Chapter 4

1. A linear map $f : V \to W$ is called an isomorphism if

   (a) there exists a linear map $g : W \to V$ with $fg = Id_w$ and $gf = Id_v$

   (b) V and W are isomorphic

   (c) for each n-tuple $(v_1, \dots, v_n)$ in V, the n-tuple $(f(v_1), \dots, f(v_n))$ is a basis for W

2. By the rank rk (f) of a linear map $f : V \to W$, one understands

   (a) dim Ker f

   (b) dim Im f

   (c) dim W

3. Here is a more subtle question. Remember dim Ker A + rk $A = n$ for $n \times n$ matrices? Good. Now let A be an $n \times n$ matrix and let $Ax = b$ have two linearly independent solutions. Then

   (a) rk $A \leq n$, and the case rk $A = n$ can occur

   (b) rk $A \leq n - 1$, and the case rk $A = n - 1$ can occur

   (c) rk $A \leq n - 2$, and the case rk $A = n - 2$ can occur

4. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?

   (a) $2f(t) + 3g(t) + 4$

   (b) $f(t) - 2g(t) + t$

   (c) $2f(t)g(t) - 3f(t)$

   (d) $f(t) - g(t)$

   (e) All of the above

   (f) None of the above

   (g) Some of the above

5. To determine whether a set of vectors is linearly independent, you form a matrix which has those vectors as columns. If the matrix is square and its determinant is zero, what do you conclude?

   (a) The vectors are linearly independent

   (b) The vectors are not linearly independent

   (c) This test is inconclusive, and further work must be done

6. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

   (a) $y_2(t) = \sin(t)\cos(t)$

   (b) $y_2(t) = 2\sin(2t)$

(c) $y_2(t) = \cos(2t - \pi/2)$

(d) $y_2(t) = \sin(-2t)$

(e) All of the above

(f) None of the above

7. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

   (a) $y_2(t) = e^{-2t}$

   (b) $y_2(t) = te^{2t}$

   (c) $y_2(t) = 1$

   (d) $y2_(t) = e^{3t}$

   (e) All of the above

   (f) None of the above

8. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if

   (a) For some constant k, $y_1(t) = ky_2(t)$ for $a < t < b$

   (b) There exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$

   (c) The equation $c_1y_1(t) + c_2y_2(t) = 0$ hold for all $t \in (a, b)$ only if $c_1 = c_2 = 0$

   (d) The ratio $y_1(t)/y_2(t)$ is a constant function

   (e) All of the above

   (f) None of the above

9. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if

   (a) There exist two constants c_1 and c_2 such that $c_1y_1(t) + c_2y_2(t) = 0$ for all $a < t < b$

   (b) There exist two constants c_1 and c_2, not both 0, such that $c_1y_1(t) + c_2y_2(t) = 0$ for all $a < t < b$

   (c) For each t in (a, b), there exists constants $c_1y_1(t) + c_2y_2(t) = 0$

   (d) For some $a < t_0 < b$, the equation $c_1y_1(t_0) + c_2y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$

   (e) All of the above

   (f) None of the above

10. The linear transformation $T(x, y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

    (a) $T(x, y) = (x, y)$

    (b) $T(x, y) = (y, x)$

    (c) $T(x, y) = (-x, y)$

    (d) $T(x, y) = (-y, x)$

    (e) None of the above

11. The linear transformation $T(x,y) = (x+2y, x-2y)$, can be written as a matrix transformation $T(x,y) = A\begin{bmatrix} x \\ y \end{bmatrix}$ where

   (a) $A = \begin{bmatrix} x & 2y \\ x & -2y \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$

   (d) It can't be written in matrix form

12. Which of the following is not a linear transformation?

   (a) $T(x,y) = (x, y+1)$
   (b) $T(x,y) = (x-2y, x)$
   (c) $T(x,y) = (4y, x-2y)$
   (d) $T(x,y) = (x, 0)$
   (e) All are linear transformations
   (f) More than one of these are not linear transformations

13. Is the transformation $T(x,y,z) = (x,y,0)$ linear?

   (a) No, it is not linear because all z components map to 0
   (b) No, it is not linear because it does not satisfy the scalar multiplication property
   (c) No, it is not linear because it does not satisfy the vector addition property
   (d) No, it is not linear for a reason not listed here
   (e) Yes, it is linear

14. If f is a function, is the transformation $T(f) = f'$ linear?

   (a) No, it is not linear because it does not satisfy the scalar multiplication property
   (b) No, it is not linear because it does not satisfy the vector addition property
   (c) No, it is not linear for a reason not listed here
   (d) Yes, it is linear

15. Let $T : \mathbf{R}^2 \to \mathbf{R}^3$ be a linear mapping, satisfying

$$T(1,2) = (1,0,1) \text{ and } T(2,5) = (0,1,1)$$

   Calculate $T(0,1)$

(a) $(0, 2, -3)$
(b) $(-1, 1, 2)$
(c) $(1, 1, 0)$
(d) $(-2, 5, 1)$
(e) $(1, -1, 0)$
(f) $(-2, 1, -1)$