

**Networks, Signals and Systems**  
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**Lecture - 22**  
**Graph Theory (Contd....)**  
**Image Impedance, Iterative Impedance and Characteristic Impedance**

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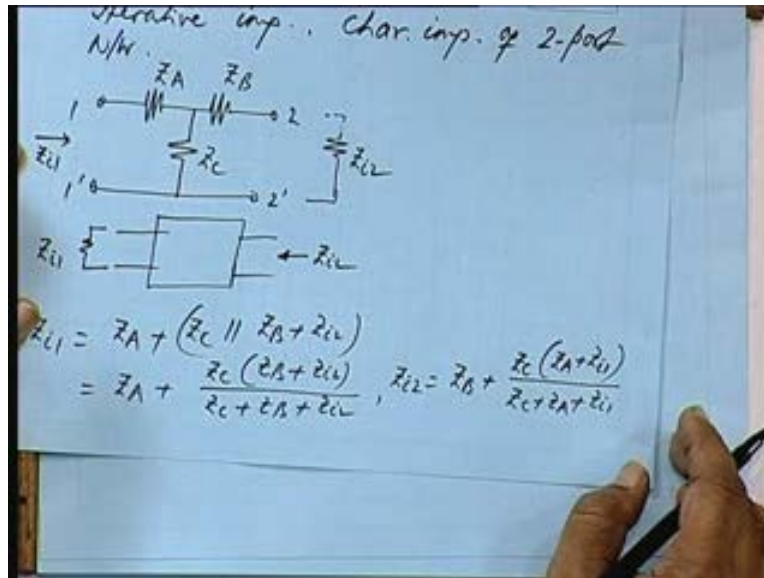


Today, we shall be discussing about image impedance, iterative impedance and finally the characteristic impedance of a 2 port network, characteristic impedance of a 2 port network. Now let us consider a general network having impedances  $Z_A$ ,  $Z_B$  and  $Z_C$  the ports are 1 and 2. Now image impedance we define, we define **image** image impedance as this if I load this side by an impedance  $Z_{i2}$  the impedance in from this side is  $Z_{i1}$  and if I load the same network, I show the network just by a block if on this side if I put  $Z_{i1}$  then impedance in from this side is  $Z_{i2}$  then  $Z_{i1}$  and  $Z_{i2}$  will be the image impedances for this network. So for a general network where  $Z_A$ ,  $Z_B$ ,  $Z_C$  are any certain values  $Z_{i1}$  and  $Z_{i2}$  will be different so there are 2 image impedances you look into the circuit from this end if I load on that side  $Z_{i2}$  then the impedance in is  $Z_{i1}$ , if I look at the network from this end and if I load it with  $Z_{i1}$  the impedance in is  $Z_{i2}$  mind you you cannot have these unique values with any set, you cannot have any combination so they are dependent on these values are dependent on  $Z_A$ ,  $Z_B$  and  $Z_C$ .

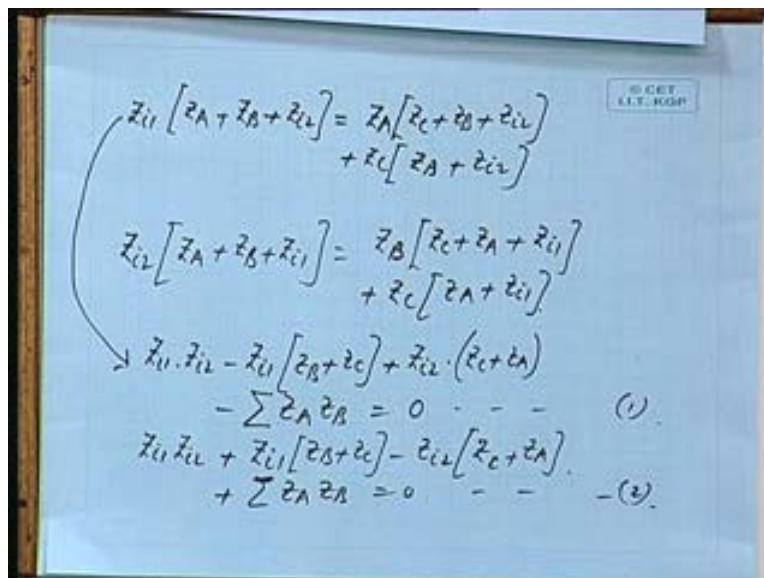
Let us what will be the relation between  $Z_{i1}$ ,  $Z_{i2}$  and these element values  $Z_A$ ,  $Z_B$  and  $Z_C$ . Now by definition you have got  $Z_{i1}$  the impedance in from this side is how much  $Z_A$  plus parallel combination of  $Z_C$  and  $Z_B$  plus  $Z_{i2}$ ,  $Z_C$  in parallel with  $Z_B$  plus  $Z_{i2}$  agreed. So

that gives me  $Z_A$  plus  $Z_C$  into  $Z_B$  plus  $Z_{i2}$  divided by  $Z_C$  plus  $Z_B$  plus  $Z_{i2}$  agreed. Similarly,  $Z_{i2}$  will be equal to  $Z_B$  plus  $Z_C$  into just replace a by b interchange a and b  $Z_C$  will be  $Z_A$  plus  $Z_{i1}$  divided by  $Z_C$  plus  $Z_A$  plus  $Z_{i1}$  agreed.

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So if I cross multiply  $Z_C$  plus  $Z_B$  plus  $Z_{i2}$  if I bring to the left side what do I get  $Z_{i1}$  sorry  $Z_{i1}$  into  $Z_A$  plus  $Z_B$  plus  $Z_{i2}$ ,  $Z_A$  plus  $Z_B$  if you add these it becomes  $Z_A$  plus  $Z_B$  plus  $Z_{i2}$  into  $Z_{i1}$  and this side you will get  $Z_A$ ,  $Z_C$  plus  $Z_B$  plus  $Z_{i2}$  plus  $Z_C$  into  $Z_B$  plus  $Z_{i2}$

correct me if I am wrong. See  $Z_{i1}$  into  $Z_C$  plus  $Z_B$ ,  $Z_{i1}$  into sorry,  $Z_{i1}$ ,  $Z_{i2}$ ,  $Z_{i1}$ ,  $Z_C$ ,  $Z_{i1}$ ,  $Z_B$  so actually this one has been written here first and similarly  $Z_{i2}$  into  $Z_A$  plus  $Z_B$  plus  $Z_{i1}$  equal to  $Z_B$  sorry  $Z_B$  into  $Z_C$  plus  $Z_A$  plus  $Z_{i1}$  plus  $Z_C$  into  $Z_A$  plus  $Z_{i1}$ . So from this you can write  $Z_{i1}$ ,  $Z_{i2}$  minus  $Z_{i1}$  into  $Z_B$  plus  $Z_C$  plus  $Z_{i2}$  into  $Z_C$  plus  $Z_A$  minus I can write sigma  $Z_A$ ,  $Z_B$  which means  $Z_A$ ,  $Z_B$ ,  $Z_B$ ,  $Z_C$  plus  $Z_C$ ,  $Z_A$ , so I am write writing in a compact form equal to 0.

Let me call it equation number1 similarly from the other1 we will get  $Z_{i1}$ ,  $Z_{i2}$  plus  $Z_{i1}$   $Z_{i1}$ ,  $Z_{i2}$  plus  $Z_{i1}$  into  $Z_B$  plus  $Z_C$  minus  $Z_{i2}$  into  $Z_C$  plus  $Z_A$  plus sigma  $Z_A$ ,  $Z_B$  equal to 0 it is so simple. So can you see you can manipulate these by adding and subtracting sorry this will be minus minus sigma  $Z_A$ ,  $Z_B$  equal to 0. So if I add these 2 what do I get twice  $Z_{i1}$ ,  $Z_{i2}$ ,  $Z_{i1}$  into  $Z_B$  plus  $Z_C$  plus  $Z_{i1}$  into  $Z_B$  plus  $Z_C$  so this will get cancelled similarly this will get cancelled. So I will get twice  $Z_{i1}$ ,  $Z_{i2}$  equal to twice sigma  $Z_A$ ,  $Z_B$  or  $Z_{i1}$ ,  $Z_{i2}$  equal to sigma  $Z_A$   $Z_B$  agreed. Similarly, subtracting 2 from1 we will eliminate these 2. So what do I get by subtracting I get  $Z_{i1}$  into  $Z_C$  plus  $Z_B$  equal to  $Z_{i2}$  into  $Z_C$  plus  $Z_A$  or  $Z_{i1}$  by  $Z_{i2}$  equal to  $Z_A$  plus  $Z_C$  divided by  $Z_C$  plus  $Z_B$  so from3 and4 if I take the product I will get  $Z_{i1}$  squared as  $Z_A$  plus  $Z_C$  by  $Z_B$  plus  $Z_C$  into sigma  $Z_A$ ,  $Z_B$  therefore  $Z_{i1}$  is square root of  $Z_A$  plus  $Z_C$  by  $Z_B$  plus  $Z_C$  into sigma  $Z_A$ ,  $Z_B$  it is so simple so you can see from here  $Z_{i2}$ .

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The image shows a person's hands writing mathematical equations on a whiteboard. The equations are as follows:

$$Z_{i2} [Z_A + Z_B + Z_{i1}] = Z_B [Z_C + Z_A + Z_{i1}] + Z_C [Z_A + Z_{i1}]$$

$$\rightarrow Z_{i1} \cdot Z_{i2} - Z_{i1} [Z_B + Z_C] + Z_{i2} \cdot (Z_C + Z_A)$$

$$- \sum Z_A Z_B = 0 \quad \dots \quad (1)$$

$$Z_{i1} Z_{i2} + Z_{i1} [Z_B + Z_C] - Z_{i2} [Z_C + Z_A]$$

$$\rightarrow \sum Z_A Z_B = 0 \quad \dots \quad (2)$$

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$$2 Z_{i1} Z_{i2} = 2 \sum Z_A Z_B$$
$$Z_{i1} Z_{i2} = \sum Z_A Z_B \quad (3)$$
$$Z_{i1} (Z_C + Z_D) = Z_{i2} (Z_A + Z_C) \quad (4)$$
$$\frac{Z_{i1}}{Z_{i2}} = \frac{Z_A + Z_C}{Z_C + Z_D}$$
$$Z_{i1} = \frac{Z_A + Z_C}{Z_A + Z_C} \sum Z_A Z_B$$
$$Z_{i1} = \sqrt{\frac{Z_A + Z_C}{Z_B + Z_C} \sum Z_A Z_B}$$

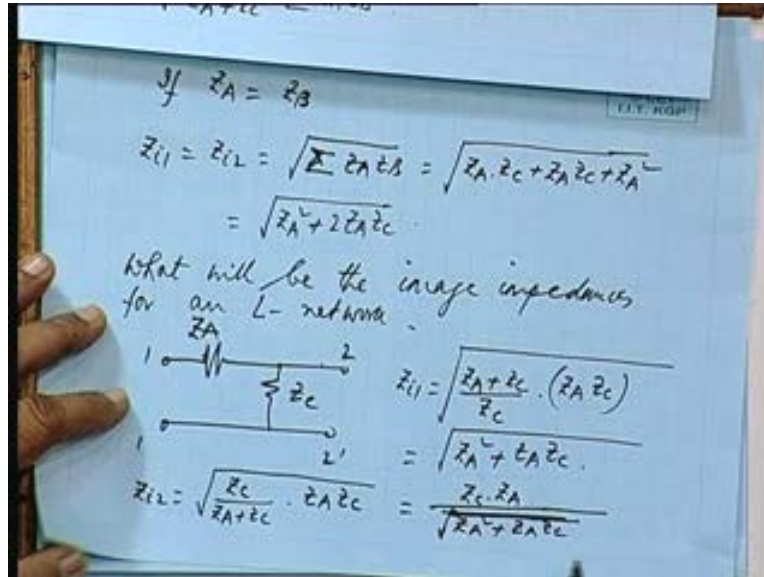
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$$\frac{Z_{i1}}{Z_{i2}} = \frac{Z_A + Z_C}{Z_C + Z_D} \quad (4)$$
$$Z_{i1} = \frac{Z_A + Z_C}{Z_A + Z_C} \sum Z_A Z_B$$
$$Z_{i1} = \sqrt{\frac{Z_A + Z_C}{Z_B + Z_C} \sum Z_A Z_B}$$
$$Z_{i2} = \sqrt{\frac{Z_A + Z_C}{Z_A + Z_C} \sum Z_A Z_B}$$

If you just look it look into the circuit from the other end its just replacing a by b, so it will be  $Z_B$  plus  $Z_C$  by  $Z_A$  plus  $Z_C$  into sigma  $Z_A$ ,  $Z_B$  agreed, so these are the 2 impedances. Now if you are having say  $Z_A$  equal to  $Z_B$ ,  $Z_A$  equal to  $Z_B$  then what do I get if  $Z_A$  is equal to  $Z_B$  if  $Z_A$  is equal to  $Z_B$  then it becomes a symmetric network so for a symmetric network here if I put  $Z_A$  equal to  $Z_B$  this will get cancel this will be 1 and sigma  $Z_A$ ,  $Z_B$  will be therefore  $Z_{i1}$  is equal to  $Z_{i2}$  will be square root of sigma  $Z_A$ ,  $Z_B$

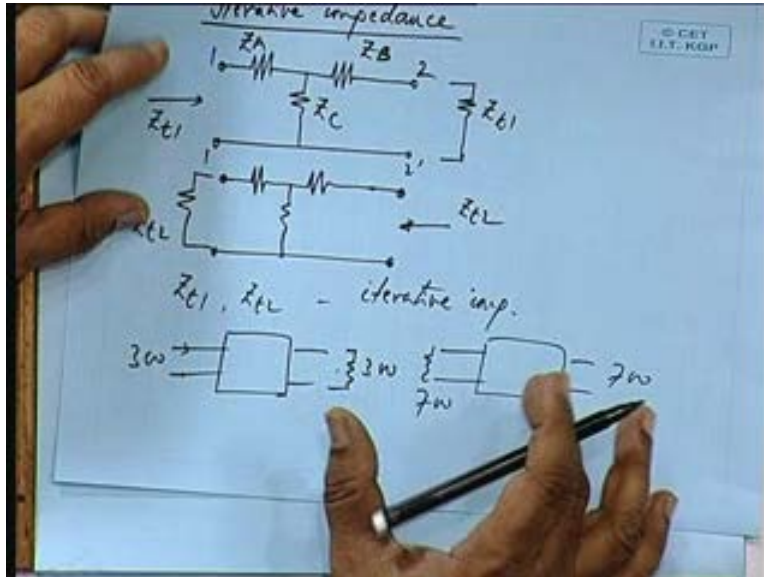
which is square root of  $Z_A, Z_C$  plus  $Z_A, Z_C$  plus  $Z_A$  squared that is  $Z_A$  squared plus twice  $Z_A, Z_C$  okay.

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Now can you find out for an asymmetrical I network what will be the image impedances for an I network, I network means suppose you are having this network what will be for an I section what will be the image impedances you are given  $Z_B$  equal to 0, so put  $Z_B$  equal to 0 here what do I get  $Z_{i1}$  will be  $Z_A$  plus  $Z_C$  by  $Z_C$  and what will be  $Z_A, Z_B$  these terms only  $Z_A, Z_C$  because  $Z_C$  into 0 is 0  $Z_A$  into 0 is 0 so multiplied by  $Z_A, Z_C$  which finally gives me  $Z_A$  squared plus  $Z_A, Z_C$  sorry, excuse me what will be  $Z_{i2}, Z_{i2}$ .

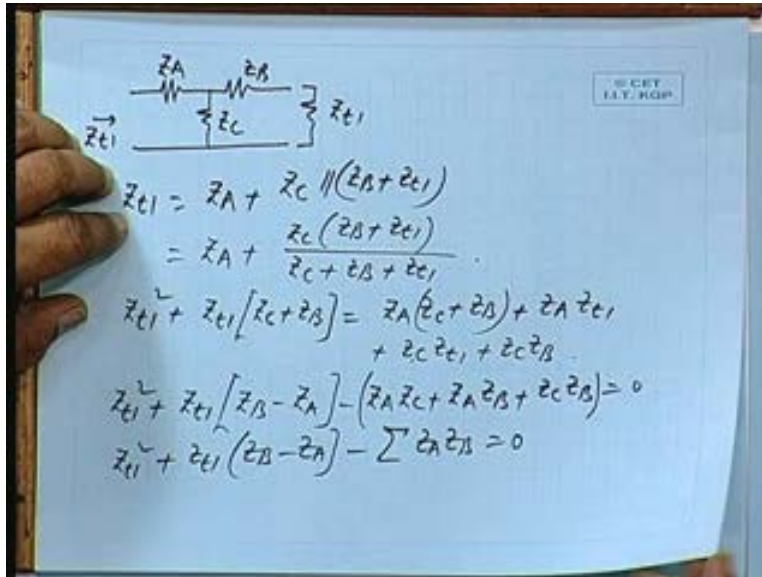
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If I put  $Z_B$  equal to 0 is  $Z_C$  by  $Z_A$  plus  $Z_C$  so it will be  $Z_C$  by  $Z_A$  plus  $Z_C$  into  $Z_A$ ,  $Z_C$  which means  $Z_C$  into  $Z_A$  I can multiply by  $Z_A$  and also multiply  $Z_A$  here in the denominator. So this will be  $Z_A$  square plus  $Z_A, Z_C$  under root is that alright. So this will be the expression for  $Z_{i2}$  this will be the expression for  $Z_{i2}$  and  $Z_{i1}$  is this much for an I section. Now let us define an iterative impedance, what do you mean by an iterative impedance for a network like this. Once again we will take a section  $Z_A, Z_B, Z_C$  by iterative impedance we mean if I terminate this side by say  $Z_{t1}$  then impedance in from this side is also  $Z_{t1}$  again if I terminate this side by  $Z_{t2}$  then impedance in from this side is  $Z_{t2}$  so  $Z_{t1}$  and  $Z_{t2}$  are the 2 iterative impedances mind you you cannot truncate you cannot terminate this by any impedance and see from this end the same impedance value, there is a particular impedance by which you can terminate and you can observe from this end the same impedance value.

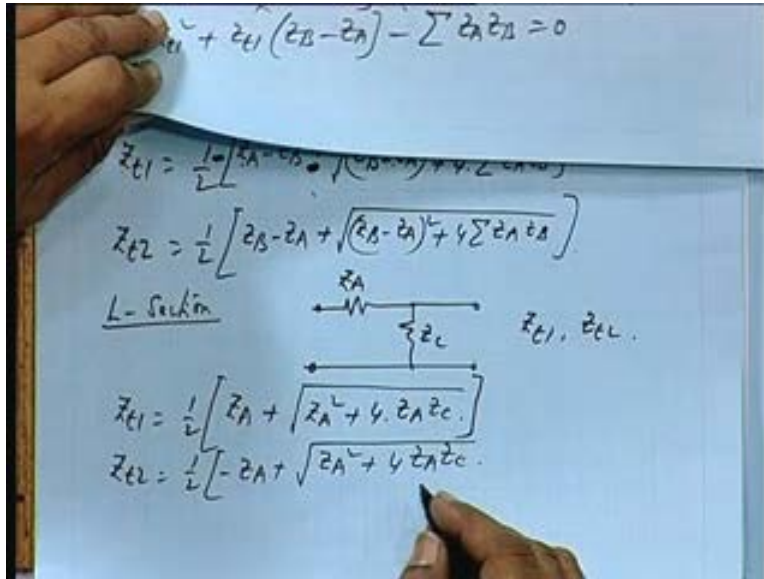
So what is that particular impedance that you want similarly for  $Z_{t2}$  what are these 2 impedance values okay so now you will be taking say we may say if I replace if I **trunc** terminate this by 300 ohms impedance in from this side is also 300 ohms if I terminate this side by 700 ohms impedance in from this side is also 700 ohms then this 300 and 700 are the 2 iterative impedance values for a given network.

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So let us take once again from definition  $Z_{t1}$  impedance in is  $Z_{t1}$  so  $Z_{t1}$  will be equal to  $Z_A$  plus parallel combination of  $Z_B$  plus  $Z_{t1}$  with  $Z_C$  okay. So  $Z_{t1}$  is equal to  $Z_A$  plus parallel combination of  $Z_C$ ,  $Z_B$  plus  $Z_{t1}$  which means  $Z_A$  plus  $Z_C$  into  $Z_B$  plus  $Z_{t1}$  divided by  $Z_C$  plus  $Z_B$  plus  $Z_C$  if I multiply again cross multiplication will give me  $Z_{t1}$  squared plus  $Z_{t1}$  into  $Z_C$  plus  $Z_B$  on this side, on this side I have got  $Z_A$ ,  $Z_C$  plus  $Z_B$  plus  $Z_A$ ,  $Z_{t1}$  plus  $Z_C$ ,  $Z_{t1}$  plus  $Z_C$ ,  $Z_B$  if I transform everything if I transfer everything on this side then you will get  $Z_{t1}$  squared plus  $Z_{t1}$  into  $Z_C$  plus  $Z_B$  minus  $Z_C$  minus  $Z_A$ . So it becomes  $Z_B$ ,  $Z_C$  will go so  $Z_B$  minus  $Z_A$  correct me if I am wrong okay minus  $Z_A$ ,  $Z_C$ ,  $Z_A$ ,  $Z_C$  minus  $Z_A$ ,  $Z_B$  minus  $Z_C$ ,  $Z_B$  equal to 0 or I can put all of them as plus okay that is  $Z_{t1}$  squared plus  $Z_{t1}$  into  $Z_B$  minus  $Z_A$  minus sigma  $Z_A$ ,  $Z_B$  equal to 0. So how much is  $Z_{t1}$  they can straight away solve these quadratic to get  $Z_{t1}$ ,  $Z_{t1}$  will be minus of b.

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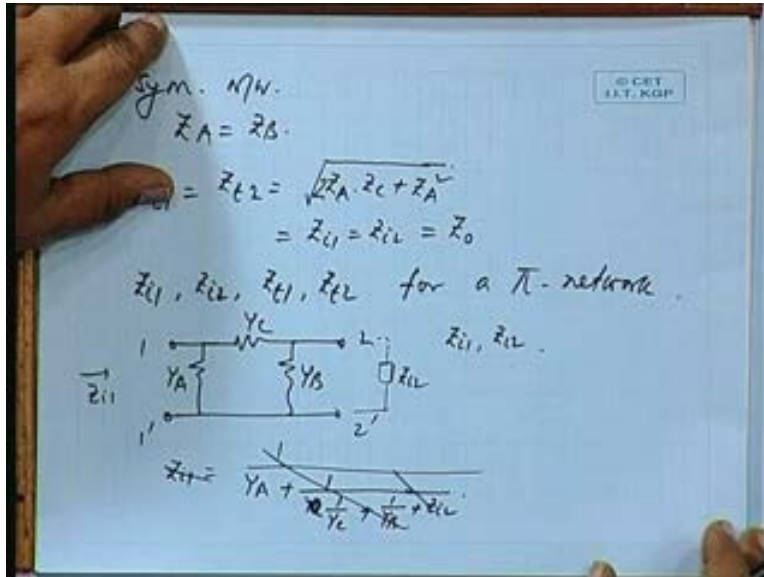
So  $Z_A$  minus  $Z_B$  plus minus root over of  $Z_B$  minus  $Z_A$  whole squared minus  $4z_C$ , so minus and minus make it will make it plus  $4 \sigma Z_A, Z_B$  divided by 2 okay. So this is the expression for  $Z_{t1}$ , now can you write in the in a similar manner the expression for  $Z_{t2}$  is very simple replace interchange  $Z_A$  and  $Z_B$ , so this will become  $Z_B$  minus  $Z_A$ . Now plus minus if I consider the minus value you can see obviously this is the quantity which is larger than  $Z_B$  minus  $Z_A$  alright. So you are extracting a quantity which is more than this. So it will be giving you negative values of  $Z_t$ , so that negative solution is ruled out, so we will consider only the plus value okay.

So similarly this one will be  $Z_B$  minus  $Z_A$  whole squared plus  $4 \sigma Z_A, Z_B$  if we have if we have an l section that means a section like this what will be this is  $Z_A$  this is  $Z_C$  what will be  $Z_{t1}$  and  $Z_{t2}$  then put  $Z_B$  equal to 0. So that will give me  $Z_{t1}$  equal to half of  $Z_A$  plus root over of  $z_s$  squared plus  $4$  into  $Z_A, Z_C, Z_{B0}$  so  $Z_A, Z_B, Z_B, Z_C, Z_C, Z_A$  that will give you just  $Z_C, Z_A$  and  $Z_{t2}$  will be half of  $Z_B$  minus  $Z_A$  so minus  $Z_A$  plus root over of  $Z_A$  squared plus  $4 Z_A, Z_C$  okay.

Now if you have a symmetric network, for a symmetric network  $Z_A$  equal to  $Z_B$ . So what you get if  $Z_A$  is equal to  $Z_B$  then this will be 0 and this will be 0, this will be 0. So it is square root of  $4 \sigma$  this and half 4 and half will get cancel. So it will be  $\sigma Z_{t1}$  equal to  $Z_{t2}$  equal to  $\sigma Z_A, Z_C$  plus 2 times  $Z_A, Z_C$  plus  $Z_A$  squared  $\sigma Z_A, Z_B$  becomes  $Z_A$  square  $Z_A$  into  $Z_A$  plus  $Z_A$  into  $Z_C$  plus  $Z_A$  into  $Z_C$ , so that gives me this which is nothing but  $Z_{i1}$  or  $Z_{i2}$ , we call this that is for a symmetrical network the image impedance and the iterative impedance will be identical and that will call as characteristic impedance some people write  $Z_o$  some people write  $Z_C$ .

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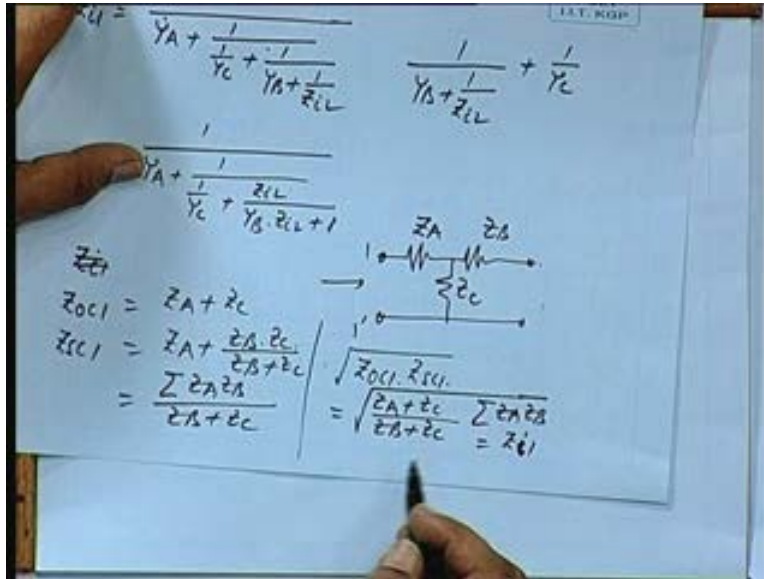




So the characteristic impedance refers to a symmetric network and that is equal to  $Z_s$  squared plus twice  $Z_A$ ,  $Z_C$  under root. You can also find out the image and iterative impedances for a phi network that is all so now it is interesting derivation for a phi network. If we consider  $Y_A$ ,  $Y_B$  and  $Y_C$  then what would be  $Z_{i1}$  and  $Z_{i2}$ , you can see for yourself any phi network can be converted to an equivalent t network and you can divide it or straight away from definition suppose this is terminated by  $Z_{i2}$  and impedance in is  $Z_{i1}$  so go from the definition  $Z_{i1}$  is equal to this admittance plus this total admittance if you invert that will be giving you  $Z_{i1}$  so  $Z_{i1}$  can be written as  $1$  by  $Y_A$  plus admittance of this which is  $1$  by impedance  $Y_C$  how much is it  $1$  by  $1$  by  $Y_C$  plus  $1$  by  $Y_B$  plus  $Z_{i2}$ .

So this impedance is  $Z_{i2}$  plus so sorry  $1$  by  $Z_{i2}$  admittances are to be added sorry, I write here separately  $Z_{i1}$  is  $1$  over the admittance  $Y_A$ , admittance you add  $Y_A$  plus this admittance and what is this admittance it is  $1$  by this impedance what is this impedance  $1$  by  $Y_C$ , is it not. This whole thing I am considering as an impedance and what is this impedance this admittance plus this admittance that is  $Y_B$  plus  $1$  over  $Z_{i2}$  is this this admittance so inverse of that is the impedance this is to be added with this impedance which is  $1$  by  $Y_C$ .

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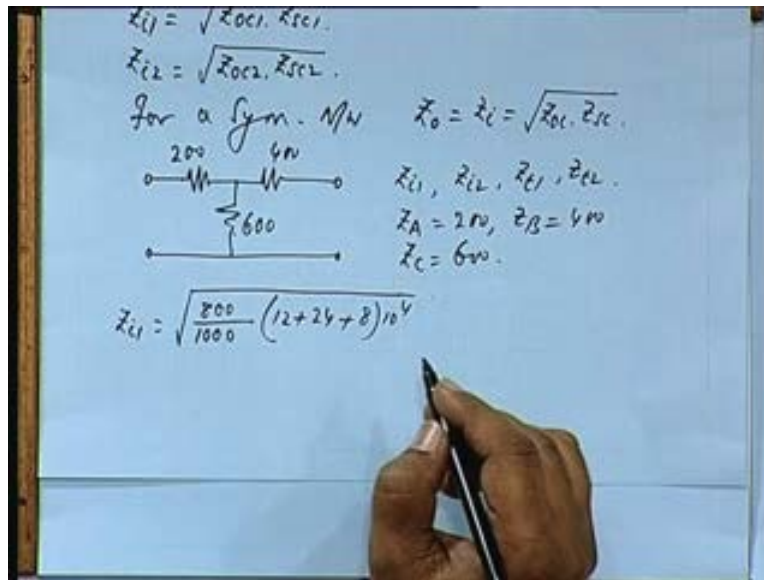


So this total impedance if I take the inverse of that that becomes the admittance, so it will be  $1/Y_C$  plus  $1/Y_B$  plus  $1/Z_{i2}$ . Now you can simplify this  $Z_{i2}$  divided by  $Y_B$ ,  $Z_{i2}$  plus 1 and so on you can simplify and then finally you get a relationship between  $Z_{i1}$  and  $Z_{i2}$  for the impedance seen from this side similarly, impedance seen from this side you establish a relation between  $Z_{i1}$  and  $Z_{i2}$  and in the same manner you can evaluate  $Z_{i1}$  and  $Z_{i2}$ .

One interesting relation is you can verify for yourself  $Z_{i1}$ , let us go back to once again to this network if I perform from this end and open circuit and short circuit test that is keep this open  $Z_A$ ,  $Z_C$  and  $Z_B$ . So  $Z$  open circuit seen from this side how much is it  $Z_A$  plus  $Z_C$  okay short circuit this, seen from this side if I short circuit it how much is it  $Z_A$  plus  $Z_B$ ,  $Z_C$  by  $Z_B$  plus  $Z_C$  agreed which means  $Z_A$ ,  $Z_B$  plus  $Z_A$ ,  $Z_C$ ,  $Z_B$ ,  $Z_C$  so  $\sigma Z_A$ ,  $Z_B$  divided by  $Z_B$  plus  $Z_C$ . If I take the product  $Z$  open circuit into  $Z$  short circuit 1 okay how much is it? How much is it? If I multiply by this it is and then take the square root  $Z_A$  plus  $Z_C$  divided by  $Z_B$  plus  $Z_C$  into  $\sigma Z_A$ ,  $Z_B$  is it not  $Z$  open circuit and  $Z$  short circuit seen from the same terminal pair gives me this product and then if I take the square root what is this you can identify this as  $Z_{t1}$  sorry  $Z_{i1}$ . Similarly, seen from this side it will be  $Z_B$  plus  $Z_C$  by  $Z_A$  plus  $Z_C$  so that will give me  $Z_{i2}$ .

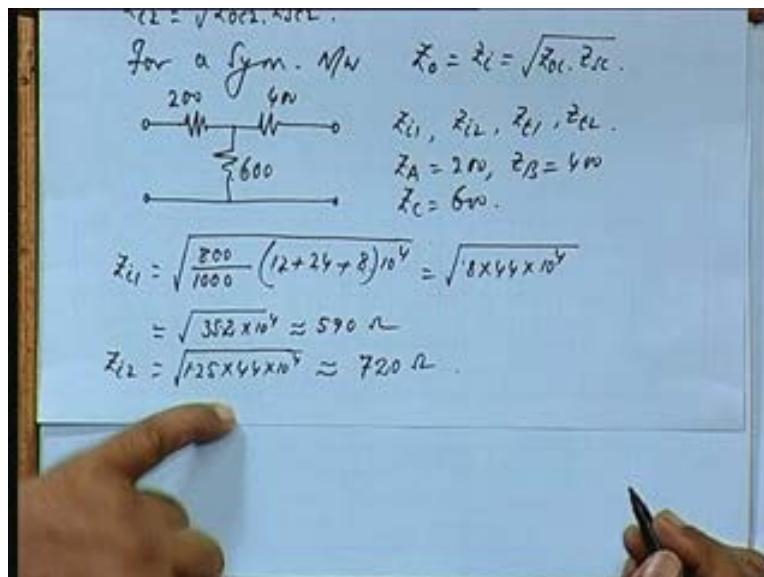
So  $Z_{i1}$  is  $Z$  open circuit 1 into  $Z$  short circuit 1  $Z_{i2}$  is  $Z$  open circuit 2 and  $Z$  short circuit 2 these are very interesting result and for a symmetric network,  $Z$  characteristic equation characteristic impedance  $Z_0$  which is  $Z_i$  that will be equal to  $Z$  open circuit into  $Z$  short circuit because  $Z_{oc1}$  and  $Z_{oc2}$  they become same for a symmetric network okay.

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Let us consider some simple values say this is 200, this is 400 and this is 600 what will be  $Z_{i1}$ ,  $Z_{i2}$ ,  $Z_{t1}$  and  $Z_{t2}$  okay. So we have got  $Z_A$  equal to 200  $Z_B$  equal to 400 and  $Z_C$  is equal to 600, so  $Z_{i1}$  will be equal to root over of  $Z_A$  plus  $Z_C$  600 plus 200, 800 divided by 600 plus 400, 1000 into sigma  $Z_A, Z_B$  be 200 into 600, 6 2's are 12, 6 4s are 24 plus 4 2s are 8 into 10 to the power 4 do you agree.

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So that gives me .8 into 12 plus 24, so 12 plus 8, 20 plus 20 into 44 into 10 to the power 4 do you agree, so that is approximately 35.2, 44 into .8 into 10 to the power 4. So 35.2 is approximately say 595.9 into 10 to the power 2 so 590 ohms approximately. Similarly  $Z_{i2}$

will be just these 2 figures you will interchange so 10 by 8 which means 1.25 into 44 into 10 to the power 4 which means approximately 44, if I assume this to be 11.

So it will be 2 20 sorry, 11, 44 into 1.25, so that gives me 5 by 4 so 55 square root of 55 is so 7.3 or 7.2, so 720 ohms. So these are the 2 values of  $Z_{i1}$  and  $Z_{i2}$  if you ask me what will be the iterative impedances  $Z_{t1}$  and  $Z_{t2}$ , so  $Z_{t1}$  if you just substitute these values in  $Z_{t1}$  and  $Z_{t2}$  how much did you get  $Z_{t1}$  was half of  $Z_A$  minus  $Z_B$  plus root over  $Z_B$  difference  $Z_A$  whole squared sorry plus 4 sigma  $Z_A$ ,  $Z_B$ . So how much will it come to half  $Z_A$  that was 200 minus 400 plus root over of 200 minus 400, so 200 squared plus 4 into  $Z_A$ ,  $Z_B$ ,  $Z_B$ ,  $Z_C$  plus  $Z_C$ ,  $Z_A$  that was 44 into 10 to the power 4.

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The image shows a whiteboard with handwritten mathematical steps for calculating iterative impedances. The steps are as follows:

$$Z_{t1} = \frac{1}{2} \left[ Z_A - Z_B + \sqrt{(Z_B - Z_A)^2 + 4 Z_A Z_B} \right]$$

$$= \frac{1}{2} \left[ (200 - 400) + \sqrt{(200)^2 + 4 \times 44 \times 10^4} \right]$$

$$= \frac{1}{2} \left[ -200 + 10 \sqrt{4 + 176} \right]$$

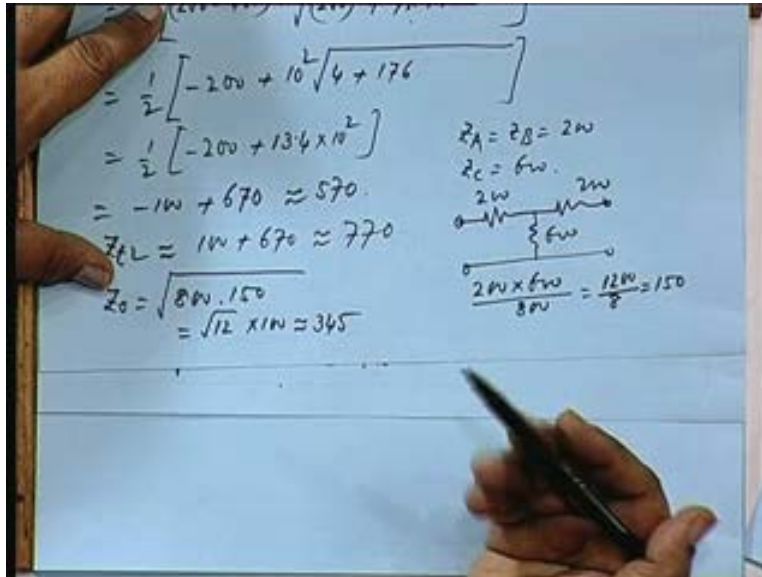
$$= \frac{1}{2} \left[ -200 + 13.4 \times 10^2 \right]$$

$$= -100 + 670 \approx 570$$

$$Z_{t2} = 100 + 670 \approx 770$$

So half of minus 200 plus I can always take 10 to the power 2 outside, inside I am left with 2 2s are 4 plus 4 into 44, 176, correct me if I am wrong. So that is half minus 200 plus 176 plus 4, 180 is that alright, 200 squared 10 to the power 4, so 176, 180 square root of 180 is 13.5, say 13.4 into 10 to the power 2. So that is this is an approximate figure minus 100 plus 680, so 13, 670, so that is 570 and  $Z_{t2}$  is everything is same except that  $Z_A$  and  $Z_B$  will be just interchanging their positions.

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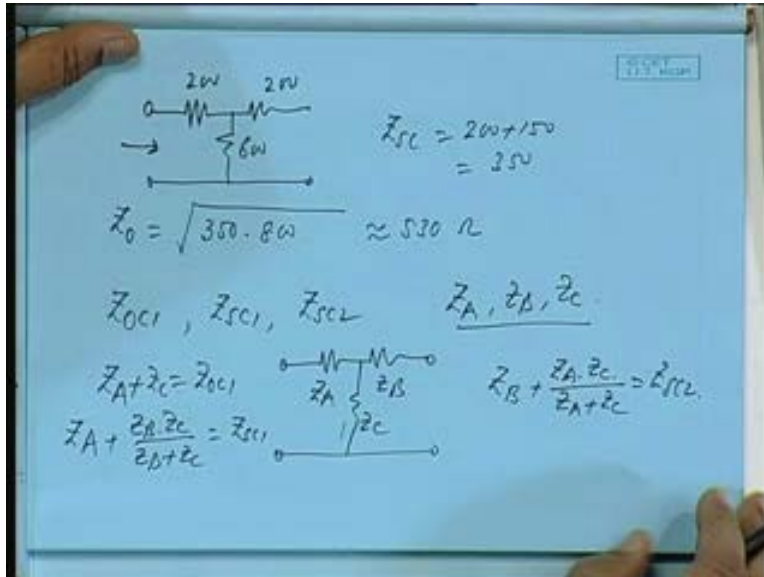


So it will be of minus 200 it will be plus 200, so it will be 100 plus 670 so that is 770. So these are the 2 values for  $Z_{t1}$  and  $Z_{t2}$  if I have  $Z_A$  equal to  $Z_B$  say 200, 200 and 600, if  $Z_A$  equal to  $Z_B$  equal to say 200 and  $Z_C$  equal to 600 then you have got a network like this what will be the characteristic impedance, this is 200, this is 200, this is 600. You can either put those values here in the expressions or you can apply short circuit and open circuit test directly.

So root over of if I have an open circuit test it is 600 plus 200, 800 and if I apply a short circuit test 200 plus parallel combination of 200 and 600. So it will be how much is 200 into 600 by this plus this is 800, so 0s will go 6 2s are 1200 by 800, 1200 by 8 so 150 so 800 into 150 square root of that okay 1, 2, 3, 4.

So 8 into 1.5 is root over of 12 into 100 that is approximately 34.5, so 345, so in that case the characteristic impedance is calculated directly from the short circuit and open circuit test, you can also substitute the values in the general expression okay. Thank you very much, we shall continue with this in the next class and then we will get into the calculation of propagation constant thank you very much. Sorry, we forgot to include this 200 ohms resistance here actually this was 600 this was 200 and this was 200, so the short circuit impedance as in from this side would be 200 plus 150.

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So it should be 350, so  $z$  characteristic equation, characteristic impedance will be 350 into 800 and that is approximately 530 ohms and not 345, I am extremely sorry please make this correction now suppose we have a question. Suppose you are given  $z$  open circuit1  $z$  short circuit 1 and  $z$  short circuit 2 these 3 values are given can you determine, can you determine  $Z_A, Z_B$  and  $Z_C$  is very simple. Here you can see the network is like this  $Z_A, Z_B$  and  $Z_C$ , so if you are measuring the short circuit impedance say open circuit impedance then  $Z_A$  plus  $Z_C$  is equal to  $Z_{oc1}$ .

Similarly,  $Z_A$  plus  $Z_B, Z_C$  parallel combination  $Z_B$  into  $Z_C$  by  $Z_B$  plus  $Z_C$  that is equal to  $z$  short circuit 1 similarly, seen from this side  $Z_B$  plus  $Z_A, Z_C$  parallel combination  $Z_A, Z_C$  by  $Z_A$  plus  $Z_C$  these are parallel combination of  $Z_A$  and  $Z_C$  so that is equal to  $z$  short circuit 2. So from these 3 given conditions you have to determine  $Z_A, Z_B$  and  $Z_C$ , so one can solve like this  $Z_B$  plus  $Z_C$  equal to you can verify for yourself  $Z$  short circuit 2 by  $z$  short circuit 1 okay,  $z$  short circuit 2 by  $Z$  short circuit 1 that will be see the numerator is  $Z_A, Z_B, Z_C, Z_C, Z_A$  so that will get cancelled so  $Z_A$  plus  $Z_C$  and  $Z_B$  plus  $Z_C$  these 2 will be involved here and then  $Z_A$  plus  $Z_C$  will get cancelled here.

So this product will give me this say  $z$  dashed and then  $Z_A$  plus  $Z_C$  is known that is  $Z_{oc1}$  so you can evaluate from here  $Z_C$  if you can little bit of manipulation will give you  $Z_C$  equal to  $Z_{oc1}$  squared into  $Z_{sc2}$  by  $Z_{sc1}$  minus  $Z_{oc1}$  into  $Z_{sc2}$ . Now I leave it to you to compute and verify this relation it is very simple, it is a question of only elimination of the terms  $Z$ , I can take common  $Z_{oc1}, Z_{sc2}$ . So I am left with  $Z_{sc1}$  by  $Z, Z_{oc1}$  by  $Z_{sc1}$  minus 1 under root of this into under root of this. So therefore  $Z_A$  can be evaluated  $Z_{oc1}$  minus  $Z_C$  which is computed from here similarly,  $Z_B$  is  $z$  dash which you have already computed minus  $Z_C$ , so this is how you can compute  $Z_A, Z_B$  and  $Z_C$ .

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$$Z_B + Z_C = Z_{oc1} \frac{Z_{sc2}}{Z_{sc1}} = Z'$$

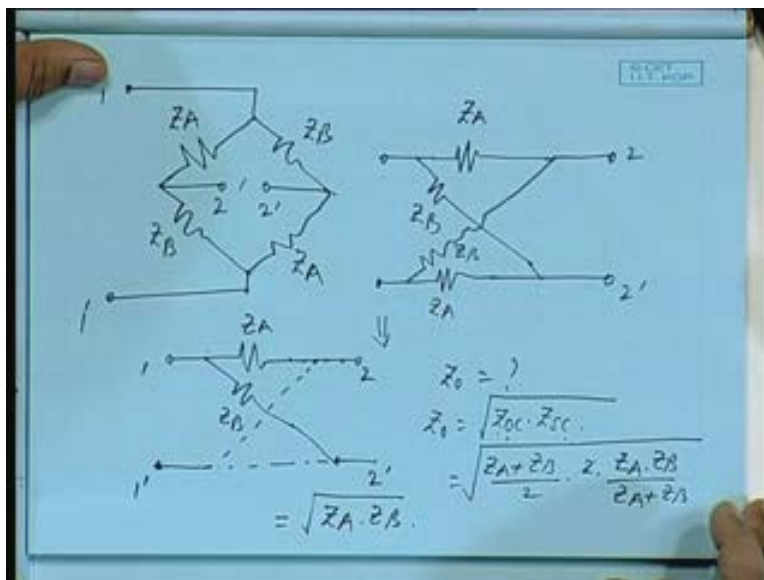
$$Z_A + Z_C = Z_{oc1}$$

$$Z_C = \sqrt{\frac{Z_{oc1} \cdot Z_{sc2}}{Z_{sc1}} - Z_{oc1} Z_{sc1}}$$

$$= \sqrt{Z_{oc1} Z_{sc1} \left( \frac{Z_{oc2}}{Z_{sc2}} - 1 \right)}$$

$$Z_A = Z_{oc1} - Z_C, \quad Z_B = Z' - Z_C$$

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Now suppose you are given a lattice network what is a lattice network, it is like this you are having an bridge network like this this is  $Z_A$ ,  $Z_B$ ,  $Z_B$ ,  $Z_A$  and you are having the output terminals here 2, 2 dashed, 1, 1 dashed okay. So this can be also shown as  $Z_A$ ,  $Z_B$ ,  $Z_A$  and  $Z_B$  these are the output terminals, this is  $Z_B$  quite often we show this as  $Z_A$ ,  $Z_B$  and rest of it is shown by dotted line that means this is  $Z_A$  and this is  $Z_B$ , it is repeated. So this is also a standard notation for a lattice network what will be  $Z$  not for this kind of a network now  $Z$  naught as you know is equal to  $Z$  open circuit into  $Z$  short circuit it is a symmetric network. So whether you look from this side or that side  $Z_{oc1}$  and  $Z_{sc1}$  they will be equal to  $Z_{oc2}$  and  $Z_{sc2}$ .

So how much is z open circuit if I keep it open it is  $Z_A$  plus  $Z_B$  and again  $Z_A$  plus  $Z_B$  in parallel so it will be  $Z_A$  plus  $Z_B$  by 2, is it not 2 ideal, 2 identical impedances in parallel. So that will be effective impedance will be half of that and what will be the short circuit impedance if I short it is  $Z_A$  and  $Z_B$  in parallel once again  $Z_A$  and  $Z_B$  in parallel, so it will be 2 times parallel combination of  $Z_A$ ,  $Z_B$  which will be  $Z_A$ ,  $Z_B$  by  $Z_A$  plus  $Z_B$ . So under root of this.

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$$\frac{I_s}{I_r}$$

$$I_r \cdot Z_A + I_r \cdot z_0 + Z_A (I_r + I_s - I_r) = V_s$$

$$I_r (z_0 + Z_A) + I_s Z_A = V_s = I_s z_0$$

$$\frac{I_s}{I_r} = \frac{Z_A + z_0}{z_0 - Z_A}$$

$$P = \log_e \left( \frac{I_s}{I_r} \right) = \log_e \left( \frac{Z_0 + Z_A}{Z_0 - Z_A} \right)$$

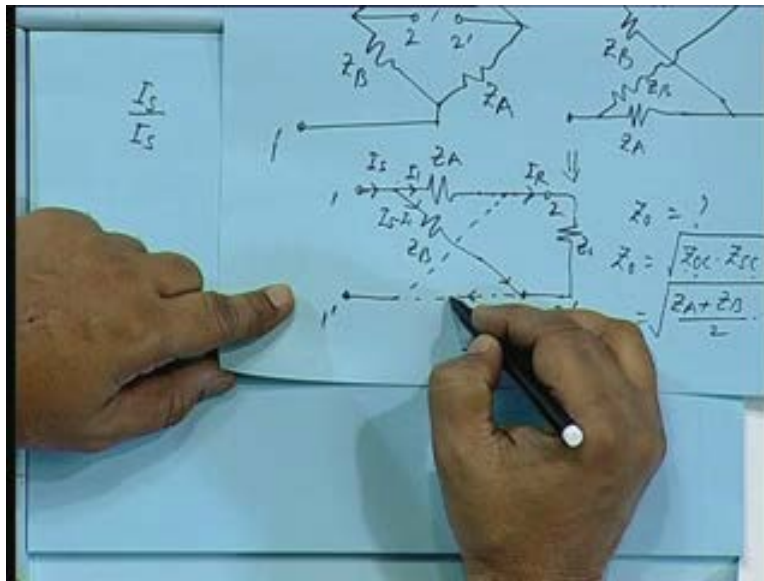
$$e^P = \frac{Z_0 + Z_A}{Z_0 - Z_A} \Rightarrow \frac{e^{P/2}}{e^{-P/2}} = \frac{Z_0}{Z_A} \Rightarrow \tanh\left(\frac{P}{2}\right) = \frac{Z_A}{Z_0}$$

So 2 will get cancel  $Z_A$  plus  $Z_B$  will get cancelled so this will be  $Z_A$  into  $Z_B$  is that okay. So this will be the characteristic impedance now if you look back what will be the propagation constant what will be the ratio of the currents  $I_s$  and  $I_r$ . Now once again I will take this diagram suppose this is  $I_s$  and when I put an impedance  $z$  not here this impedance the current through this impedance is  $I_r$  okay let this current be  $I_1$  then the current through this is  $I_s$  minus  $I_1$  is it not and this current is  $I_r$  this gets added with this, so this final current will be  $I_r$  plus  $I_s$  minus  $I_1$  okay. Now what will be the voltage drop if you look at the voltage here  $v_s$  is equal to this total drop. Now let us compute the drops it will be  $I_1$  into  $Z_A$ , so I can write  $I_1$  into  $z$   $I_1$  into  $Z_A$  plus  $I_r$  into  $Z$  naught  $I_r$  into  $Z$  naught plus  $Z_A$  into  $Z_A$  into this current plus this current  $I_r$  plus  $I_s$  minus  $I_1$  so  $I_r$  plus  $I_s$  minus  $I_1$  this entire drop will be equal to  $v_s$  okay.

So we can write  $I_r$  into  $Z$  naught plus  $Z_A$   $Z$  naught plus  $Z_A$  plus  $I_s$  into  $Z_A$  and  $I_1$   $Z_A$  will get cancelled with this  $I_1$ ,  $Z_A$  equal to  $v_s$  and what is  $v_s$ ?  $V_s$  is again  $I_s$  into  $Z$  naught the impedance in from this side, so  $I_s$  into  $Z$  naught do you all agree therefore if I transfer this to this side  $I_r$  into  $Z$  naught plus  $Z_A$  is equal to  $I_s$  into  $Z$  naught minus  $Z_A$ . So that gives me  $I_r$ ,  $I_s$  by  $I_r$  as equal to  $Z_A$  plus  $Z$  naught divided by  $Z$  naught minus  $Z_A$  okay.

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The image shows a whiteboard with two handwritten equations:

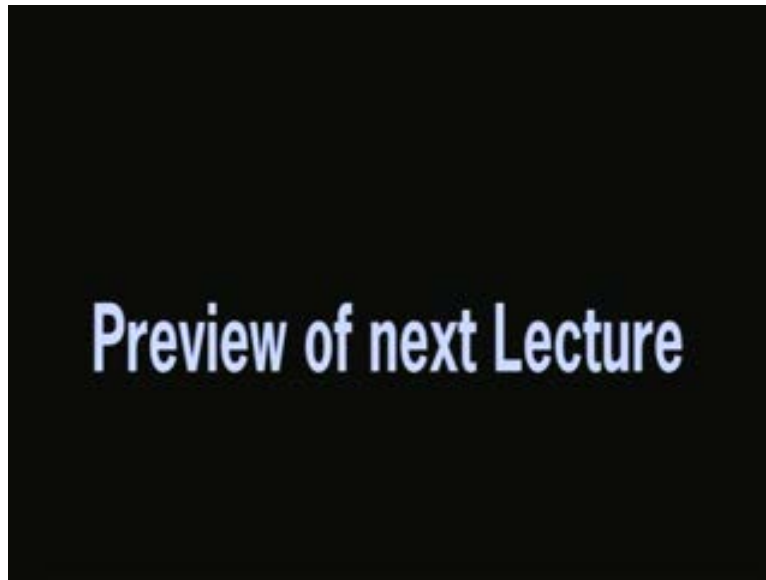
$$Z_A = Z_0 \tanh\left(\frac{pL}{2}\right)$$

$$Z_B = Z_0 \coth\left(\frac{pL}{2}\right)$$

So the propagation constant which was  $\log$  of  $I_s$  by  $I_r$  will be equal to  $\log$  of  $Z_{naught} + Z_A$  by  $Z_{naught} - Z_A$  which means  $e$  to the power  $p$  is equal to  $Z_{naught} + Z_A$  by  $Z_{naught} - Z_A$  and that is equal to that give me if I take component or dividend though  $e$  to the power  $p$  plus 1 by  $e$  to the power  $p$  minus 1 is equal to  $Z_{naught}$  by  $Z_A$  and what is this? This is nothing but  $\coth$  hyperbolic  $p$  by 2 if I invert it so that will be giving me  $\tanh$  hyperbolic  $p$  by 2 is equal to  $Z_A$  by  $Z_{naught}$  or  $\tanh$  hyperbolic  $p$  by 2 is  $Z_A$  by  $Z_{naught}$   $Z_{naught}$  is equal to root over of  $Z_A$  into  $Z_B$ . So that gives me root over of  $Z_A$  by  $Z_B$  okay so we get  $Z_A$  is equal to  $Z_{naught} \tanh$  hyperbolic  $p$  by 2 I may write like this and  $Z_B$  equal to  $\coth$  hyperbolic  $p$  by 2 either way you can write alright

okay. We will stop here for today; we will take up some problems in the next class, thank you very much.

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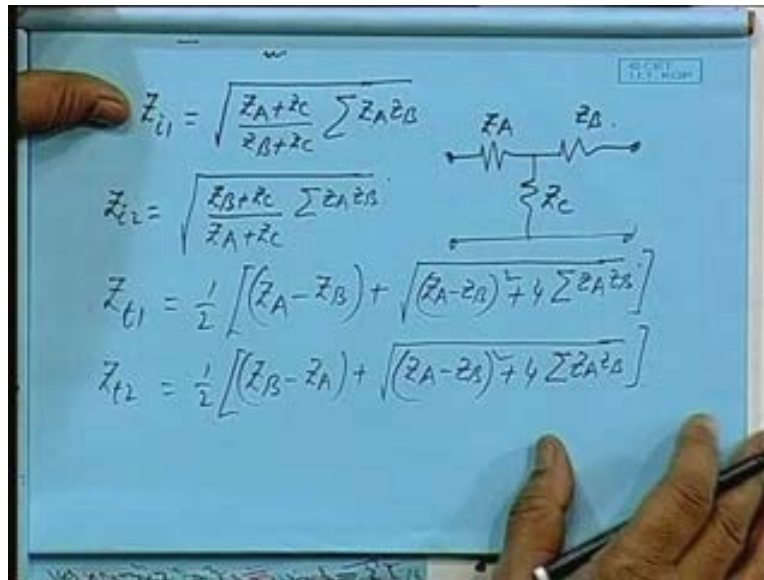


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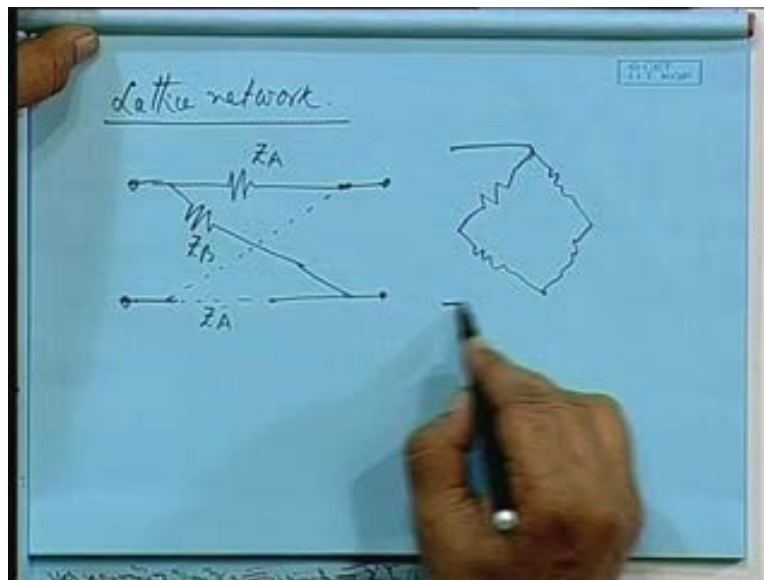


Last time we are discussing about the image impedances and iterative impedances and we derive these relations for a T network that was  $Z_A$  plus  $Z_C$  by  $Z_B$  plus  $Z_C$  into  $Z_A$ ,  $Z_B$  this means combination of 2 impedances at a time. So it was a structure like this  $Z_A$ ,  $Z_C$  and this was  $Z_B$  similarly,  $Z_{i2}$  we derived as  $Z_B$  plus  $Z_C$  divided by  $Z_A$  plus  $Z_C$  into  $Z_A$ ,  $Z_B$  this we derived last time.

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We also derived for the iterative impedances  $Z_{t1}$  as half of  $Z_A$  minus  $Z_B$  plus under root  $Z_A$  minus  $Z_B$  whole squared plus 4 times that sigma term  $Z_A, Z_B$ . Similarly,  $Z_{t2}$  was  $Z_{t2}$  was half  $Z_B$  minus  $Z_A$  plus  $Z_A$  minus  $Z_B$  whole squared the same expression okay. Then we discussed about a network that is a lattice network where we considered  $Z_A, Z_B, Z_A$  this was the notation for the counter parts, this is  $Z_B$  and this is  $Z_A$ , this can also be seen as a bridge network okay like this this was see.