Radiation-Matter Interaction II

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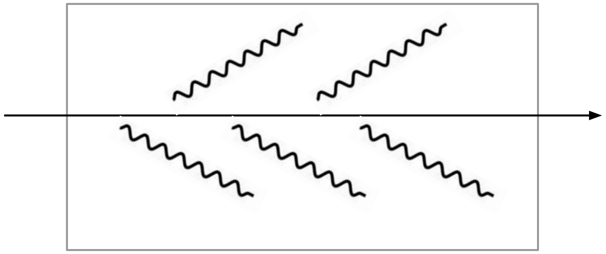
Radiation Length

- Energy loss for irradiation: $-\left(\frac{dE}{dx}\right)^{rad} = \frac{E}{X_0}$

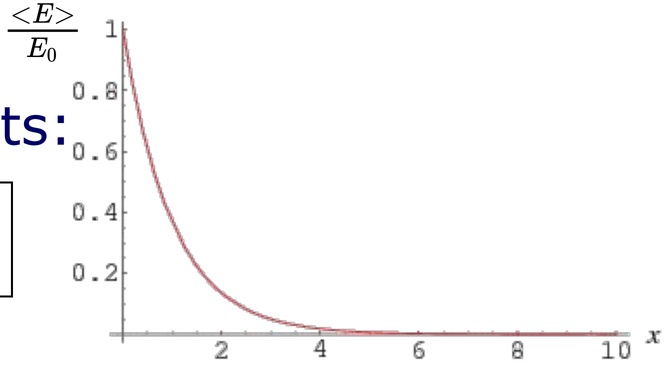


$$\frac{1}{X_0} = \frac{4\alpha N_A Z(Z+1) r_e^2 \log\left(183 Z^{-1/3}\right)}{A} \Rightarrow X_0 \propto Z^2$$

- Integrating on the dE/E spectrum one gets:

$$\langle E \rangle = E_0 \cdot e^{-\frac{x}{x_0}}$$

initial energy



<u>Radiation length</u> *X_0= material thickness that reduces the mean energy of a beam of electrons by a factor 1/e (for only radiative losses)*

- If the material thickness is given in terms of X_0 : $X' = X/X_0$

$$-\frac{dE}{dX'} \approx E_0 \longrightarrow$$

Independent from the material

Energy loss via radiation

- It is possible to show that

$$\left(-\frac{dE}{dx}\right)_{rad} = \frac{E}{X_0} \Rightarrow E(x) \propto \exp\left(-\frac{x}{X_0}\right)$$

with

$$X_0 \propto m^2$$

- Therefore

$$\frac{\left(-dE/dx\right)^{\mu}_{rad}}{\left(-dE/dx\right)^{e}_{rad}} = \left(\frac{m_e}{m_\mu}\right)^2 \approx 2\times10^{-5} \quad !!$$

- Bremsstrahlung typically very important for electrons but not for muons

Ionization & Bremsstrahlung

IONIZATION

- → dE/dx goes with log(E) and is proportional to Z
- → Dominant at energies smaller than the critical one
- → Energy is released to the material

BREMSSTRAHLUNG

- → dE/dx is proportional to E e to Z^2
- → Dominant at energies > Ec
- → Energy is given to the photons

Cherenkov Radiation

Charged particle in a material

↓

Atomic polarization

↓

Temporal variation of the electric dipole

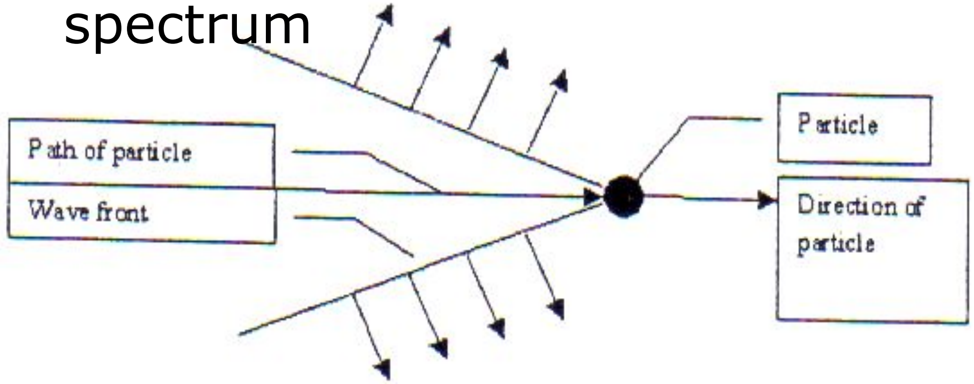
↓

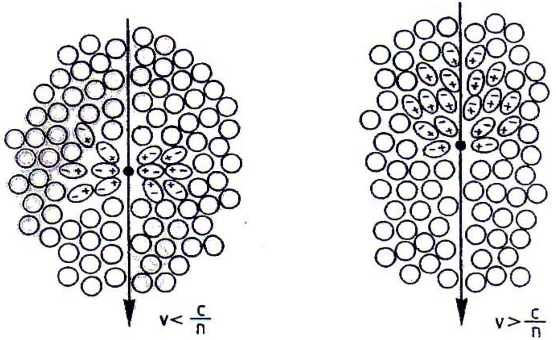
Electromagnetic radiation emitted in a very short time (ps) in a cone of angular semi-aperture:

$$\cos \theta_c = 1/\beta n$$

refraction index of the material

Photons emitted in the visible spectrum

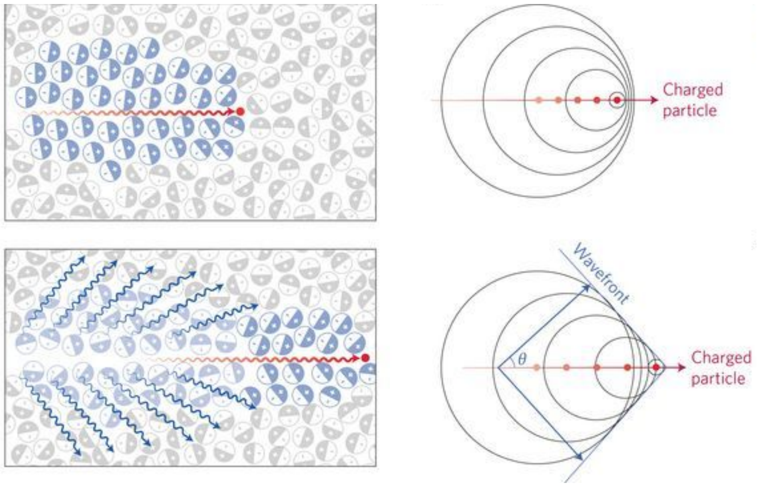




v > c/n

Symmetric dipole around the

Broken symmetry



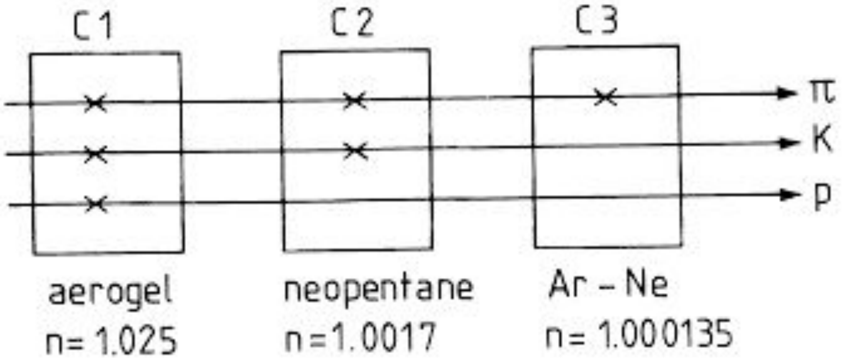
- Very low efficiency (few photons emitted)
- Little energy lost

Cherenkov Detector

Threshold Č

Threshold velocity: v > c/n

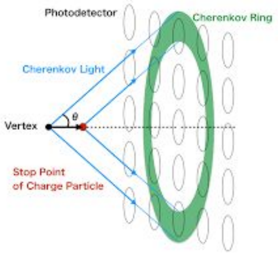
Discriminates *relativistic* particles with ***same momentum***
and ***different mass***



Differential Č

Radiation is emitted
at a given an $\cos \theta_c = 1/ßn$

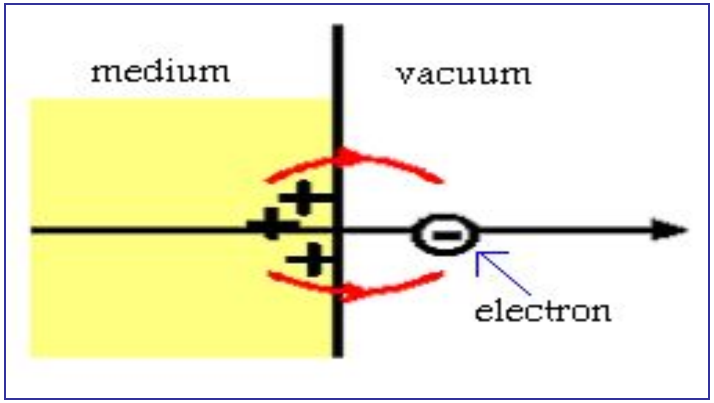
By ***measuring θ$_c$***:
- get momentum by assuming mass
- get mass (particle id.) by knowing mor



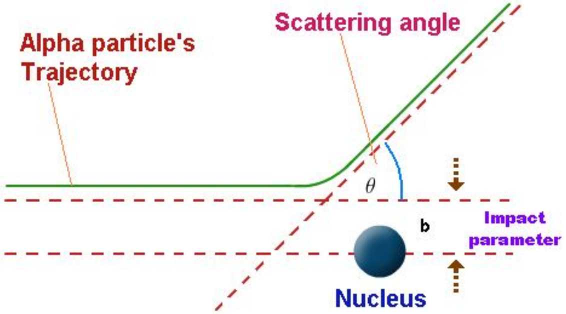
Transition Radiation

Radiation emitted when a charged particle crosses the *boundary* between two materials with a *different index of refraction*

- The particle forms an **electric dipole** with its image (depending on time)
  and produces electromagnetic radiation
- The ***energy loss is small***: nr. of photons emitted per separating surface N**γ** ~ 1/137
- Irradiated energy I ~ **γ :**
  identification of particles with a high **γ** (electrons)
- Angle of emission: θ ~ 1/**γ**

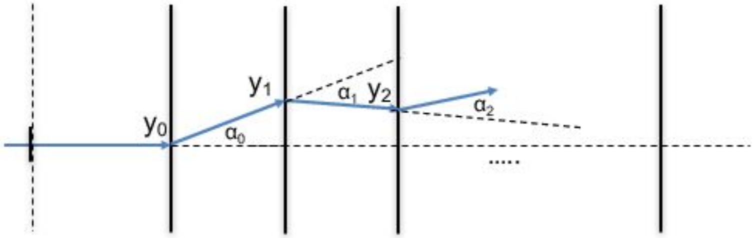


Multiple Coulomb Scattering



Particle scattering produced by the diffusion from the coulomb field of the nuclei

Described by the Rutherford formu



$$d\sigma/d\Omega=(Zzr_em_ec^2)^2/4p^2v^2sen^4(\theta/2)$$

θ = scattering angle

Probability of interaction

Large for small angle

Large for small speed

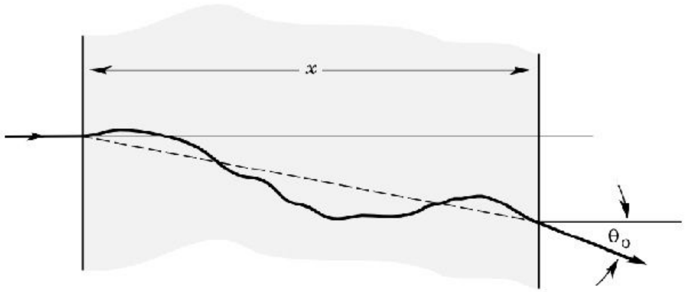
Many collisions at small angles

★ *Small loss of energy*

Multiple Coulomb Scattering

Scattering →
- single (thin absorber)
- more
- multiple

⇒ statistically treated:

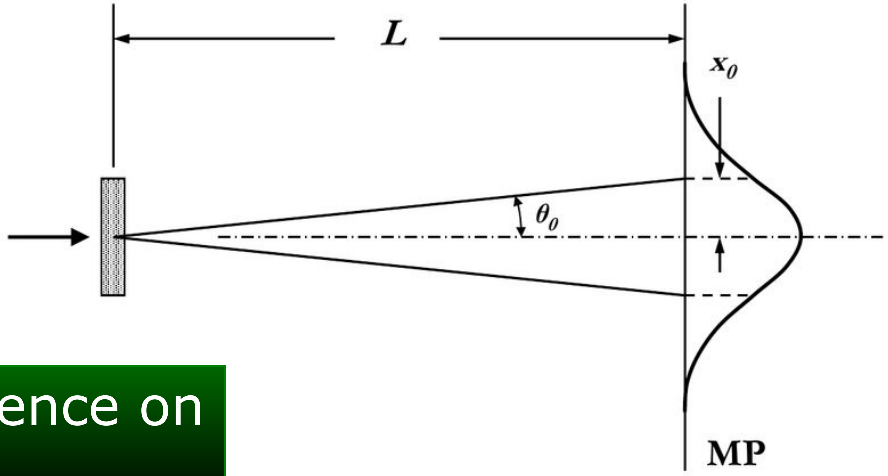


Standard deviation: $\sqrt{\langle\theta^2\rangle} = \frac{E_s\, z}{p\cdot v}\sqrt{\frac{X}{X_0}}$

Es ~ 21 Mev, z = charge of parti

X_0= radiation length

(defined for radiative processes)



Scattering is ***not*** a radiative

→

Dependence on X_0

Multiple Coulomb Scattering

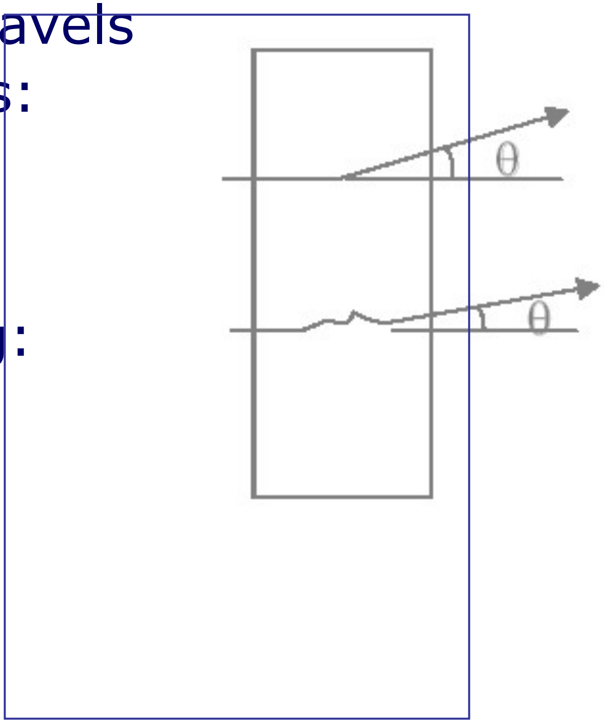
For a relativistic particle $\beta \sim 1$ with $z = 1$ in a magnetic field $B = 1.5$ T which travels $X = 1$ m in iron ($X_0 = 0.02$ m) one has:

$$pc\,[\mathrm{Mev}] = 300\,BR$$

$$\theta = X / R = 300\,BX / pc$$

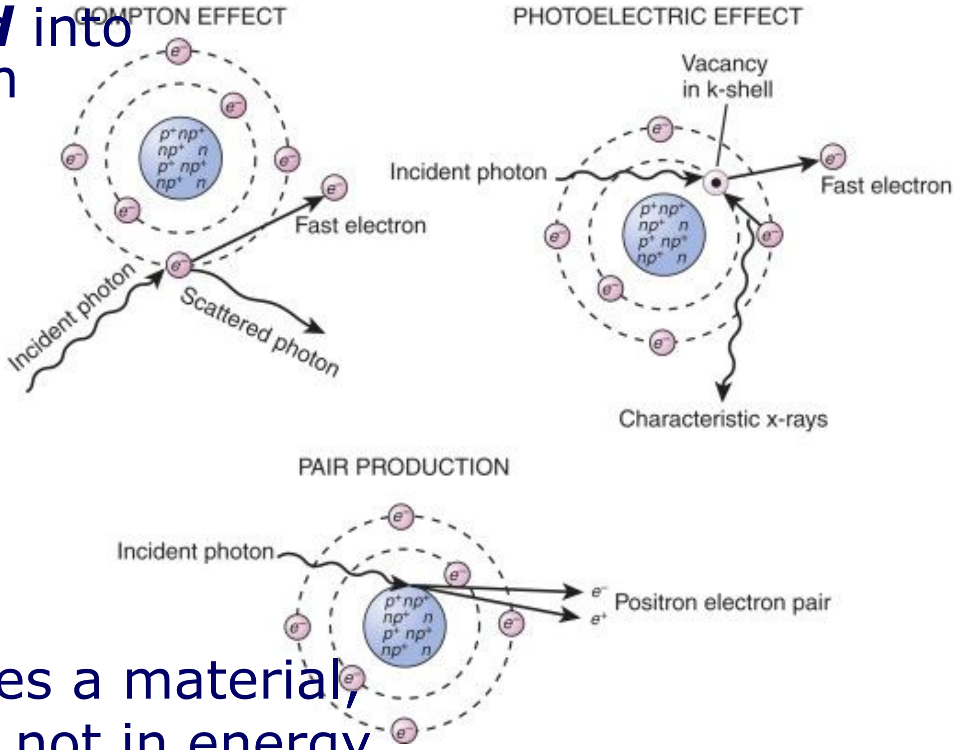
Uncertainty due to multiple scattering:
$\theta^{RMS} / \theta \sim 0.25$

Precision Limit to precision of particle direction measurement!



Photon – matter interaction

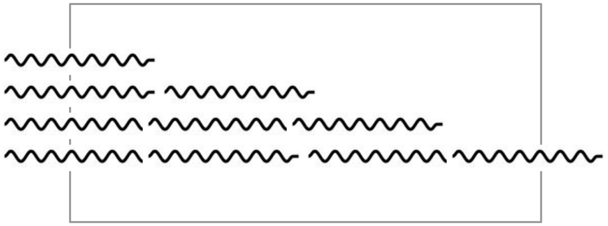
- No charge □ no collisions with atomic e$^-$
- They have to be ***converted*** into charged particles to be seen
- More ***penetrating*** than charged particles



- Three main effects:
  - *Photoelectric*
  - *Compton*
  - *Pairs production*
- When a photon beam crosses a material, it is degraded in ***intensity***, not in energy

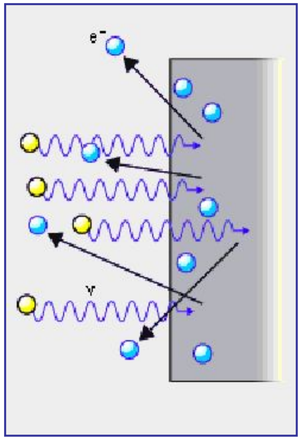
$I(x) = I_0$

Where x is the material thickness
and μ is the absorption coefficient



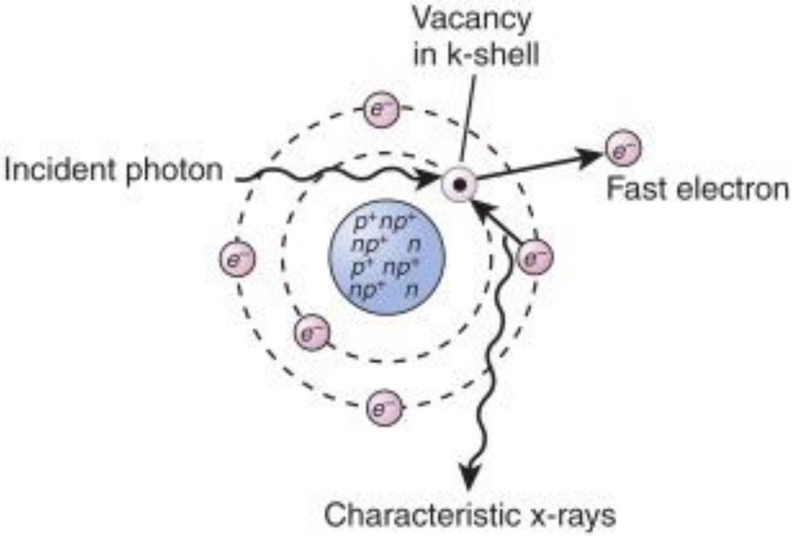
Photoelectric Effect

$\gamma A \rightarrow A^{+} e^{-}$



Photons, colliding with atoms, transfer to the peripheral
photons an energy which can allow their emission

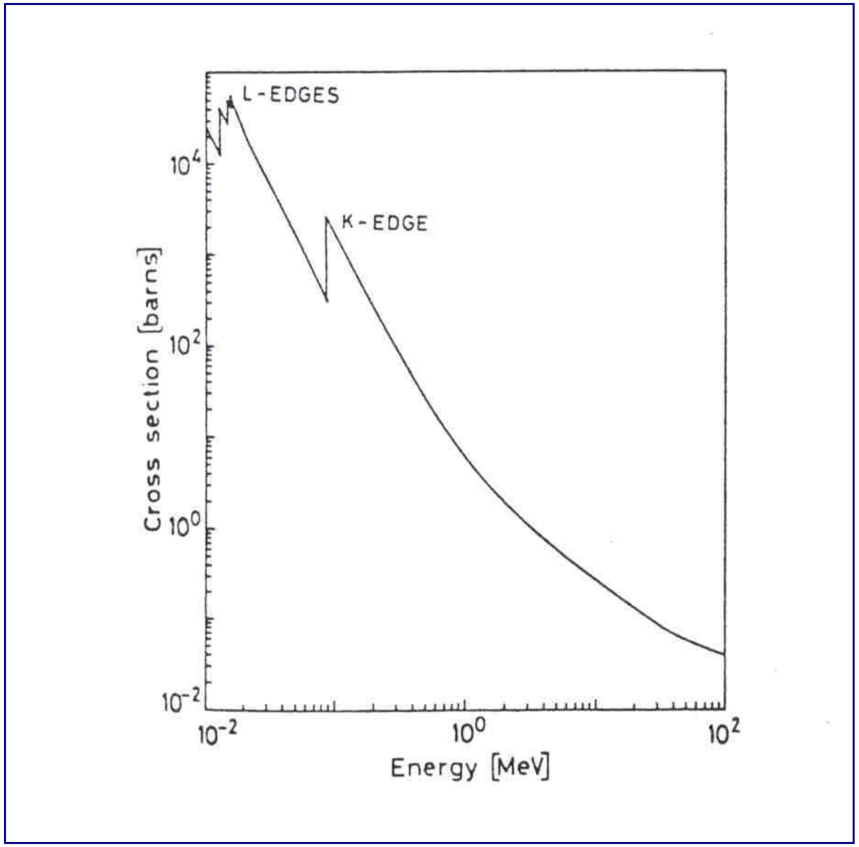
- It is an over-threshold ef $E_\gamma > E_{bound}$
- The atom remains in an ***excited state*** and decays emitting fluorescence
other electrons
- The ***e⁻ direction is random*** with
to the direction of the incident ph



- *The **nr.** of emitted e^- depends on the **intensity** of the beam*
- *The e^- **energy** depends in the beam **energy**: $T_e = h\nu - E_{bound}$*

Photoelectric Effect

Process cross section $\sigma \div Z^{4\text{-}5} / E_{\gamma}^{7/2}$ (for Eγ > shell k)



It depends mainly on atomic number Z of the target material and on photon energy

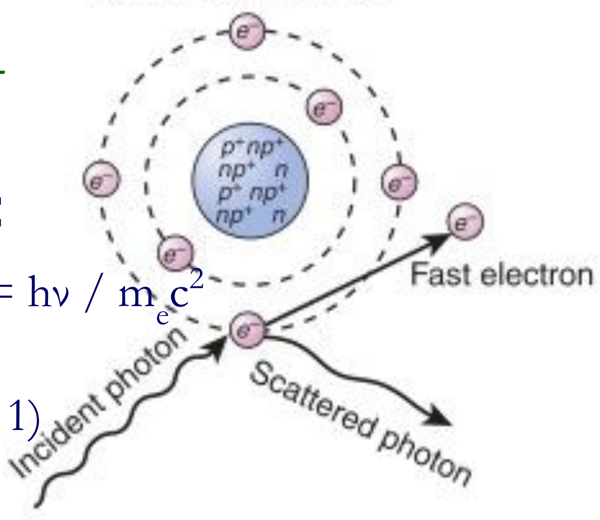
Cross section goes down when energy grows and when Z grows

Peaks correspond to atomic orbitals

Compton Effect

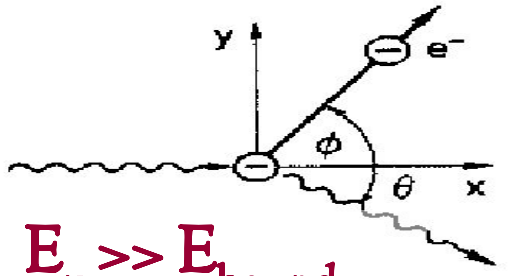
$\gamma e^- \rightarrow \gamma e^-$

Scattering of a photon on a free e⁻



Applying energy & momentum conservation:

- Energy of scattered γ: $h\nu' = h\nu / (1+ \gamma (1 - \cos \theta))$, $\gamma = h\nu / m_e c^2$
- Energy of scattered e⁻: $T = h\nu - h\nu'$
- Direction of scattered γ: $\cos \theta = 1- 2 / ((1+ \gamma)^2 \, tg^2 \varphi + 1)$
- Direction of scattered e⁻: $\cos \varphi = (1+ \gamma)\, tg (\theta/2)$



$E_\gamma >> E_{bound}$

Absorbtion coefficient:

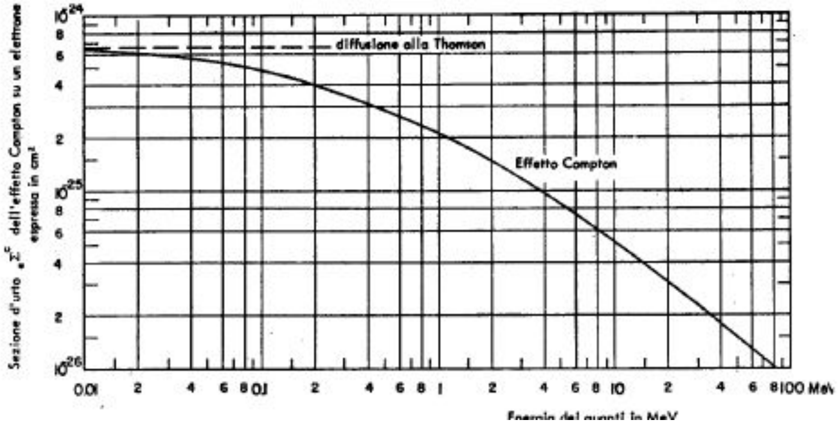
$$\mu c = n_e \sigma_c = \frac{NaZ}{A} \rho \sigma_c$$

Only part of the energy of the photon gets transferred to the electron

Compton Effect

The process cross section is given by the Klein-Nishina formula:

$$d\sigma / d\Omega = (r_e^{\,2} / 2) \; * \; (1 / [1 + \gamma\,(1 - \cos\varphi)]^2) \; * \; [1 + \cos^2\varphi + \gamma^2 (1 - \cos\varphi)^2 / (1 + \gamma(1 - \cos\varphi))]$$



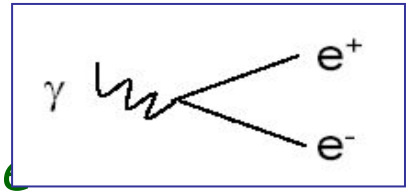
Total Cross section vs energy

Integrating, one gets a cross section which is proportional to Z/E

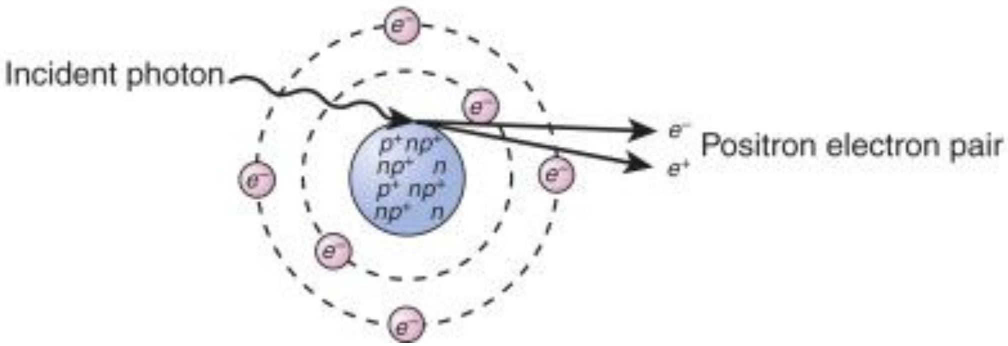
$$\sigma \div Z/E$$

Pair Production

$$\gamma N \rightarrow N e^- e^+$$



Transformation of a photon in a pair e^+e



- Threshold reaction: $E_\gamma > 2\,me$
- Over a certain energy explains all the photons absorption
- Takes place in the presence of a nucleus for the momentum conservation, but no energy is released to the material.

Pair Production

The cross section of the process is:

→ for high energies: $\sigma \div Z^2$ □ independent from E

→ for low energies : $\sigma \div Z^2 \ln E$ □ weak dependency from E

Absorbtion coefficient:

$$\mu pp = \frac{Na\rho}{A}\sigma pp = \frac{7}{9X0}$$

The pair production phenomenon is linked to bremsstrahlung emission

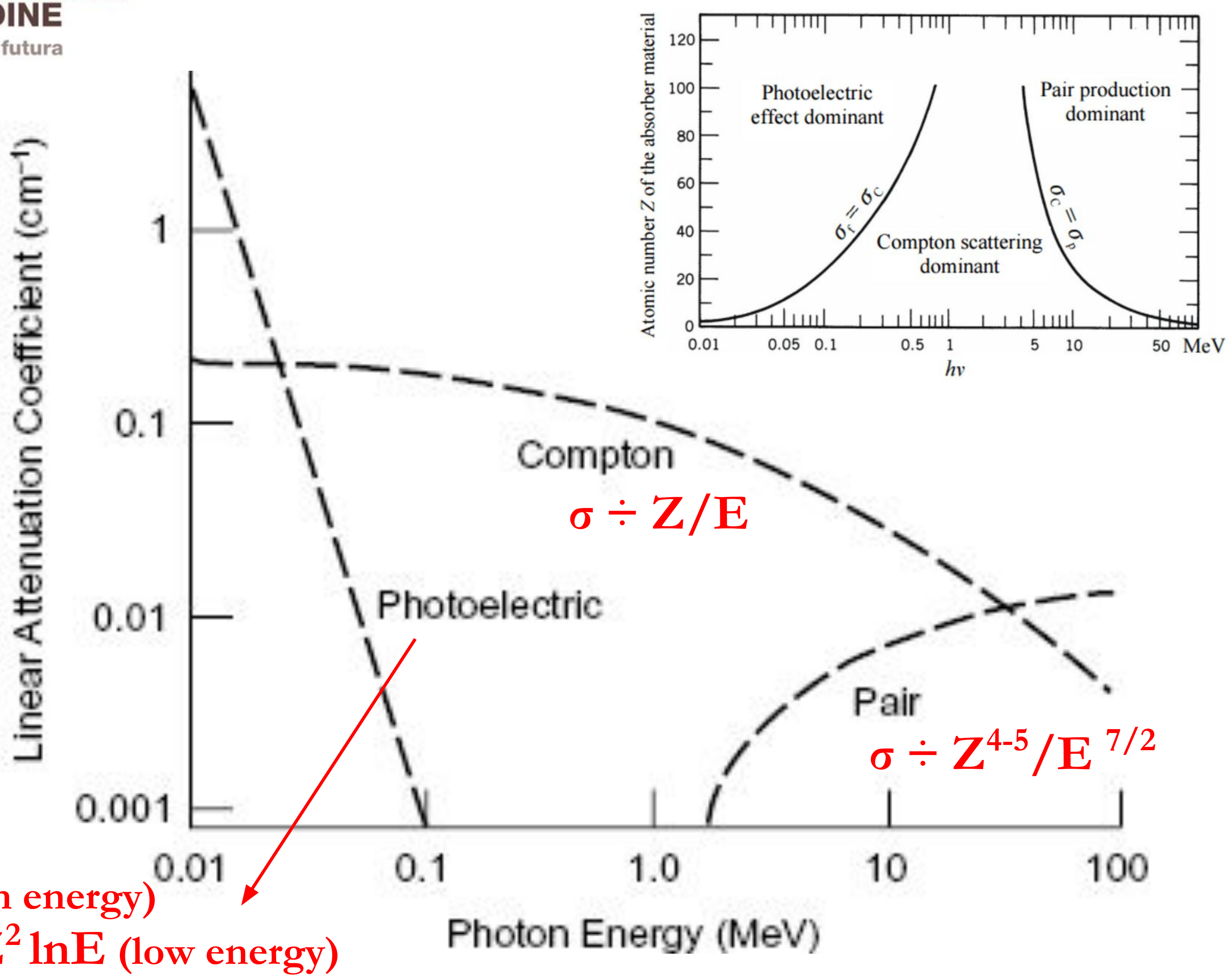
⬇

Intensity attenuation of a photon beam of initial intensity I_0 for pair production:

➡

$$\boxed{I = I_0 \exp(-\frac{7}{9} \cdot \frac{X}{X_0})}$$

Cross sections comparison



$\sigma \div Z/E$

$\sigma \div Z^{4\text{-}5}/E^{7/2}$

$\sigma \div Z^2$ (high energy)

$\sigma \div Z^2 \ln E$ (low energy)

Electromagnetic Shower

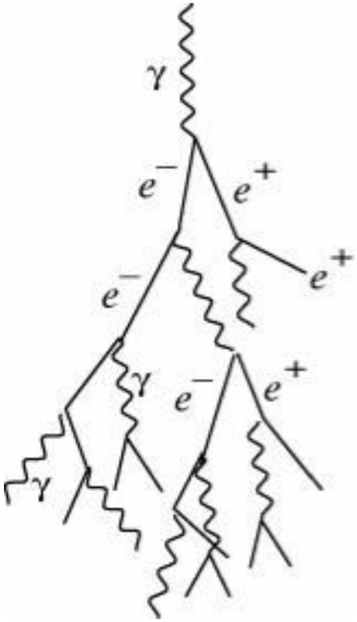
Can be started by a photon or an electron

After about (7/9) X_0

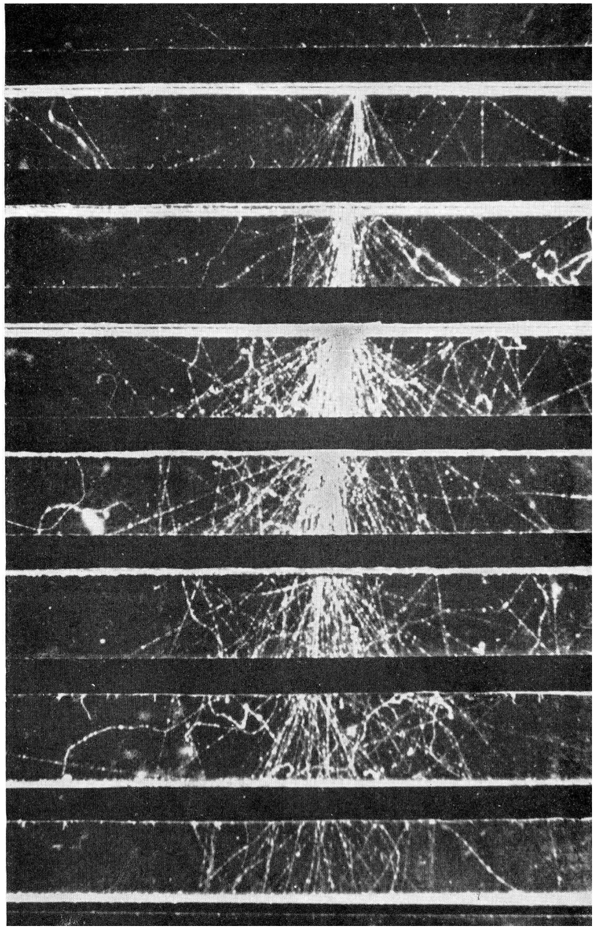
$$\gamma + N \to e^+ + e^- + N$$

After about one X_0

$$e^\pm + N \to e^\pm + N + \gamma$$

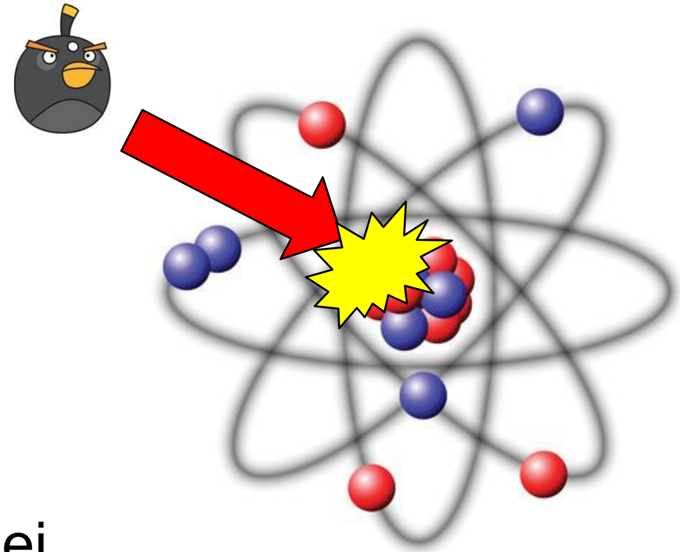


- ➔ The number of electrons and positrons initially increase and then reaches a maximum
- ➔ When the particle energy is reduced to E_c, starts to decrease



Pb gas

Hadronic Energy Loss



- ○ Hadrons = particles made of quarks
  - ➜ protons
  - ➜ neutrons
  - ➜ pions
  - ➜ kaons...

- ○ Beside EM interactions, hadrons can have strong interaction with atomic nuclei

- ○ Typically, nucleus mass > hadron mass
  ⇒ hardon stopped
  ⇒ possible production of secondary hadrons (depending on energy)

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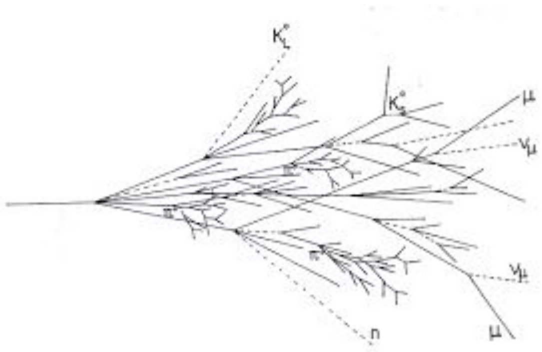
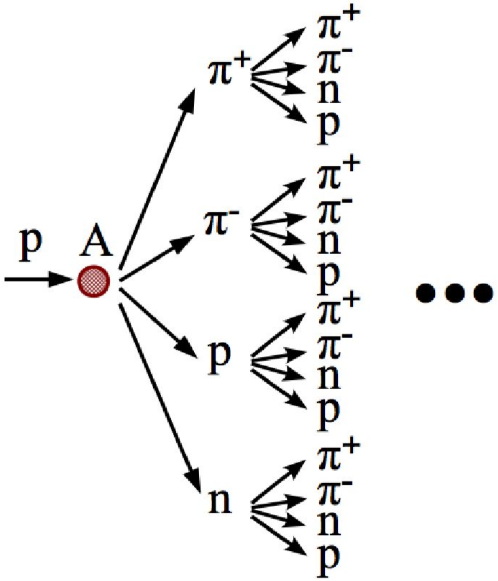
Hadronic Energy Loss

- Interaction length λ_0 = mean distance before a neutron beam suffers attenuation 1/e
  - ➜ weak dependence on energy
  - ➜ typical values (for E > 100 MeV)

air $\lambda_0 \approx 750$ m H_2O $\lambda_0 \approx 0.85$ m
C $\lambda_0 \approx 0.38$ m   Fe $\lambda_0 \approx 0.7$ m   Pb $\lambda_0 \approx 0.17$ m

- If energy high enough
  ⇒ hadronic collisions initiate hadronic shower
  - ➜ distance between interactions ~ λ_0





Summary

Charged particles
- Ionization
- Bremsstrahlung (negligible for heavy charged particles)
- Multiple Coulomb scattering
- Cherenkov emission
- Transition radiation (important for electrons)

Photons
- Photoelectric effect
- Compton effect
- Pair production

Hadrons
- Interaction with nuclei
- + same as charged particles if charge

Backup