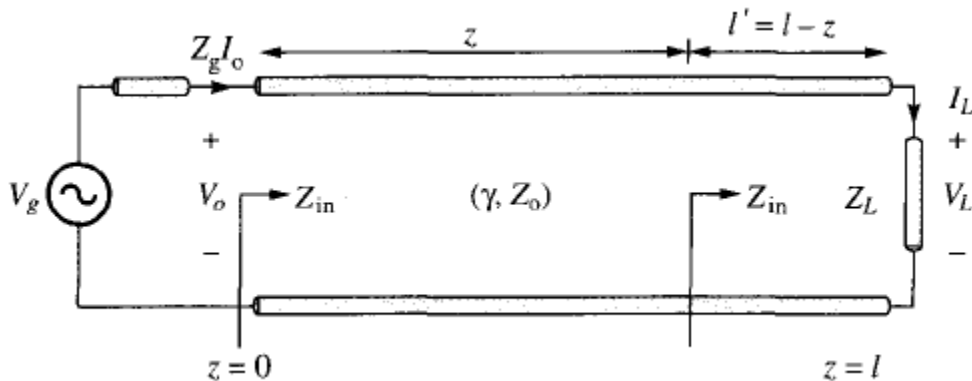
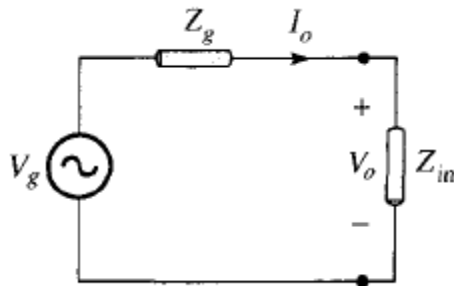


figure: a



**figure: b**

**Figure a:** input impedance due to a line terminated by a load;

**(b)** Equivalent circuit for finding  $V_o$  and  $I_o$  in terms of  $Z_{in}$  at the input.

Substituting these into equations  $V_s(z)$  and  $I_s(z)$  results in

$$V_o^+ = \frac{1}{2} (V_o + Z_o I_o)$$

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If the input impedance at the input terminals is  $Z_{in}$ , the input voltage  $V_o$  and the input current  $I_o$  are easily obtained from Figure (b) as On the other hand, if we are given the conditions at the load, say

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \quad I_o = \frac{V_g}{Z_{in} + Z_g}$$

On the other hand, if we are given the conditions at the load, say

$$V_L = V(z = l), \quad I_L = I(z = l)$$

Substituting these into equations  $V_s(z)$  and  $I_s(z)$  gives

$$V_o^+ = \frac{1}{2} (V_L + Z_o I_L) e^{\gamma \ell}$$

$$V_o^- = \frac{1}{2} (V_L - Z_o I_L) e^{-\gamma \ell}$$

Next, we determine the input impedance  $Z_{in} = V_s(z)/I_s(z)$  at any point on the line. At the generator, for example, equations  $V_s(z)$  and  $I_s(z)$  yield

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-}$$

Substituting in above two equations and utilizing the fact that

$$\frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2} = \cosh \gamma\ell, \quad \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} = \sinh \gamma\ell$$

Or

$$\tanh \gamma\ell = \frac{\sinh \gamma\ell}{\cosh \gamma\ell} = \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{e^{\gamma\ell} + e^{-\gamma\ell}}$$

We, get

$$Z_{in} = Z_o \left[ \frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell} \right]$$

Although above equation has been derived for the input impedance  $Z_{in}$  at the generation end, it is a general expression for finding  $Z_{in}$  at any point on the line. To find  $Z_{in}$  at a distance  $l$  from the load as in Figure a, we replace  $l$  by  $l'$ . A formula for calculating the hyperbolic tangent of a complex number, required in above equation is found in.

For a lossless line,  $\Gamma = j\beta$ ,  $\tanh j\beta l = j \tan \beta l$ , and  $Z_o = R_o$ , so above equation becomes

$$Z_{in} = Z_o \left[ \frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell} \right]$$

Showing that the input impedance varies periodically with distance  $l$ , from the load the quantity  $\beta l$  in above equation is usually referred to as the electrical length of the line and can be expressed in degrees or radians.

We now define  $\Gamma_L$  as the voltage reflection coefficient (at the load).  $\Gamma_L$  is the ratio of the voltage reflection wave to the incident wave at the load, that is,

$$\Gamma_L = \frac{V_o^- e^{\gamma\ell}}{V_o^+ e^{-\gamma\ell}}$$

Substituting  $V_o^+$  and  $V_o^-$  equation into above equation and incorporating  $V_L = Z_L I_L$  gives

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

**4. Explain the step to solve the single stub-matching?**

**Ans: Goal:** Design a single-stub matching network such that

$$Y_{IN} = Y_{STUB} + Y_A = Y_0$$

Convert the load to a normalized admittance:  $Y_L = g + jb$

Transform  $y_L$  along constant  $\Gamma$  towards generator until  $y_A = 1 + jb$

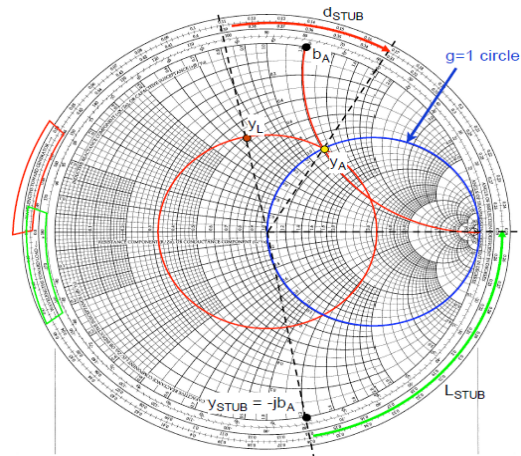
This matches the network's conductance to that of the transmission line and determines  $d_{STUB}$

3) Find  $y_{STUB} = -jb_A$  on Smith Chart

Transform  $y_{STUB}$  along constant  $\Gamma$  towards load until we reach PSC (for short-circuit stub) or POC (for open-circuit stub)

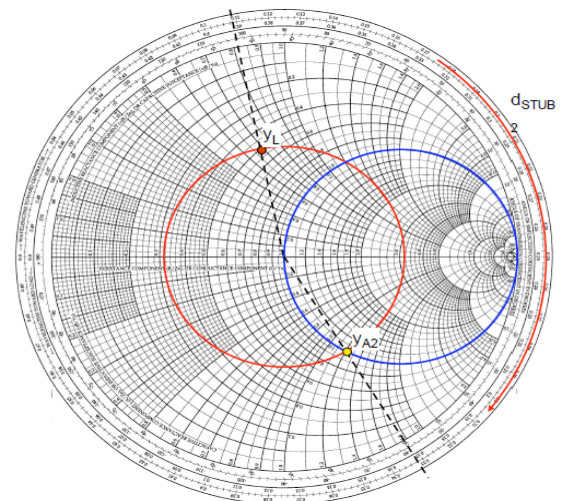
This cancels susceptance from (2) and determines  $L_{STUB}$

- Find  $y_L$
- Transform  $y_L$  to  $y_A = 1 + jb_A$
- Find  $y_{STUB} = -jb_A$
- Transform  $y_{STUB}$  to PSC (or POC)



There is a second solution where the  $\Gamma$  Circle and  $g=1$  circle intersect. This is also a solution to the problem, but requires a longer  $d_{STUB}$  and  $L_{STUB}$  so is less desirable, unless practical constraints require it.

- Find  $y_L$
- Rotate towards generator until intersection with  $g=1$  circle ( $d_{STUB}$ )
- Find  $b_{STUB}$
- Read off  $b_A$



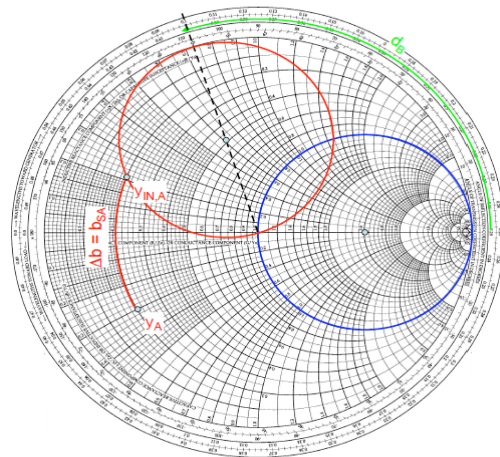
Rotate towards load until stub termination is reached ( $L_{STUB}$ )

**5. Explain the Steps to Solve a Double-Stub Matching Problem?**



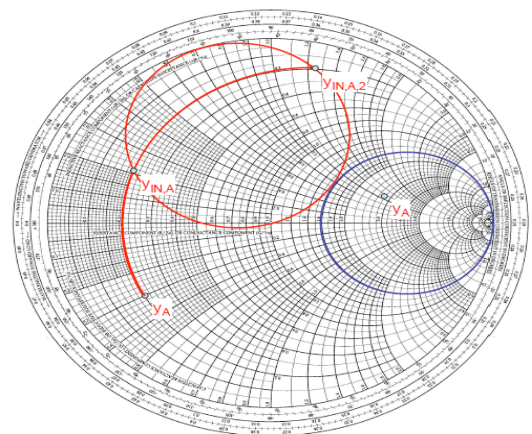
Goal: Design a double-stub matching network such that  $Y_{IN, A} = Y_0$

- Convert the load to a normalized admittance:  $y_L = g + jb$
- Transform  $y_L$  along constant  $\Gamma$  towards generator by distance  $d_A$  to reach  $y_A = g_A + jb_A$
- Draw auxiliary circle (pivot of  $g=1$  circle by distance  $d_B$ )
- Add susceptance ( $b$ ) to  $y_A$  to get to  $y_{IN, A}$  on auxiliary circle. The amount of susceptance added is equal to  $-b_{SA}$ , the input susceptance of stub A.
- Find  $y_{SA} = -jb_{SA}$  Determine  $L_A$  by transforming  $y_{SA}$  along constant  $\Gamma$  towards load until we reach PSC (for short-circuit stub) or POC (for open-circuit stub).
- Transform  $y_{IN, A}$  along constant  $\Gamma$  towards generator by distance  $d_B$  to reach  $y_B$  on auxiliary circle. The susceptance of  $y_B$  ( $b_B$ ) is equal to  $-b_{SB}$ , the input susceptance of stub B.
- Find  $y_{SB} = -jb_{SB}$  Determine  $L_B$  by transforming along constant  $\Gamma$  towards load until we reach PSC (for short-circuit stub) or POC (for open-circuit stub).



To solve a double-stub tuner problem:

- Find the  $g=1$  circle. All possible solutions for  $y_B$  must fall on this circle
- Rotate the  $g=1$  circle a distance  $d_B$  towards the load.
- These are the values at the input to the A junction that will transform to the  $g=1$  circle at junction B
- Find  $y_A$  on chart
- Rotate along the constant  $g$  circle to find the intersection with the rotated  $g=1$  circle. The change in  $b$  to do this is



the susceptance at the input to the stub at junction A

To find the admittance at junction B ( $y_B$ ) rotate  $IN, A$  towards the generator by  $dB$ . If we've drawn everything right, this will intersect the  $g=1$  circle.

Read off the value  $y_B$  for  $b_B$ . This is  $-b$  SB for the stub at junction B

Calculate the length of the B stubby rotating towards the load from  $b$  SB to the appropriate stub termination (PSC or POC)

Calculate the length of the A stub in the same way starting from BSA

Similar to the single stub network, there are multiple lengths for the stubs that will work

There is a range of  $y_A$  that cannot be matched. Irregard less of the short/open stub properties, we will never intersect the rotated  $g=1$  circle.

**Multiple choice questions:**

- 1. The constant  $x$ -circles of Smith chart becomes smaller due to increase in the value of 'x' from [     ]**  
a. 0 to  $\pi$                       b. 0 to  $2\pi$                       c. 0 to  $\pi/2$                       d. 0 to  $\infty$
- 2. According to Smith diagram, where should be the position of reflection coefficient value for a unity circle with unity radius? [     ]**  
A. On or inside the circle                      b. outside the circle

- c. Both a and b                                      d. None of the above
- For a matched line, the input impedance will be equal to [      ]  
a) Load impedance    b) Characteristic impedance    c) Output impedance    d) Zero
  - Identify the material which is not present in a transmission line setup [      ]  
a) waveguides                  b) cavity resonator    c) antenna                      d) oscillator
  - The Smith chart is a polar chart which plots [      ]  
a) R vs Z                                  b) R vs  $Z_{norm}$     c) T vs Z                                  d) T vs  $Z_{norm}$
  - The best stub selection for the transmission line will be [      ]  
a) Series open                  b) Series short                  c) Shunt open                  d) Shunt short
  - The open circuit impedance of the transmission line is given by [      ]  
a)  $Z_{OC} = j Z_o \tan \beta l$     b)  $Z_{OC} = -j Z_o \tan \beta l$     c)  $Z_{OC} = j Z_o \cot \beta l$     d)  $Z_{OC} = -j Z_o \cot \beta l$
  - The short circuit impedance of the transmission line is given by [      ]  
a)  $Z_{SC} = j Z_o \tan \beta l$     b)  $Z_{SC} = -j Z_o \tan \beta l$     c)  $Z_{SC} = j Z_o \cot \beta l$     d)  $Z_{SC} = -j Z_o \cot \beta l$
  - The open circuit line will have a reflection coefficient of [      ]  
a) 0                          b) 1                          c) -1                          d)  $\infty$
  - The short circuit line will have a reflection coefficient of [      ]  
a) 0                          b) 1                          c) -1                          d)  $\infty$

**Answer key**

1	2	3	4	5	6	7	8	9	10
d	a	b	d	b	d	d	A	c	b

**Fill in the blanks:**

- If the quarter line is short-circuited, then it acts as \_\_\_\_\_
- The characteristic impedance of a quarter wave transformer with load and input impedances given by 30 and 75 respectively is\_\_\_\_\_
- For a matched line, the input impedance will be equal to\_\_\_\_\_

4. Given that the reflection coefficient is 0.6. Find the SWR\_\_\_\_\_
5. Find the reflection coefficient of the wave with SWR of 3.5\_\_\_\_\_
6. The Smith chart is a polar chart which plots\_\_\_\_\_
7. The Smith chart is graphical technique used in the scenario of transmission lines. State true/false\_\_\_\_\_
8. The Smith chart consists of the constant\_\_\_\_\_
9. The circles in the Smith chart pass through which point\_\_\_\_\_
10. Moving towards the clockwise direction in the Smith chart implies moving\_\_\_\_\_

**Answer key**

1	2	3	4	5	6	7	8	9	10
Insulator	43.47	Zo	4	0.55	Rvs Znorm	true	R & X	1, 0	Towards generator

## 17. beyond syllabus Topics with material

### Foster's Reactance Theorem

For a positive real rational function  $Z(s)=1/Y(s)$  to be realizable as the driving point impedance of a lossless one-port, the necessary and sufficient condition is that it should be expressible in the form

$$Z(s) \text{ or } Y(s) = \frac{[a_n \cdot (s^2 + \omega_1^2) \cdot (s^2 + \omega_3^2) \dots]}{[b_m \cdot (s^2 + \omega_2^2) \cdot (s^2 + \omega_4^2) \dots]}$$

where  $a_n$  and  $b_m$  are constants and

1.  $0 \leq \omega_1 < \omega_2 < \omega_3$  (Interlacing poles and zeros, all on  $j\omega$  axis)
2. Foster's Theorem further restricts the degrees of the numerator,  $n$ , and denominator,  $m$ , by requiring that they must differ by unity. In other words, if the numerator is an even degree, the denominator is odd, and vice versa.

From these conditions, the following properties can be deduced:

1. Unity degree difference between numerator and denominator implies that  $Z(s)$  must have either a single pole or a single zero at both  $s=0$  and  $s=\infty$ . Therefore the function  $Z(s)$  or  $Y(s)$  will belong to one of the four types:
  - a. Pole at  $s=0$  and pole at  $s=\infty$
  - b. Pole at  $s=0$  and zero at  $s=\infty$
  - c. Zero at  $s=0$  and pole at  $s=\infty$
  - d. Zero at  $s=0$  and zero at  $s=\infty$
2.  $Z(j\omega)$  is purely reactive. Therefore it can be written as

$$Z(j\omega) = jX(\omega)$$

where  $X(\omega)$  is the input reactance with

$$X(\omega) \text{ or } \frac{1}{X(\omega)} = \frac{[a_n \cdot (\omega_1^2 - \omega^2) \cdot (\omega_3^2 - \omega^2) \dots]}{[b_m \cdot \omega \cdot (\omega_2^2 - \omega^2) \cdot (\omega_4^2 - \omega^2) \dots]}$$

Alternation of poles and zeros leads to the property

$$0 < \frac{|X(\omega)|}{\omega} < \frac{d}{d\omega} X(\omega)$$

In other words, the reactance  $X(\omega)$  is always an increasing function of frequency. The rational functions satisfying these requirements are called Foster functions.

3. Since all poles of  $Z(s)$  and  $Y(s)$  are on the  $s=j\omega$  axis, they can always be expanded as

$$Z(s) = \frac{k_0}{s} + k_\infty \cdot s + 2 \cdot \sum \frac{k_i}{s^2 + \omega_i^2}$$

and

$$Y(s) = \frac{h_0}{s} + h_\infty \cdot s + 2 \cdot \sum \frac{h_i}{s^2 + \omega_i^2}$$

where the constants "k" and "h" are residues of the respective poles. Physically, they correspond to simple network elements, as follows:

- o If  $Z(s)$  has a pole at  $s=0$ , it can be extracted as a series capacitor:



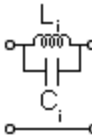
$$C_0 = \frac{1}{k_0} = \frac{1}{s \cdot Z(s)} \Big|_{s=0}$$

- If  $Z(s)$  has a pole at  $s=\infty$ , it can be extracted as a series inductor:



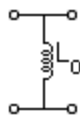
$$L_\infty = k_\infty = \frac{Z(s)}{s} \Big|_{s=\infty}$$

- If  $Z(s)$  has a pole at  $s=j\omega_i$ , it can be extracted as a parallel resonator in series:



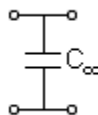
$$C_i = \frac{1}{2 \cdot k_i} = \frac{s}{(s^2 + \omega_i^2) \cdot Z(s)} \Big|_{s=j\omega_i} \quad L_i = \frac{1}{\omega_i^2 \cdot C_i}$$

- If  $Y(s)$  has a pole at  $s=0$ , it can be extracted as a shunt inductor:

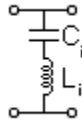


$$L_0 = \frac{1}{k_0} = \frac{1}{s \cdot Y(s)} \Big|_{s=0}$$

- If  $Y(s)$  has a pole at  $s=\infty$ , it can be extracted as a shunt capacitor:



$C_{\infty} = k_{\infty} = \frac{Y(s)}{s} \Big|_{s=\infty}$  If  $Y(s)$  has a pole at  $s=j\omega_i$ , it can be extracted as a series resonator to ground:



$$L_i = \frac{1}{2 \cdot k_i} = \frac{s}{(s^2 + \omega_i^2) \cdot Y(s)} \Big|_{s=j\omega_i} \quad C_i = \frac{1}{\omega_i^2 \cdot L_i}$$

Note that if a pole of  $Z(s)$  at  $s=0$  or  $s=\infty$  is extracted; a zero appears at that frequency automatically in the remaining impedance function, which acts as a pole of the remaining admittance function. Hence, given  $Z(s)$ , one can synthesize a variety of circuits all having the same input impedance but with different structures by extracting elements in different orders from impedance or admittance functions.



## **16. COURSE ATTAINMENT**

### 17. CO-PO MAPPING

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO 1	PSO 2	PSO 3	PSO4
CO1	3	2										1	3	2	2	1
CO2	3	2										1	3	2	1	
CO3	2	3										1	2	1		
CO4	3	2	2									1	3	2		
CO5	3	2	2									1	3	2	1	

Legend

1: Slight(Low) 2: Moderate(Medium) 3: Substantial(High)

## 18. BEYOND SYLLABUS TOPICS WITH MATERIAL

### A: Micro waves

Besides wave propagation, transmission lines, waveguides, and antennas, there are several other areas of applications of EM. These include microwaves, electromagnetic interference and compatibility, fibre optics, satellite communication, bio electromagnetic, electric machines, radar meteorology, and remote sensing. Due to space limitation, we shall cover the first three areas in this chapter: microwaves, electromagnetic interference and compatibility, and fibre optics. Since these topics are advanced, only an introductory treatment of each topic will be provided. Our discussion will involve applying the circuit concepts learned in earlier courses and the EM concepts learned in earlier chapters.

At the moment, there are three means for carrying thousands of channels over long distances:(a) microwave links, (b) coaxial cables, and (c) fibre optic, a relatively new technology, to be covered later.

**Microwaves are EM waves whose frequencies range from approximately 300 MZ TO 1000 MZ.**

For comparison, the signal from an AM radio station is about 1 MHz, while that from an FM station is about 100 MHz The higher frequency edge of microwaves borders on the optical spectrum. This accounts for why microwaves behave more like rays of light than ordinary radio waves. You may be familiar with microwave appliances such as the microwave oven, which operates at 2.4 GHz, the satellite television, which operates at about 4 GHz, and the police radar, which works at about 22 GHz.

Features that make microwaves attractive for communications include wide available band widths (capacities to carry information) and directive properties of short wavelengths. Since the amount of information that can be transmitted is limited by the available band-width, the microwave spectrum provides more communication channels than the radio and TV bands. With the ever increasing demand for channel allocation, microwave communications has become more common.

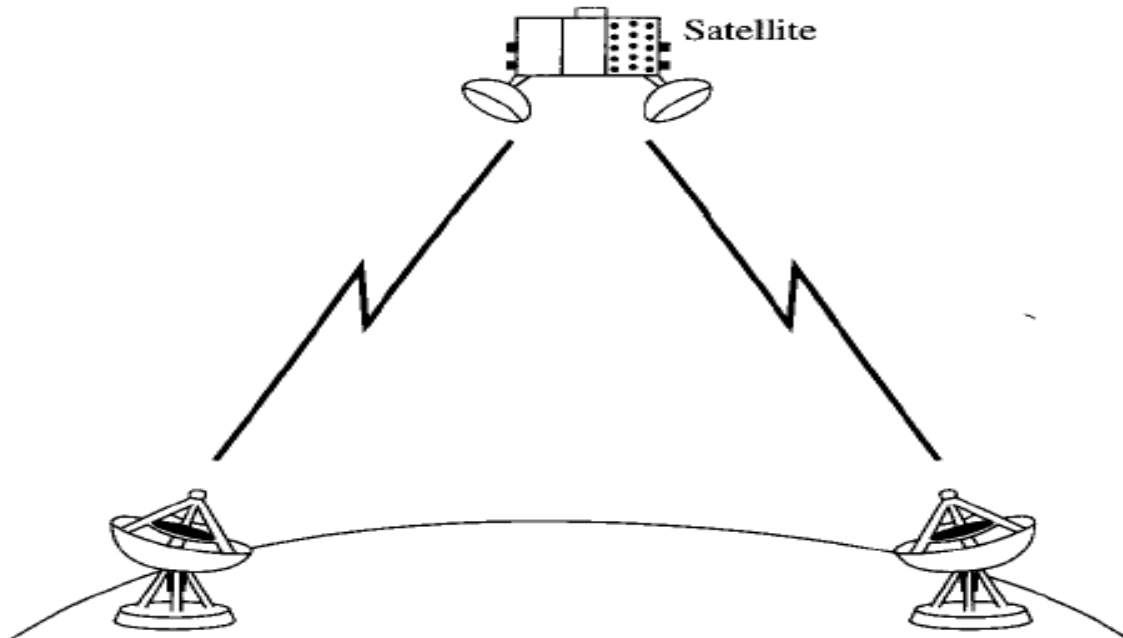
A microwave system normally consists of a transmitter (including a microwave oscillator, waveguides, and a transmitting antenna) and a receiver subsystem (including a receiving antenna, transmission line or waveguide, microwave amplifiers, and a receiver). A microwave network is usually an interconnection of various microwave components and devices. There are several microwave components and variations of these components. Common microwave components include:

- Coaxial cables, which are transmission lines for interconnecting microwave components
- Resonators, which are usually cavities in which EM waves are stored
- Waveguide sections, which may be straight, curved or twisted
- Antennas, which transmit or receive EM waves efficiently
- Terminators, which are designed to absorb the input power and therefore act as one port Attenuators, which are designed to absorb some of the EM power passing through it and thereby decrease the power level of the microwave signal
- Directional couplers, which consist of two waveguides and a mechanism for coupling signals between them
- Isolators which allow energy flow only in one direction
- Circulators, which are designed to establish various entry/exit points where power can either be fed or extracted
- Filters, which suppress unwanted signals and/or separate signals of different frequencies.

The use of microwaves has greatly expanded. Examples include telecommunications, radio astronomy, land surveying, radar, meteorology, UHF television, terrestrial microwave links, solid-state devices, heating, medicine, and identification systems. We will consider only four of these.

**1. Telecommunications:** (the transmission of analog or digital information from one point to another) is the largest application of microwave frequencies. Microwaves propagate long a straight line like a light ray and are not bent by the ionosphere as are lower frequency signals. This makes communication satellites possible. In essence, a communication satellite is a microwave relay station that is used to link two or more ground-based transmitters and receivers. The satellite receives signals at one frequency, repeats or amplifies it, and transmits it at another frequency. Two common modes of operation for satellite communication are portrayed in Figure.

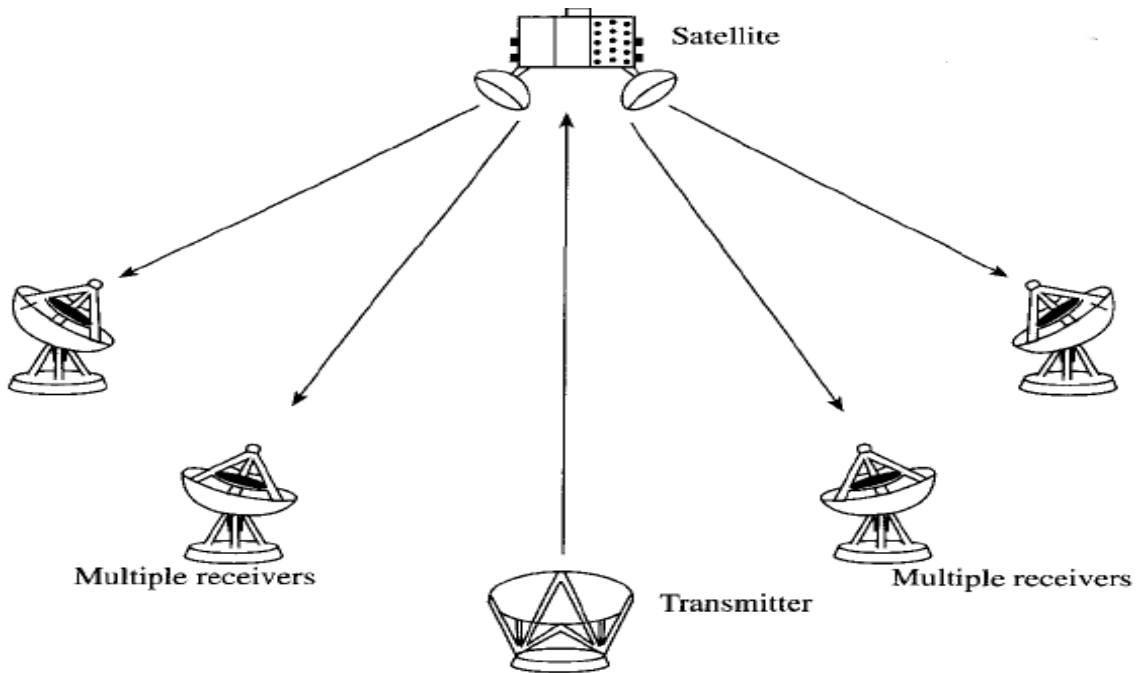
The satellite provides a point-to-point link in Figure (a), while it is being used to provide multiple links between one ground based transmitter and several ground-based receivers in Figure (b).



(a) Point-to-point link via satellite microwave

(b) Broadcast link via satellite microwave

**B:**



## **Electromagnetic interference and compatibility**

Every electronic device is a source of radiated electromagnetic fields called radiated emissions. These are often an accidental by-product of the design.

**Electromagnetic interference** (EMI) is the degradation in the performance of a device due to the fields making up the electromagnetic environment.

The electromagnetic environment consists of various apparatuses such as radio and TV broadcast stations, radar, and navigational aids that radiate EM energy as they operate. Every electronic device is susceptible to EMI. Its influence can be seen all around us. The results include "ghosts" in TV picture reception, taxicab radio interference with police radio systems, power line transient interference with personal computers, and self-oscillation of a radio receiver or transmitter circuit.

**Electromagnetic compatibility** (EMC) is achieved when a device functions satisfactorily without introducing intolerable disturbances to the electromagnetic environment or to other devices in its neighbourhood. EMC<sup>2</sup> is achieved when electronic devices coexist in harmony, such that each device functions according to its intended purpose in the presence of, and in spite of, the others. EMI is the problem that occurs when unwanted voltages or currents are present to influence the performance of a device, while EMC is the solution to the problem. The goal of EMC is to ensure system or subsystem compatibility and this is achieved by applying proven design techniques, the use of which ensures a system relatively free of EMI problems. EMC is a growing field because of the ever-increasing density of electronic circuits in modern systems for computation, communication, control, etc. It is not only a concern to electrical and computer engineers, but to automotive engineers as well. The increasing application of automotive electronic systems to improve fuel economy, reduce exhaust emissions, ensure vehicle safety, and provide assistance to the driver has resulted in a growing need to ensure compatibility during normal operation. We will consider the sources and characteristics of EMI. Later, we will examine EMI control techniques

## **19. Result Analysis-Remedial/Corrective Action**

## **20. Record of Tutorial Classes**



## **21. Record of Remedial Classes**

## **22. Record of guest lecturers conducted**