SPC 407
Sheet 4
Compressible Flow – Oblique Shock wave

1. It is claimed that an oblique shock can be analyzed like a normal shock provided that the normal component of velocity (normal to the shock surface) is used in the analysis. Do you agree with this claim?

   Solution

   Yes, the claim is correct. Conversely, normal shocks can be thought of as special oblique shocks in which the shock angle is $\beta = \pi/2$, or 90°. The component of flow in the direction normal to the oblique shock acts exactly like a normal shock. We can think of the flow parallel to the oblique shock as "going along for the ride" – it does not affect anything.
2. How do oblique shocks occur? How do oblique shocks differ from normal shocks?

   Solution

   Oblique shocks occur when a gas flowing at supersonic speeds strikes a flat or inclined surface. Normal shock waves are perpendicular to flow whereas inclined shock waves, as the name implies, are typically inclined relative to the flow direction. Also, normal shocks form a straight line whereas oblique shocks can be straight or curved, depending on the surface geometry.

   In addition, while a normal shock must go from supersonic (Ma > 1) to subsonic (Ma < 1), the Mach number downstream of an oblique shock can be either supersonic or subsonic.

3. For an oblique shock to occur, does the upstream flow have to be supersonic? Does the flow downstream of an oblique shock have to be subsonic?

   Solution

   Yes, the upstream flow has to be supersonic for an oblique shock to occur.

   No, the flow downstream of an oblique shock can be subsonic, sonic, and even supersonic.

   The latter is not true for normal shocks. For a normal shock, the flow must always go from supersonic (Ma > 1) to subsonic (Ma < 1).

4. Can the Mach number of a fluid be greater than 1 after a normal shock wave? Explain.

   Solution

   No, the second law of thermodynamics requires the flow after the shock to be subsonic.

5. Consider supersonic airflow approaching the nose of a two-dimensional wedge and experiencing an oblique shock. Under what conditions does an oblique shock detach from the nose of the wedge and form a bow wave? What is the numerical value of the shock angle of the detached shock at the nose?

   Solution

   A When the wedge half-angle δ is greater than the maximum deflection angle θ_{max}, the shock becomes curved and detaches from the nose of the wedge, forming what is called a detached oblique shock or a bow wave. The numerical value of the shock angle at the nose is $\beta = 90°$.

   When δ is less than θ_{max}, the oblique shock is attached to the nose.

6. Consider supersonic flow impinging on the rounded nose of an aircraft. Is the oblique shock that forms in front of the nose an attached or a detached shock? Explain.

   Solution

   When supersonic flow impinges on a blunt body like the rounded nose of an aircraft, the wedge half-angle δ at the nose is $90°$, and an attached oblique shock cannot exist, regardless of Mach number. Therefore, a detached oblique shock must occur in front of all such blunt-nosed bodies, whether two-dimensional, axisymmetric, or fully three dimensional.

   Since $\delta = 90°$ at the nose, δ is always greater than θ_{max}, regardless of Ma or the shape of the rest of the body.

7. Consider supersonic airflow approaching the nose of a two-dimensional wedge at a Mach number of 5. Using the Fig., determine the minimum shock angle and the maximum deflection angle a straight oblique shock can have.

Solution

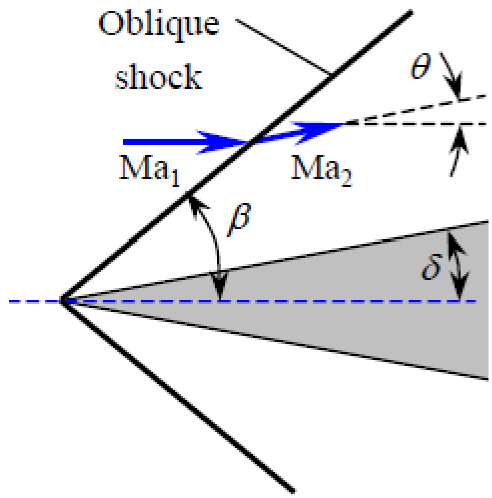
Assumptions: Air is an ideal gas with a constant specific heat ratio of k = 1.4.

For Ma = 5, we read from β curve,

Minimum shock (or wave) angle: β_{min} = 12°

Maximum deflection (or turning) angle: θ_{max} = 41.5°

Note that the minimum shock angle decreases and the maximum deflection angle increases with increasing Mach number Ma_1.



8. Air at 12 psia, 30 °F, and a Mach number of 2.0 is forced to turn upward by a ramp that makes an 8° angle off the flow direction. As a result, a weak oblique shock forms. Determine the wave angle, Mach number, pressure, and temperature after the shock.

Solution

Assumptions 1. The flow is steady.

2. The boundary layer on the wedge is very thin.

3. Air is an ideal gas with constant specific heats.

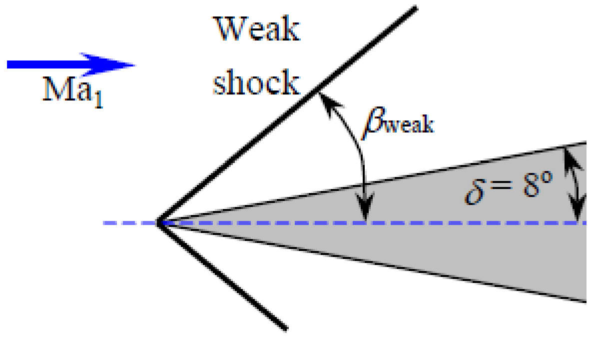
Properties: The specific heat ratio of air is k = 1.4.

On the basis of Assumption #2, we take the deflection angle as equal to the ramp, i.e., $\theta \approx \delta = 8°$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2} \quad \rightarrow \quad \tan 8° = \frac{2(2^2 \sin^2\beta - 1)/\tan\beta}{2^2(1.4 + \cos 2\beta) + 2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or from the $\beta - \theta$ curve. It gives β_{weak} = 37.21° and β_{strong} = 85.05°. Then for the case of weak oblique shock, the upstream “normal” Mach number $Ma_{1,n}$ becomes

$$\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 37.21° = 1.209$$



Also, the downstream normal Mach numbers $Ma_{2,n}$ become

$$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.209)^2 + 2}{2(1.4)(1.209)^2 - 1.4 + 1}} = 0.8363$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,n}^2 - k + 1}{k+1} = (12\text{ psia})\frac{2(1.4)(1.209)^2 - 1.4 + 1}{1.4+1} = \mathbf{18.5psia}$$

$$T_2 = T_1 \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1}\frac{2 + (k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = (490\text{ R})\frac{18.5\text{ psia}}{12\text{ psia}}\frac{2 + (1.4-1)(1.209)^2}{(1.4+1)(1.209)^2} = \mathbf{556R}$$

The downstream Mach number is determined to be

$$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.8363}{\sin(37.21° - 8°)} = \mathbf{1.71}$$

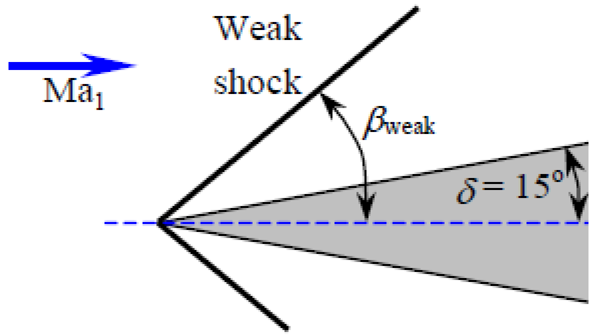
Note that $Ma_{1,n}$ is supersonic and $Ma_{2,n}$ is subsonic. However, Ma_2 is supersonic across the weak oblique shock (it is subsonic across the strong oblique shock).

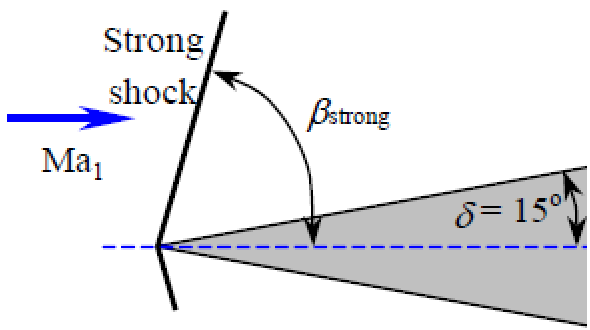
9. Air r flowing at 8 psia, 480 °R, and Ma_1 = 2.0 is forced to undergo a compression turn of 15°. Determine the Mach number, pressure, and temperature of air after the compression.

Solution

Assumptions 1. The flow is steady.
2. The boundary layer on the wedge is very thin.
3. Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is k = 1.4.





On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15°$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k+\cos 2\beta)+2} \quad \rightarrow \quad \tan 15° = \frac{2(2^2\sin^2\beta - 1)/\tan\beta}{2^2(1.4+\cos 2\beta)+2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or using $\beta - \theta$ curves. It gives β_{weak} = 45.34° and β_{strong} = 79.83°. Then the upstream "normal" Mach number $Ma_{1,n}$ becomes

Weak shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 45.34° = 1.423$

Strong shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 79.83° = 1.969$

Also, the downstream normal Mach numbers $Ma_{2,n}$ become

$$\textit{Weak shock:}\quad \mathrm{Ma}_{2,n}=\sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2+2}{2k\mathrm{Ma}_{1,n}^2-k+1}}=\sqrt{\frac{(1.4-1)(1.423)^2+2}{2(1.4)(1.423)^2-1.4+1}}=0.7304$$

$$\textit{Strong shock:}\quad \mathrm{Ma}_{2,n}=\sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2+2}{2k\mathrm{Ma}_{1,n}^2-k+1}}=\sqrt{\frac{(1.4-1)(1.969)^2+2}{2(1.4)(1.969)^2-1.4+1}}=0.5828$$

The downstream pressure and temperature for each case are determined to be

$$\textit{Weak shock:}\quad P_2=P_1\frac{2k\mathrm{Ma}_{1,n}^2-k+1}{k+1}=(6\text{ psia})\frac{2(1.4)(1.423)^2-1.4+1}{1.4+1}=17.57\cong\mathbf{17.6psia}$$

$$T_2=T_1\frac{P_2}{P_1}\frac{\rho_1}{\rho_2}=T_1\frac{P_2}{P_1}\frac{2+(k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2}=(480\text{ R})\frac{17.57\text{ psia}}{8\text{ psia}}\frac{2+(1.4-1)(1.423)^2}{(1.4+1)(1.423)^2}=609.5\text{ R}\cong\mathbf{610R}$$

$$\textit{Strong shock:}\quad P_2=P_1\frac{2k\mathrm{Ma}_{1,n}^2-k+1}{k+1}=(8\text{ psia})\frac{2(1.4)(1.969)^2-1.4+1}{1.4+1}=34.85\cong\mathbf{34.9\ psia}$$

$$T_2=T_1\frac{P_2}{P_1}\frac{\rho_1}{\rho_2}=T_1\frac{P_2}{P_1}\frac{2+(k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2}=(480\text{ R})\frac{34.85\text{ psia}}{8\text{ psia}}\frac{2+(1.4-1)(1.969)^2}{(1.4+1)(1.969)^2}=797.9\text{ R}\cong\mathbf{798R}$$

The downstream Mach number is determined to be

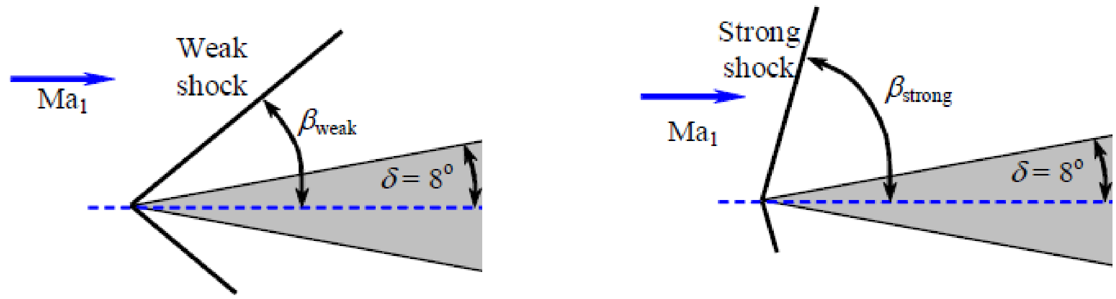
$$\textit{Weak shock:}\quad \mathrm{Ma}_2=\frac{\mathrm{Ma}_{2,n}}{\sin(\beta-\theta)}=\frac{0.7304}{\sin(45.34^\circ-15^\circ)}=\mathbf{1.45}$$

$$\textit{Strong shock:}\quad \mathrm{Ma}_2=\frac{\mathrm{Ma}_{2,n}}{\sin(\beta-\theta)}=\frac{0.5828}{\sin(79.83^\circ-15^\circ)}=\mathbf{0.644}$$

Note that the change in Mach number, pressure, temperature across the strong shock are much greater than the changes across the weak shock, as expected. For both the weak and strong oblique shock cases, $Ma_{1,n}$ is supersonic and $Ma_{2,n}$ is subsonic. However, Ma_2 is supersonic across the weak oblique shock, but subsonic across the strong oblique shock.

10. Air flowing at 60 kPa, 240 K, and a Mach number of 3.4 impinges on a two-dimensional wedge of half-angle 8°. Determine the two possible oblique shock angles, β_{weak} and β_{strong}, that could be formed by this wedge. For each case, calculate the pressure, temperature, and Mach number downstream of the oblique shock.

Solution



Assumptions 1. The flow is steady.
2. The boundary layer on the wedge is very thin.
3. Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is k = 1.4.

On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 8°$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(Ma_1^2 \sin^2\beta - 1)/\tan\beta}{Ma_1^2(k+\cos 2\beta)+2} \quad \rightarrow \quad \tan 8° = \frac{2(3.4^2 \sin^2\beta - 1)/\tan\beta}{3.4^2(1.4+\cos 2\beta)+2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or $\beta - \theta$ curve. It gives β_{weak} = 23.15° and β_{strong} = 87.45°. Then the upstream "normal" Mach number $Ma_{1,n}$ becomes

Weak shock: $$Ma_{1,n} = Ma_1 \sin\beta = 3.4\sin 23.15° = 1.336$$

Strong shock: $$Ma_{1,n} = Ma_1 \sin\beta = 3.4\sin 87.45° = 3.397$$

Also, the downstream normal Mach numbers $Ma_{2,n}$ become

Weak shock: $$Ma_{2,n} = \sqrt{\frac{(k-1)Ma_{1,n}^2 + 2}{2kMa_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.336)^2 + 2}{2(1.4)(1.336)^2 - 1.4 + 1}} = 0.7681$$

Strong shock: $$Ma_{2,n} = \sqrt{\frac{(k-1)Ma_{1,n}^2 + 2}{2kMa_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(3.397)^2 + 2}{2(1.4)(3.397)^2 - 1.4 + 1}} = 0.4553$$

The downstream pressure for each case is determined to be

Weak shock: $$P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k+1} = (60\text{ kPa})\frac{2(1.4)(1.336)^2 - 1.4 + 1}{1.4+1} = \mathbf{115.0 kPa}$$

Strong shock: $$P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k+1} = (60\text{ kPa})\frac{2(1.4)(3.397)^2 - 1.4 + 1}{1.4+1} = \mathbf{797.6 kPa}$$

The downstream Mach number is determined to be

Weak shock: $$\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.7681}{\sin(23.15° - 8°)} = \mathbf{2.94}$$

Strong shock: $$\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.4553}{\sin(87.45° - 8°)} = \mathbf{0.463}$$

Note that the change in Mach number and pressure across the strong shock are much greater than the changes across the weak shock, as expected. For both the weak and strong oblique shock cases, $\text{Ma}_{1,n}$ is supersonic and $\text{Ma}_{2,n}$ is subsonic. However, Ma_2 is supersonic across the weak oblique shock, but subsonic across the strong oblique shock.

11. Calculate the maximum surface pressure (in newtons per square meter) that can be achieved on the forward face of a wedge flying at Mach 3 at standard sea level conditions ($p_l = 1.01 \times 10^5$ N/m^2) with an attached shock wave.

Solution

From the θ-β-M diagram, at $M_1 = 3$, $\theta_{max} = 34.1°$ and $\beta = 66°$. Hence, the wedge can be no larger than a half-angle of 34.1°.

$M_{n_1} = M_1 \sin\beta = (3.0)\sin 66° = 2.74$

From Table: $p_2/p_1 = 8.592$

$p_1 = 1.01 \times 10^5$ N/m_2 (standard sea level pressure)

$$p_2 = \frac{p_2}{p_1} p_1 = 8.592\,(1.01 \times 10^5) = 8.678 \times 10^5 \ \frac{\text{N}}{\text{m}^2}$$

This is the maximum pressure. Any higher pressure would correspond to a wedge half-angle > θ_{max}, the shock would be detached.