Applied Optics
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Lecture 48
Normal and Oblique Incidence

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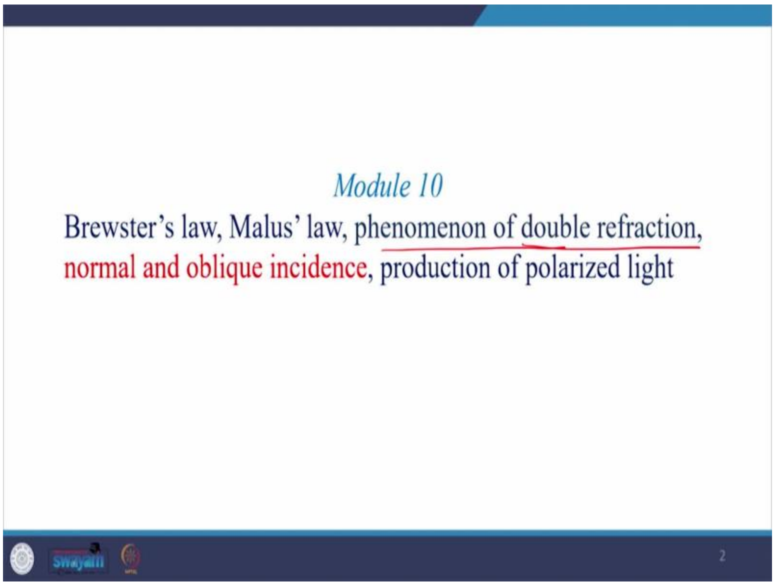

Hello everyone, welcome to my class. Today we will talk about normal and oblique incidence, in the last class we talked about double refraction. And we also talked about the different kinds of double refracting media birefringent media, which we categorized as positive and negative birefringent media and then we also sub categories them as uniaxial crystals and the biaxial crystal. Today, we will learn refraction in these double refracting media. And in this class first of all we will talk about normal incidence and then it would be followed by oblique incidence.

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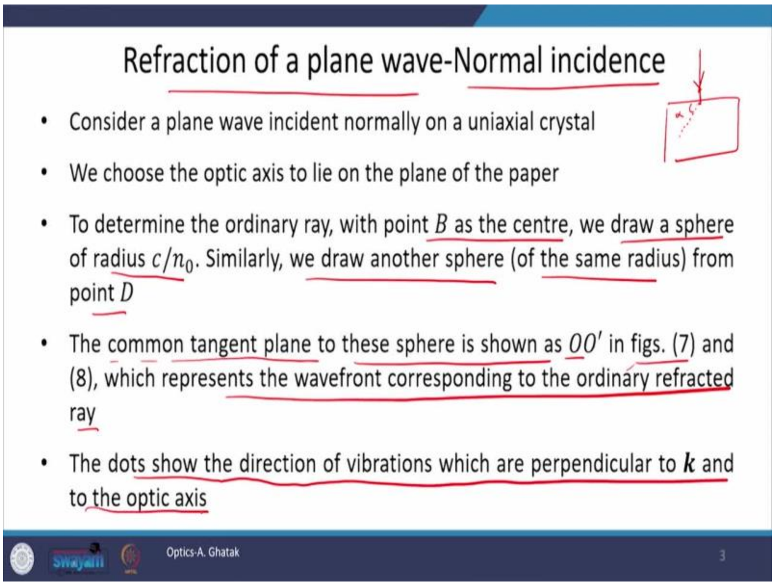

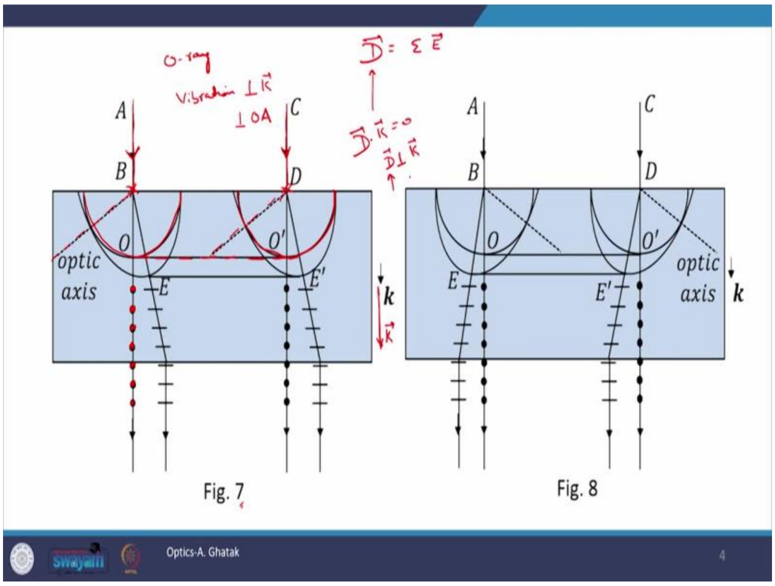

Now the refraction of a plane wave for normal incidence is discussed first. In this case, we will consider a plane wave and we will assume that this wave is made to incident normally on a uniaxial negative crystal, like calcite. Now we choose the optic axis to lie on a plane of the paper, suppose if this is the crystal and this is the incident plane wave then the optic axis is assumed to be in the plane of this paper in some direction say making an angle α with the horizontal.

Now to determine mind the ordinary refraction or to determine the direction of refracted ray for ordinary ray, what we will do is that, as shown here in this figure, we will assume that a ray is made to incident here on this negative uniaxial crystal and we assume that there are 2 rays

AB and CD which are made to incident on this crystal normally and the optic axis is denoted by this dashed line which is in the plane of this paper. And the point of incidence is B and D.

Now to determine the ordinary ray path with point B as the center what we will do is that we will draw a sphere of radius c/n_0, where n_0 is the refractive index of the medium for ordinary ray. Similarly, we draw another sphere of same radius from point D because, we have assumed that 2 rays AB and CD are falling on the crystal normally. Therefore, treating point B and D as a center we draw a sphere which is this sphere and the radius of this sphere is c/n_0.

Now after this we will draw a common tangent to this sphere, now the common tangent plane to these sphere is soon as OO' in figure 7, let us go again to figure 7 and this line is the common tangent to these is sphere. This common tangent represents the wavefront corresponding to the ordinary refracted ray, a plane wave is falling under a double refracting crystal normally and then from there we suppose that we are given 2 rays and these 2 rays falls at point B and D on the interface of birefringent material and treating point B and D as a center, we draw 2 spheres and then we draw a common tangent to these spheres and this common tangent represent the wavefront which corresponds to the ordinary refracted ray. Now the dots show the direction of vibration which are perpendicular to k and to the optic axis.

Now you see that in this figure 7 these are the dots and these dots are the polarization direction or the direction of vibration. Now as you see that these are dots therefore, they are going inside the paper, they are perpendicular to the plane of the paper and the oscillations is like this up and down and these oscillations are perpendicular to the optic axis as well as the wave vector $\vec{k}$which is in this direction. Therefore, make it a point for o ray, the vibration direction is perpendicular to vector $\vec{k}$ as well as it is perpendicular to optic axis. Now for isotropic medium the direction of vibration is associated with $\vec{E}$ field electric field, but for anisotropic medium it is $\vec{D}$ which is perpendicular to $\vec{k}$, $\vec{D}$ is displacement vector and $\vec{D}$ is related to $\vec{E}$ through relation which is expressed as are termed as $\vec{D} = \epsilon\vec{E}$, where ϵ is permittivity, and $\vec{E}$ is electric field, $\vec{D}$ is displacement vector and $\vec{D}$ is related to $\vec{E}$ through this relation.

Now we associate vibration now with the direction of $\vec{D}$ therefore, these dots represents the direction of the vibration, $\vec{D}$ is vibrating in a direction which is given by this dot in this particular case and this is also the direction of polarization. And this definition is more correct for anisotropic material or I should say that this is more generalized definition because isotropic material are usual material which does not exhibit any birefringent, the $\vec{D}$ and $\vec{E}$ they are in the same direction. But for anisotropic material that $\vec{D}$ is not in the direction of $\vec{E}$ and therefore, $\vec{D}$ is preferred as direction of vibration, why? Because $\vec{D}$ is perpendicular to $\vec{k}$, the dot product of $\vec{D}$ and $\vec{k}$ is equal to 0 therefore, $\vec{D}$ is perpendicular to $\vec{k}$ and this is why we associate polarization with $\vec{D}$ vector.

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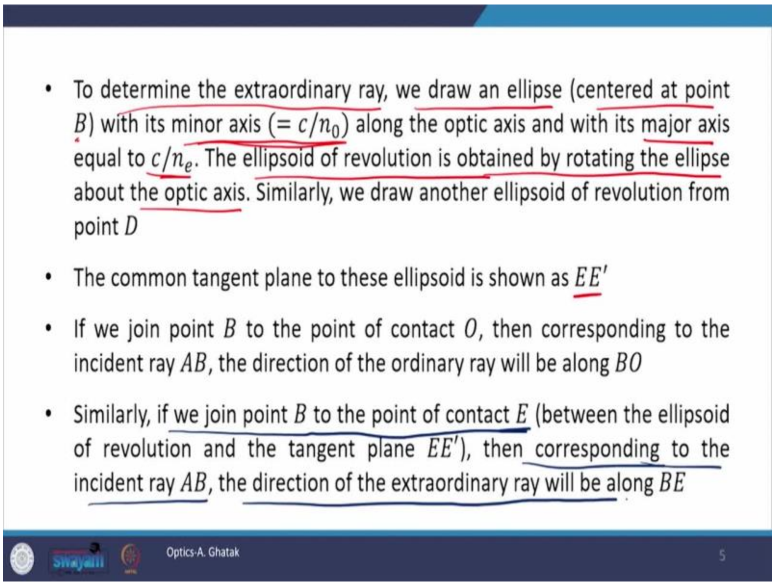

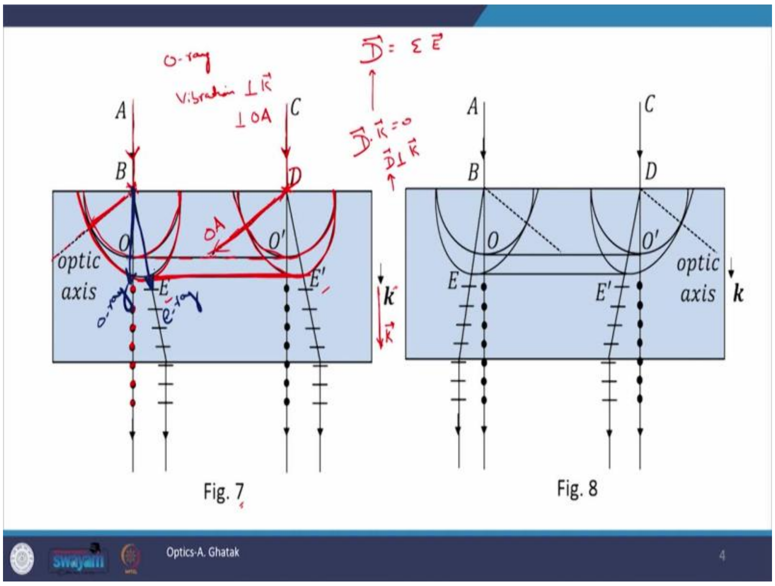

Now to determine the extraordinary ray, till now we have just talked about ordinary ray, now to determine extraordinary ray, we draw an ellipse centered at point B. Why do we draw an ellipse? Because we know in case of extraordinary ray the velocity is direction dependent and therefore, at point B for extraordinary ray we draw an ellipse. Since it is a negative uniaxial crystal, we know that the velocity of extraordinary ray is larger than that of ordinary ray therefore, the sphere would be inside this ellipse and since it is negative uniaxial crystal, the semi minor axis of the ellipse would be along optic axis and along optic axis the sphere and ellipse will touch and this is why the ellipse is drawn in such a way that semi minor axis is in this direction along the optic axis. Once semi minor axis is decided we can easily draw the ellipse.

Now we know that for extra ordinary ray we will have to draw the ellipsoid of revolution, to draw the ellipsoid of revolution this ellipse is rotated around the optic axis and this will give the ellipsoid of revolution. The similar procedure would be repeated for e ray at starting from point D. Here too we will draw an ellipse and then we will orient the ellipse in such a way that the minor axis of the ellipse is along the optic axis which is in this direction and once it is drawn then here again we will draw a common tangent and in this particular case, this line would serve as a common tangent to the 2 ellipse and this line is EE' line, EE' line now represents the wavefront after e ray, extraordinary ray.

Now you see that the wavefront of ordinary ray and extraordinary ray they both are parallel and the way vector for the both rays are perpendicular to this wavefront. Now since along the optic axis both rays travel with the same velocity therefore, minor axis of the ellipse would be equal to c/n_0 which is the radius of the sphere. Now major axis of the ellipse would be decided by this relation c/n_e, this would be the length of major axis. The ellipsoid of revolution as I stated before is obtained by rotating the ellipse about the optic axis and similarly we draw another ellipsoid of revolution starting from point D and common tangent would be EE'.

Now if we join point B to the point of contact O, I mean this point, let me pick a different color, if we joined B with O, then this line represents the direction of propagation of o ray ordinary ray. Similarly, if we join point B to the point of contact E then corresponding to the incident ray AB the direction of extraordinary ray will be along BE which means this would be the direction of extraordinary ray and this would be the direction of ordinary ray. What we did is that, we drew common tangent and the common tangent touches the sphere at point O, then we joined B with O and then BO would be the direction of propagation of o ray similarly, BE would the direction of propagation of e ray.

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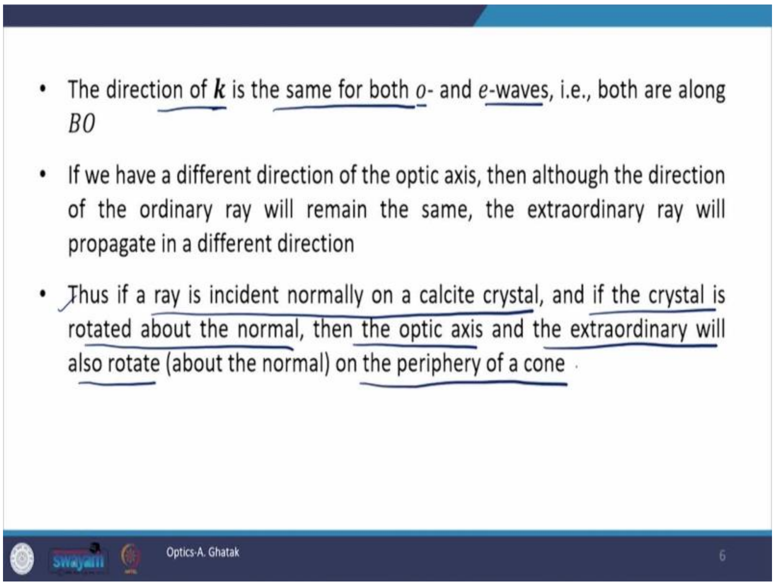

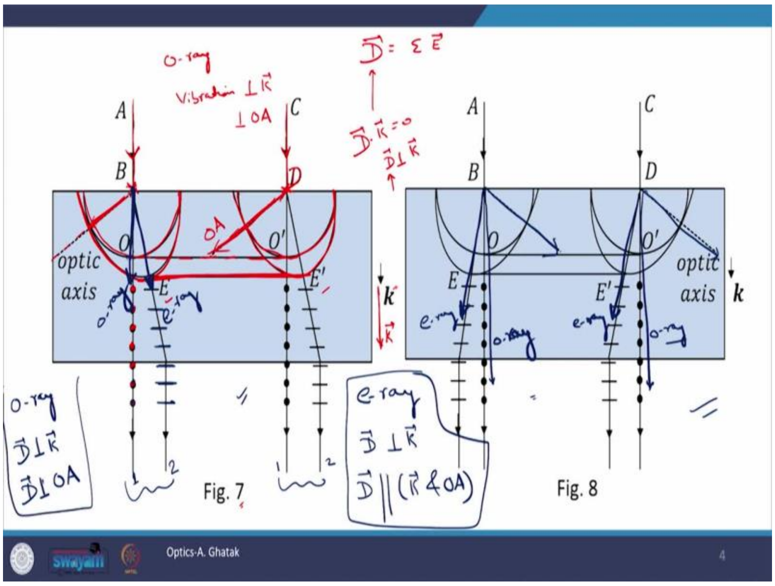

The direction of wave vector $\vec{k}$ is same for both o and e waves that is both are along BO along the direction of propagation of ordinary ray. If we have different direction of optic axis, then although the direction of ordinary ray will remain the same, the extraordinary ray will propagate in a different direction which is shown in this figure, here you see that the optic axis is now directed here along this direction. Now if we changed the direction of optic axis, the direction of o ray remain as it is, o ray did not change its direction of propagation while the e ray direction got changed. In the first case the e ray was on the right hand side of O now here in the second case the e ray is on the left hand side of o ray and this would be your o ray. It means that direction of e ray depends upon the orientation of optic axis.

Now thus if a ray is incident normally on a calcite crystal and if the crystal is rotated about the normal then the optic axis and the extraordinary ray will also rotate about the normal and this rotation would be on the periphery of a cone. And now you see here at the output we have 2 rays this is ray number 1, which is for ordinary ray and this is the ray number 2 for the extra ordinary ray, simply here too.

Now in ordinary ray we see only particular type of polarization which is perpendicular to that plane of the paper while in e ray, the polarization is in the plane of this paper, this horizontal lines represents the direction of vibration for e ray. Therefore, what you can see is that in case of o ray, the vibration, the direction of $\vec{D}$ is perpendicular to optic axis as well as wave vector $\vec{k}$ for o ray, the $\vec{D}$ is perpendicular to wave vector $\vec{k}$ and $\vec{D}$ is also perpendicular to optic axis, while for e ray what we see is that the $\vec{D}$ is although perpendicular to $\vec{k}$, but $\vec{D}$ is in the plane containing $\vec{k}$ and optic axis, $\vec{D}$ is in the plane which have wave vector $\vec{k}$ and optic axis both, this is for e ray. This is the difference between the o and e ray and these are very important points.

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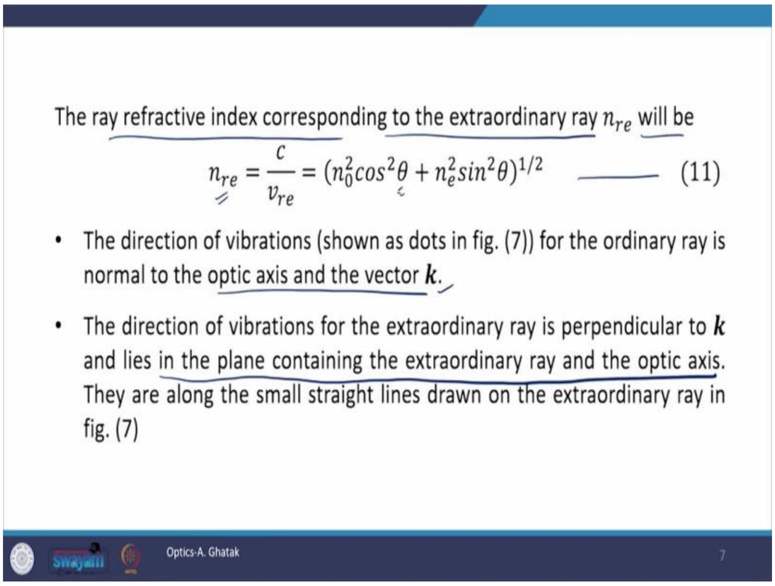

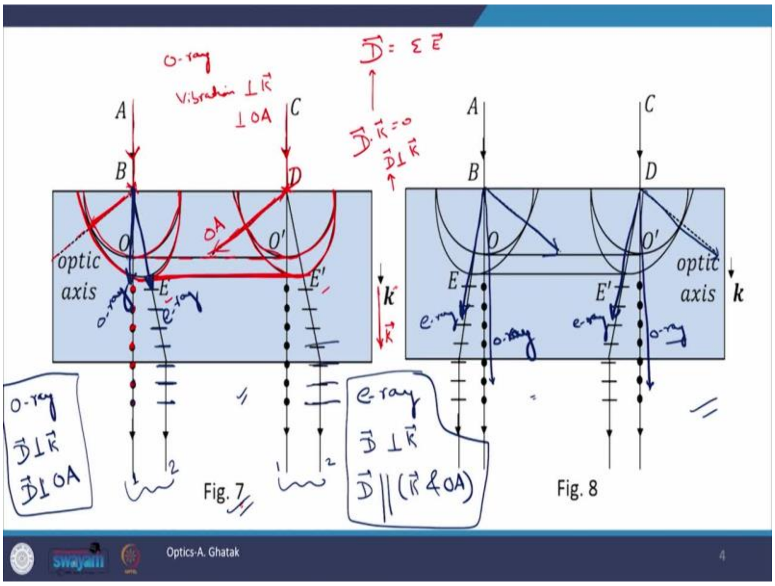

Now the ray refractive index corresponding to the extraordinary ray is given by equation number 11 which we have already seen. Now once we know the dependence of refractive index on θ which is angle made by the ray with optic axis. Once this relation is known using Fermat principle also you can decide the direction of propagation of e ray. Now the direction of vibration for o rays normal to the optic axis and the wave vector $\vec{k}$, this we have already discussed and the direction of vibration for e ray is perpendicular to $\vec{k}$ and lies in the plane containing the e ray and the optic axis, they are along small straight lines drawn on the extraordinary ray in figure 7, as is visible here, these are the direction of vibration in case of e ray. Now this is how we decide the direction of o and e ray after refraction on normal incidence in a birefringent medium.

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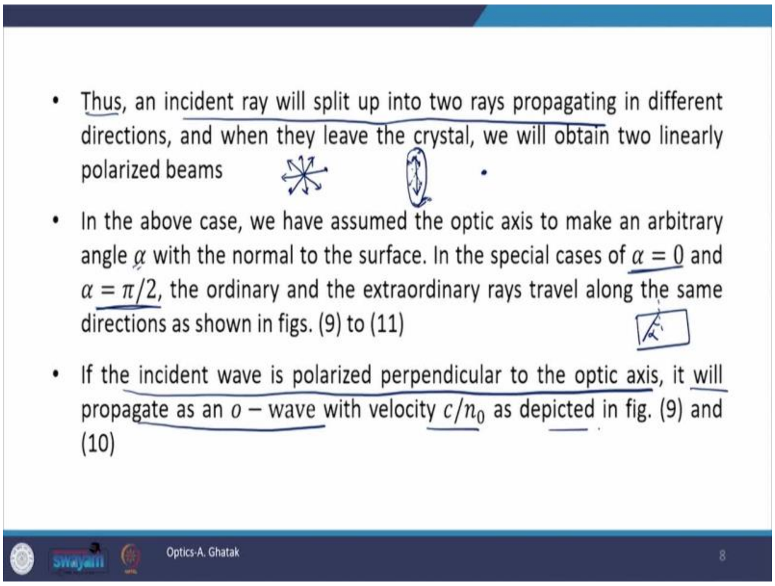

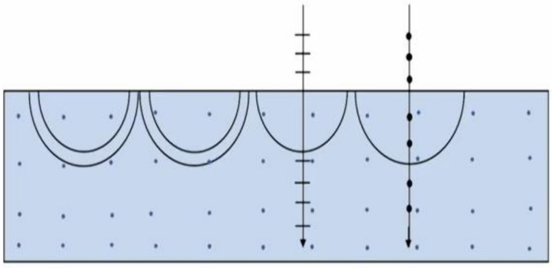

optic axis normal to the page

Fig. 10

- On the other hand, if the incident wave is polarized parallel to the optic axis, it will propagate as an $e-wave$ with velocity c/n_e. In fig. (11) the optic axis is normal to the surface, and both waves travel with the same velocity
- In configurations shown in figs. (9) and (10), although both waves travel in the same direction, they propagate with different velocities. This phenomena is used in the fabrication of quarter and half wave plates

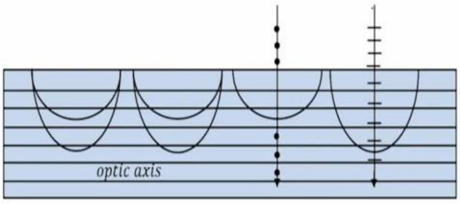

Fig. 9

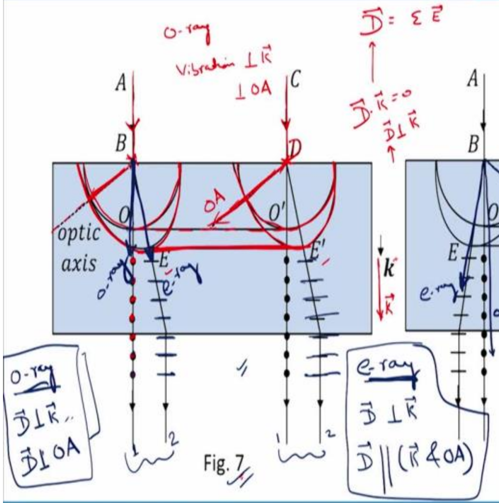

Fig. 7

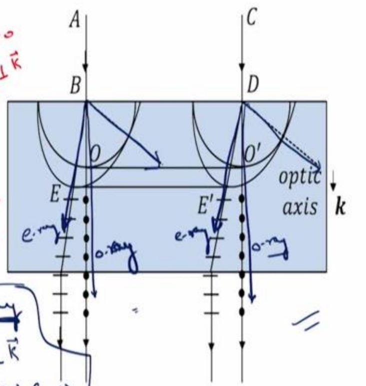

Fig. 8

Now we see that we can conclude that an incident ray, therefore, will split up into 2 rays propagating in different direction and when they leave the crystal, we will obtain 2 linearly polarized beams. Why 2 linearly polarized beam? Because we launched an non-polarized light and o ray contains polarization which is vibrating perpendicular to the plane of the paper while e ray contains a polarization which is vibrating in the plane of the paper. I would like to make 2 points very clear here and we have already discussed this also that polarized light is represented by this symbol and this double headed arrow represents the random direction of polarization the random orientation of the vibration plane of the electric field, the vibration direction of the electric field here.

Now whenever we say that light is vertically polarized or light is linearly polarized oscillating in the vertical direction then we represent it with this arrow vertical arrow. Now whenever we say linear polarization then we assume that the polarization which is perpendicular to this linear direction is removed, it may so happen that they are some polarization magnitude of electric field are present in these directions which are not along this dominant direction. But still since the dominant field is oscillating in a vertical direction we call it a linearly polarized light.

Similarly, the field which is vibrating perpendicular to the plane of the paper it is not so that only one field is there only one vector is there which is oscillating in that direction, there would be a component which is slightly deviated, but still that it would be assumed that the dominant direction of polarization is perpendicular to the plane of the paper. Now in the case above which we have discussed till now we have assumed that the optic axis makes an arbitrary angle with the normal to the surface. Say this angle is α, now this was the medium and this was the normal and optics axis is pointing in this direction and say this angle is α.

Now there are 2 special cases, the first case is when $\alpha = 0$ and the second case is when $\alpha = \pi/2$. Now in these 2 special cases, when optic axis is along the normal to the interface and when the optic axis is perpendicular to the normal to the interface, in these 2 cases o ray as well as e ray, they both travel along the same direction. And this is also shown in figure number 9 and 10 which we will discuss in detail.

Now if the incident wave is polarized, perpendicular to the optic axis, it will propagate as an o wave with velocity c/n_0 and this is also depicted in the figure number 9 and 10 and this is also clear from our previous analysis. Here too we have listed the properties of e ray as well as o ray, where it says that the direction of vibration would be perpendicular to $\vec{k}$ as well as OA for o ray, while direction of vibration will be perpendicular to $\vec{k}$ and it will lie in the plane which is made by wavevector $\vec{k}$ in optic axis, these are the properties for e ray.

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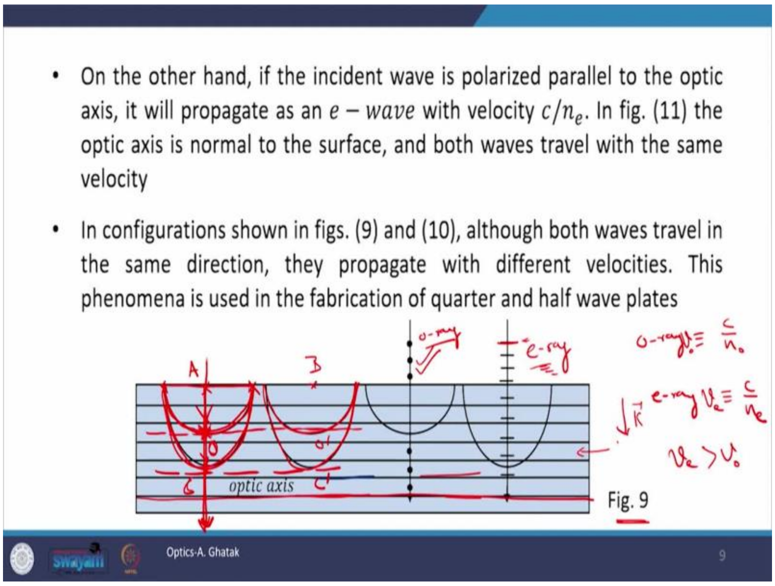

Now with this suppose we have a situation in which optic axis is along this line, these horizontal lines represent the optic axis direction, the orientation of optic axis. Now in this case what we see is that, since the optic axis is in horizontal direction if we launch an unpolarized light then what will happen is that along this horizontal direction the velocity of o and e ray would be the same. Therefore, if the birefringent medium is uniaxial negative crystal then from the point of incidence we will draw an sphere as well as ellipsoid of revolution and the ellipsoid would be oriented in such a way that its minor axis would be in the horizontal direction, why?

Because along this optic axis, the 2 velocities, velocities of o and e waves would be the same therefore, the sphere would touch the ellipse at these 2 points the horizontal point. The ellipse is related to o ray and in o ray we know the vibration D is perpendicular to $\vec{k}$ as well as optic axis and since optic axis is in the horizontal direction and $\vec{k}$ vector is in forward direction therefore, the only possibility for direction of vibration is perpendicular to the plane of the paper which is given here. Similarly for e ray, the direction would be in such a way that it is perpendicular to $\vec{k}$ and it lie in the plane which is created by optic axis and the wave vector $\vec{k}$ and this horizontal position of this vibration satisfies these 2 criteria, therefore this ray is o ray and the second ray is e ray.

Now here in this figure in figure number 9 you see that, in this direction both o and e ray, they both travel in this direction in the downward direction, the direction of propagation is downward, how to calculate? Suppose this is our point A and this is our point B, we draw 2 spheres considering A and B as a center and then draw a common tangent and then join A with this tangent point and say this point is O and this point is O' then join it with O and this would be the direction of OA.

Similarly draw 2 ellipsoid of revolution, draw a common tangent and say this is C and C' then you join A with C then AC direction will now represent the direction of e ray. It means both o ray and e ray, they are travelling in the same direction but o ray is travelling with a speed which is equal to c/n_0, while e ray is travelling with a speed which is equal to c/n_e. And we know that in negative uniaxial crystal, the velocity of e ray is larger this is v_e and this is v_o. Since v_e is larger than v_o therefore, a sphere would be within the ellipse.

Now it is also clear from the figure is that the 2 velocities are different and therefore, there is a difference between point O and C, O is earlier than C. And using this property using the differential velocities, we can fabricate quarter and half wave plates, which we will discuss in coming lecture.

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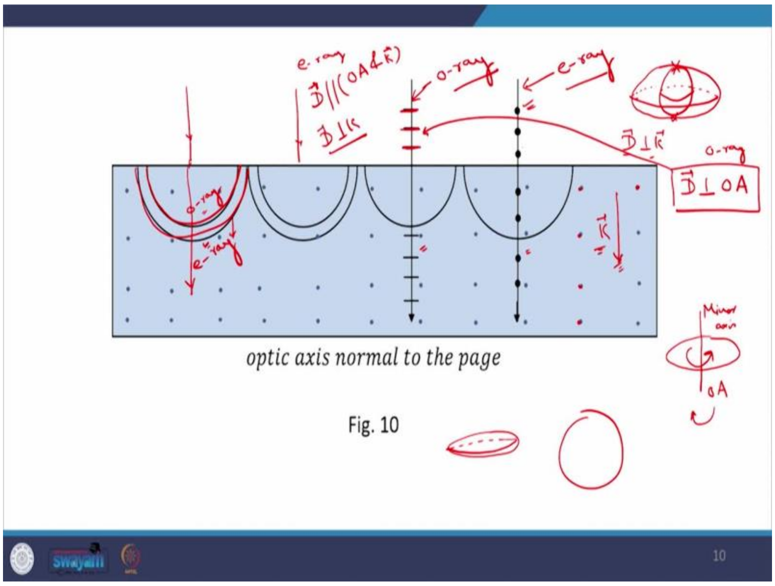

Now consider different case wherein the optic axis of the bifringent medium is pointing normal to the plane of the paper. Now these dots, they represents the direction of optic axis, which is normal to the plane of the paper which is perpendicular to the plane of the paper. Now in this case what happens is that, we will launch the waves and then for o ray we will draw a sphere while for e ray we will draw an ellipsoid of revolution. Now since optic axis is pointing inside the plane of the paper, in that inside direction only the 2 rays will exhibit the same velocities. Therefore, if we want to draw this in 3D then it will look like this, the figure will look something like this.

Now you see that these points are the points where the sphere and the ellipse will touch, while if you take the cross section of this then what you will get is inner circle and outer circle, inner circle is for o ray, while outer circle is for e ray. I repeat the inner circle is for o ray, we have a sphere, we have a ellipsoid of revolution and the minor axis of the ellipsoid is along the optic axis. And now you will to create the ellipsoid of revolution you will have to rotate the ellipse, we have an ellipse which is like this and this is the direction of minor axis, now to create ellipsoid of revolution, we will have to rotate the ellipse around optic axis and since minor axis is pointing along optic axis, the rotation would be like this or like this.

After this rotation, we will create if you see it on the top it will look like a circle and if you see it from the side then it will look like this. Now if you take the cross section of this ellipsoid of revolution you will get a bigger circle here which represents e ray. Now here too what you see is that both the rays are propagating in the same direction both o ray as well as e ray, you see here. Now which one will be the o ray and which one will be the e ray, for o ray the polarization the $\vec{D}$ should be perpendicular to $\vec{k}$ and $\vec{k}$ is in this direction and this is satisfied by both o and e ray.

For o ray, the $\vec{D}$ must also be perpendicular to the optic axis and this condition is being satisfied by this ray, because this vibration is perpendicular to both direction of $\vec{k}$ as well as the optic axis therefore, this ray is o ray now, while this ray you see that the oscillation direction is along optic axis and we know that for e ray $\vec{D}$ must be in plane which is spanned by optic axis and $\vec{k}$ and $\vec{D}$ must be perpendicular to $\vec{k}$ and for this ray.

We see that the vibration is perpendicular to the plane and which of course would be perpendicular to the $\vec{k}$ vector and this vibration is also contained in a plane which is spanned by $\vec{k}$ and optic axis, which contains both optic axis and vector $\vec{k}$ in this particular case is a plane which is like this which is perpendicular to the plane of the paper. If this is the plane of the paper, then the plane which contains both optic axis and $\vec{k}$ would be like this, this would be the plane which contains both OA optic axis or OA and wave vector $\vec{k}$. And you know that the direction of vibration is in this plane only therefore, the second ray would be e ray, this ray is now e ray. We will always exercise these 2 conditions to check whether the ray is o or e.

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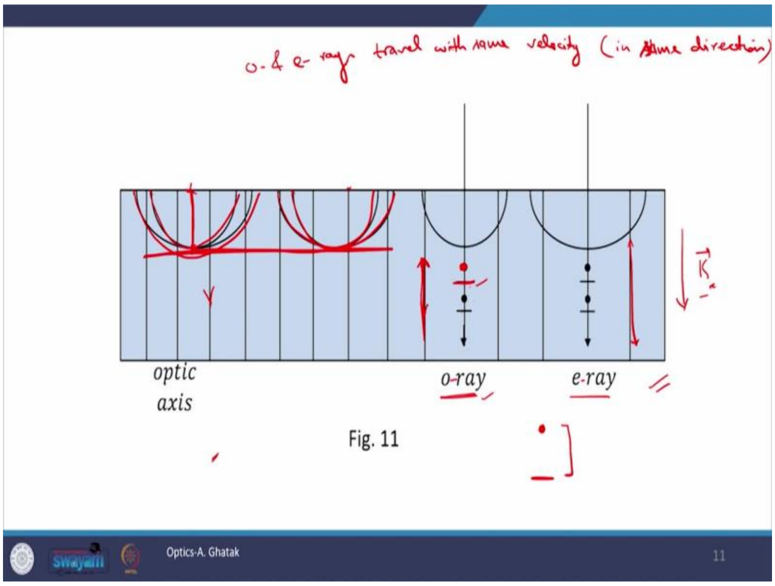

Now let us consider the third case where the optic axis is along this direction it is going through the material media. Now if the optic axis is pointing in this direction, and since wave will incident normally where vector $\vec{k}$ would be in this direction as is written here. Now since the optic axis direction is this, the velocities of o and e would be same at this point and this would be the semi minor axis of the ellipse. We will again draw a sphere and ellipse, this is the ellipse, and we will then draw a common tangent to this and now we see that the tangent which is common to both spheres is also common to both ellipsoid of revolution.

Therefore, in this particular case, in this particular orientation of optic axis both o and e ray will travel with the same velocity, o and e rays travel with same velocity and of course in same direction. Therefore, in this particular case what we understood is that both o and e ray travel with the same velocity and with the same direction. Now what you see that since the optic axis is pointing here in this particular direction and if we take a polarization which is perpendicular to the plane of the paper then this polarization would be perpendicular to the optic axis and this polarization is also perpendicular to the $\vec{k}$.

And you talk about this horizontal polarization, then horizontal polarization is also perpendicular to the optic axis as well as to the direction of $\vec{k}$, it means irrespective whether we are talking about dot polarization or horizontal line polarization, irrespective whether we talk about this polarization or this polarization, they both satisfies the criteria of being o ray as well as e ray. Therefore, in this particular case, we will have both type of polarization in o ray as well as e ray.

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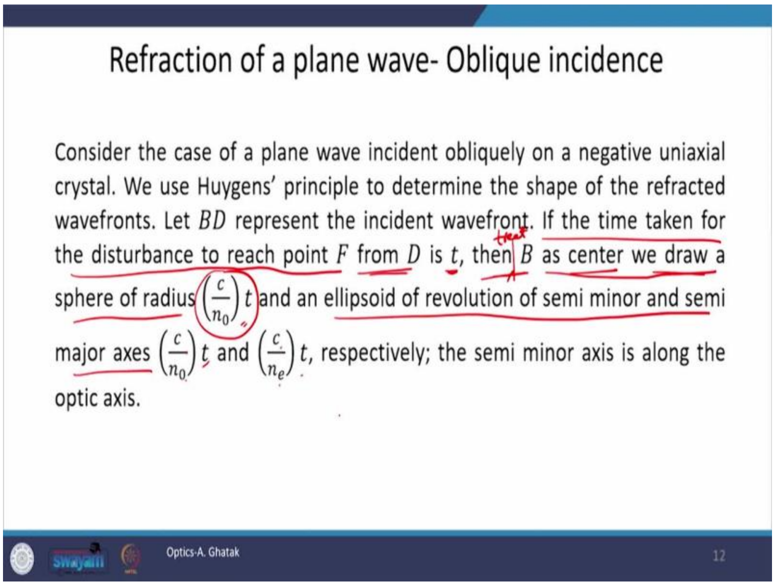

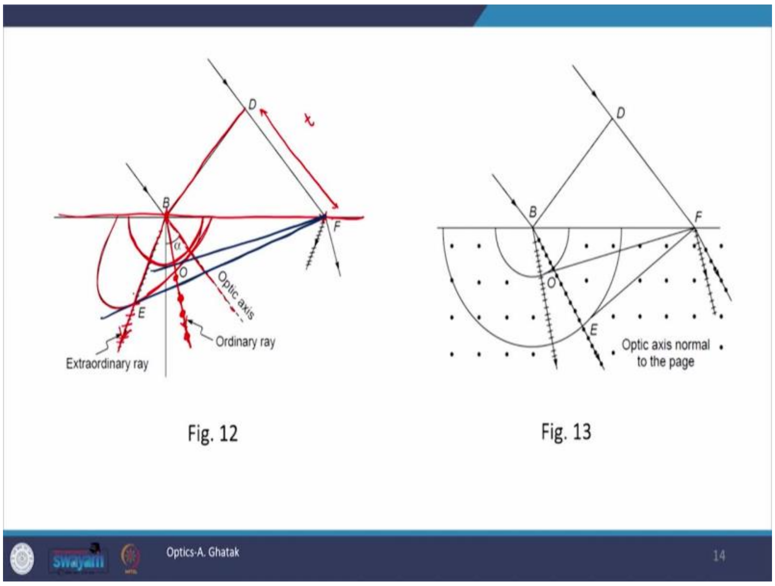

This is all for normal incidence, now we will look into oblique incidence a bit more complicated case. In this case, we will consider oblique incidence of a plane wave in on a negative uniaxial crystal and again similar to the previous case we will use Huygens principle to determine the shape of the refracted wavefront.

Now let us go to the picture. Now in this picture you see let us suppose that BD is the wavefront which is met to incident obliquely on this interface, BD is met to incident at some angle which is nonzero angle. And due to this oblique incidence, the point B of the wavefront falls earlier as compared to point D, say point D take some time, say this is equivalent to time t in reaching point F or in reaching the interface, a wave front a plane wavefront is made to incident obliquely at the interface of a double refracting medium and since wavefront is inclined, the lower part of the wavefront touches the interface first and the upper portion of the wavefront touches the interface a bit later. And say the upper portion of the wavefront touches the extreme upper portion of the wavefront touches the interface t time later.

Now if the time taken for the disturbance or for the wavefront to reach point F from D is t then B as a center we draw a sphere of radius $(c/n_0)t$. What we will do is that that we treat B as a center and then draw a sphere of radius $(c/n_0)t$ because by the time the D reaches F and the wave which has already incident at point B it would have travelled within the birefringent material medium. For o ray we will draw a sphere, the sphere is drawn here and say the direction of optic axis is along this dashed line. Therefore, for e ray for extraordinary ray, we will again draw ellipsoid of revolution and the ellipsoid would be such that the minor axis of the ellipsoid is along optic axis.

Now the ellipsoid of revolution of semi minor and semi major axes as $(c/n_0)t$ and $(c/n_e)t$ would be drawn, this is easy to draw once you know the length of the semi minor axis and major axis and we also know that the minor axis is along optic axis. With this what we will do is that we will follow the same procedure which we followed in the last topic, from point F we will draw tangent to the sphere as well as to the ellipsoid of revolution.

For a tangent let us pick different colors, this is tangent to the sphere and the second line this line is tangent to the ellipsoid. Now join point B to O, this line will represent the direction of propagation of o ray and this dot represents the vibration direction. Similarly, this line represents the direction of propagation of e ray and this horizontal line represents the direction of vibration.

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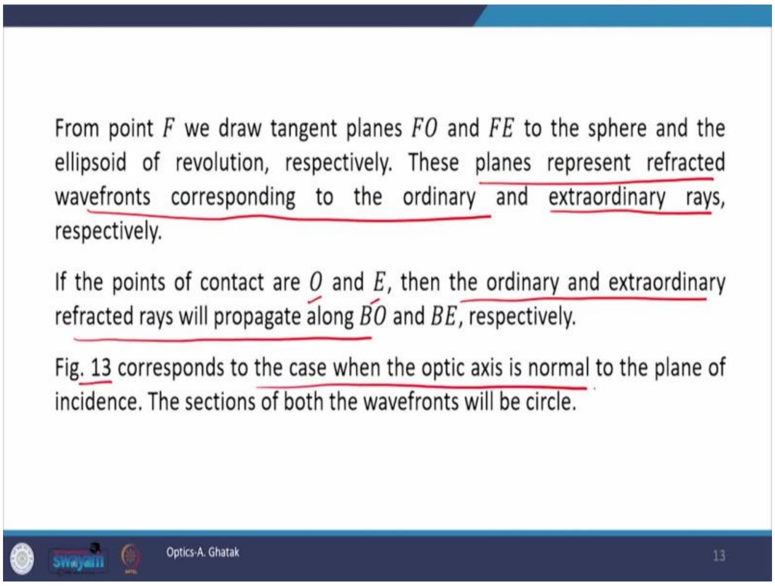
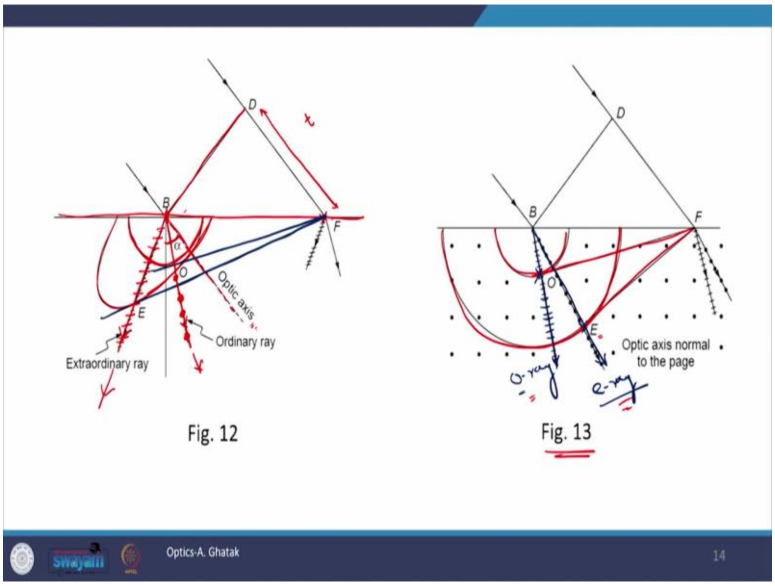




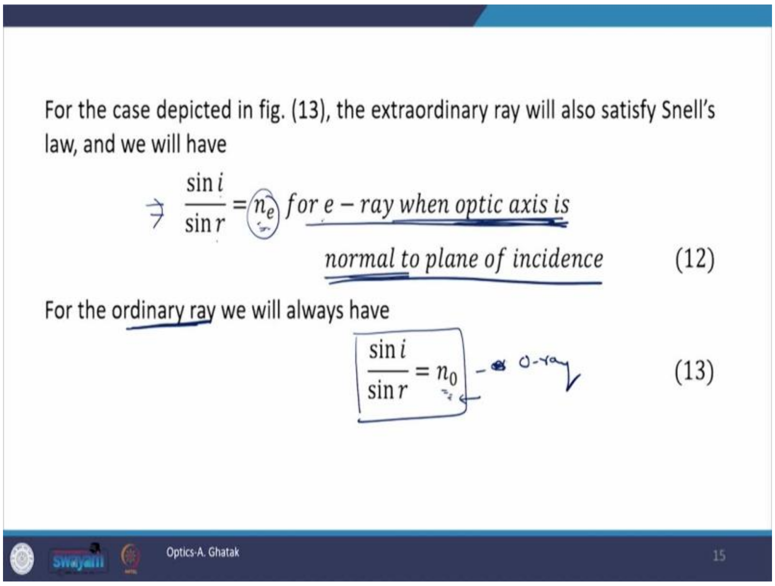
Now these planes, the tangent plane which is FO and FE, these planes represents the refracted wavefront corresponding to ordinary and extraordinary rays respectively. Now if the points of context are O and E, then the ordinary and extraordinary refracted ray will propagate along BO and BE respectively, which are shown here in this figure here, this is the direction of o ray propagation and this is direction of e ray propagation.

Now figure 13 correspond to the case when the optic axis is normal to the plane of the incidence this figure. Now in this particular case the optic axis was in the plane of the paper, this optic axis was oriented at angle α with respect to the normal to the interface. But now in the second figure, in figure number 13, we are considering a case where optic axis is again normal to the plane of the paper and we have already discussed this case in our previous slides.

And here we know that for both e and o ray we will get circles and for e ray the circle radius would be bigger, we will again draw a tangent from point F to these 2 different circles. The contact point for the inner circle is O while the contact point for the outer circle is E, if we join B with O and this represents the direction of propagation of e ray, when we join B with E then it represents the direction of propagation of e ray.

Now you see here the tangent from point F to the inner circle is at a point O, this is the tangent and therefore, this ray represents the direction of propagation of o ray, while the tangent to the outer circle from point F is FE and the contact point is E therefore, this direction represents the direction of e ray. Now you see here is that in o ray the vibration direction is represented by this horizontal line instead of the dots which represents the vibration into the plane of the paper. Now the polarization vibration direction is swiped which we have already discussed in the previous slides.

(Refer Slide Time: 38:49)



Now further case of figure 13, where optic axis is inside the plane of the paper, where optic axis is normal to the plane of the paper, the extra ordinary ray will also satisfy Snell's law and therefore, we will have this relation where $sini/sinr = n_e$ for e ray when optic axis is normal to the plane of the incidence, here make it a point, this is only true when the optical axis is normal to the plane of the paper or normal to the plane of the incidence. While the ordinary ray Snell's law is always valid and we can easily write $sini/sinr = n_0$. Observe the difference here on the right hand side we have n_e, while here in the case of o ray we have n_o here, this is for o ray and this is for e ray. Now this is all for refraction through birefringent medium. I conclude my lectures with this. Thank you for being with me. See you in the next class.